

# Study of Thermal Instability in Nanofluids

**BABASAHEB  
BHIMRAO  
AMBEDKAR  
UNIVERSITY**



• LUCKNOW •  
प्रज्ञा शील करुणा  
ESTABLISHED 1996

ABSTRACT

OF

THESIS SUBMITTED FOR DEGREE OF  
**DOCTOR OF PHILOSOPHY**  
IN  
**APPLIED MATHEMATICS**

BY

**Vineet Kumar**

UNDER SUPERVISION OF

**Prof. B.S. Bhaduria**

DEPARTMENT OF APPLIED MATHEMATICS

SCHOOL FOR PHYSICAL SCINECES

BABASAHEB BHIMRAO AMBEDKAR UNIVERSITY

(A CENTRAL UNIVERSITY)

LUCKNOW - 226025

INDIA

Enrollment No. 056/13

Year 2018

## ABSTRACT

Owing to the suspension of nanosize particles of metal in the base fluid, the nanofluids are being used as heat energy carriers due to their enhanced abilities over ordinary fluids. The presence of a few quantity of nanoparticles in base fluids causes a significant enhancement in the heat transfer. This heat transfer characteristic of nanofluids depends on both the thermo-physical properties of the base fluid and the suspended nanoparticles. The nanofluids found a wide range of applications in industrial, commercial, residential, medical and transportation sectors.

An important virtue of nanofluids is the thermal conductivity enhancement, introduced first by Masuda et al. (1993). Choi (1995) was the first to propose the term “nanofluid”. The ballistic nature of heat transport within nanoparticles was studied by Chen (2001), Eastman et al. (2001) found that 40% thermal conductivity of ethylene-glycol increases with 0.3% volume of copper nanoparticles of size 10nm, and Das et al. (2003) found (10 – 30)% increase of the effective thermal conductivity in alumina-water nanofluids with (1 – 4)% of alumina. Based on the above study Buongiorno and Hu (2005) suggested the possibility of the use of nanofluids in advanced nuclear systems as a coolant. The nanofluid flow can be used in the delivery of nano-drug, due to Kleinstreuer et al. (2008).

The **first chapter** consists of basic terminology used in thesis, definitions relevant to physical phenomenon, hydrodynamic equations and laws, applied analytical and numerical methods, literature review and real world applications in precise way.

In the **second chapter** the effect in thermal instability of nanofluid saturated porous layer confined between two horizontal surface in presence of internal heat source and rotation about vertical axis with revised boundary condition is studied. The revised boundary conditions are in practical because the value of temperature at boundaries can be adjusted but not the concentration in case of nanoparticles, and so

at boundaries the nanoparticle flux is assumed to be constant. Further, linear stability analysis is performed subject to these boundary conditions. Darcy-Brinkman model is studied using Galarkin method to carried out linear stability analysis by depicting the behavior of Rayleigh number with respect to wave number taking different values of the other parameters. The effect of internal heat source is destabilising, the effect of rotation is stabilising the system whereas the instability of vertical throughflow depends on the direction of nanofluid flow.

The **third chapter** deals with the thermal instability of non-Newtonian power-law nanofluid saturated porous layer, confined between two horizontal surfaces in presence of throughflow. Extended Darcy's model together with the Oberbeck-Boussinesq approximation has been employed. Linear stability analysis is performed subject to various boundary conditions. The effect of parameters corresponding to the throughflow (Péclet number  $Pe$ ), and the power-law index of the nanofluid has been investigated. These parameters shift the position of the neutral stability curve and also the value of critical Rayleigh number.

In the **fourth chapter** thermal instability in a nanofluid saturated porous medium under the effects of gravity modulation and internal heating is investigated. The non-uniform vertical vibrations of the system, which can be realized by oscillating the system vertically is considered to vary sinusoidally with time. Linear and non-linear stability analyses have been performed to investigate onset of convection and heat/mass transfer in the system. Linear stability analysis is made using Venezian approach, however, for nonlinear stability analysis truncated Fourier series expansions is used. The effects of various physical parameters have been investigated on heat and mass transfer. Linear system shows that there is a particular range of frequency of modulation where the system is stabilizing and destabilizing. The dual effect of this nature is due to the presence of internal heating of the system. It is found that gravity modulation can be used effectively to regulate the stability of the system. Further, the effect of internal Rayleigh number is to destabilize the system

The **fifth chapter** deals with linear and non-linear stability analyses of the convective flow in a porous layer saturated by nanofluid, under rotational speed modulation rotating about a vertical axis. The revised boundary conditions are used as they are more realistic. Perturbation method has been adopted to execute the linear stability analysis, while a truncated Fourier series method has been used for weak non-linear analysis. The amendment in the critical Rayleigh number due to rotational speed modulation has been noticed using linear stability analysis. The concentration and thermal Nusselt numbers are obtained by performing a weak non-linear stability analysis, and their behavior is explored by solving the time dependent (Non-autonomous in nature) finite amplitude equations using fourth order Runge-Kutta method. The obtained results are depicted graphically, and discussed in details. It is observed that the modulated rotational speed have stabilizing effect on the system for different values of the modulation frequency.

In the **sixth chapter**, the variation of thermal instability in an electrically conducting nanofluid layer under magnetic field modulation is studied. Time periodic magnetic field (i.e., magnetic field modulation) is associated in vertical direction together with certain boundary conditions. The perturbation technique has been adopted to perform both linear stability analysis and weak-nonlinear analysis. The non autonomous Ginzburg-Landau equation is derived under the assumptions of Boussinesq approximation and small-scale convective motion. In all convection measuring quantities (Nusselt number, nanoparticle concentration Nusselt number, Rayleigh number), the effect caused by magnetic field modulation has been obtained. Finally, the results have been discussed in detail, and depicted graphically.

In the **last chapter**, the heat transfer in a horizontal nanofluid layer is investigated by means of weakly non-linear stability analysis. A set of new boundary conditions for the nanoparticle fraction, which is physically more realistic has been considered. The new boundary condition is based on the assumption that the nanoparticle fraction

adjusts itself so that the nanoparticle flux is zero on the boundaries. The governing equations for this problem are reduced to Ginzburg–Landau equation and solved by homotopy analysis method (HAM), and numerical method Mathematica NDSolve. The obtained results are valid for the whole solution domain with high accuracy. Nusselt number and Nanoparticle Nusselt number are calculated for different values of parameters. The results have been depicted graphically.

Each chapter contains brief introduction, mathematical model, method of solution, conclusions and references relevant to discipline. All the references are placed without chronological error, in a separate section **BIBLIOGRAPHY**, at the end of the thesis.

Portion of above works are accepted/communicated in different journals, the list is as follows:

### **Accepted**

1. Palle Kiran, B.S. Bhadauria, **Vineet Kumar**, “Thermal convection in a nanofluid saturated porous medium with internal heating and gravity modulation”.  
**Journal of nanofluid (American Scientific Publishers)**, Vol. 5, No. 3, pp. 328-339(12), (2016).
2. B.S. Bhadauria, **Vineet Kumar**, Brajesh K. Singh, “Weak nonlinear stability analysis of thermal convection in an electrically conducting nanofluid layer under magnetic field modulation”.  
**Recent Advances in Mathematical & Computational Science**, Book Chapter in BBAU conference proceedings. ISBN : 9789384337674, (2015).
3. B.S. Bhadauria, **Vineet Kumar**, Brajesh K. Singh, I. Hashim, “Study of convective thermal instability in nanofluid saturated porous media in the presence of vertical throughflow, internal heat source and rotation”.

**VIJNANA BHARATHI, Bangalore University Journal of Science**, Vol. 1, No. 2, pp. 120-140, ISSN : 0971-6882, (2016).

4. Alok Srivastava, **Vineet Kumar**, B.S. Bhadauria, I. Hashim, “Study of heat transfer in a nanofluid layer using homotopy analysis method”.

**International Journal of Science, Technology and Society**, Vol 3, No. 1, pp. 27-39, (2017).

5. Alok Srivastava, **Vineet Kumar**, B.S. Bhadauria, I. Hashim, “Nonlinear study of heat transfer in nanofluid saturated horizontal porous medium”.

**International Journal of Science, Technology and Society**, Vol 3, No. 2, pp. 41-53, (2017).

6. B.S. Bhadauria, **Vineet Kumar**, “Convective motion in nanofluid under variable rotational speed”.

**Advanced Science Engineering and Medicine(American Scientific Publishers)**, Vol. 10, 717-723, (2018).