CLASSICAL AND BAYESIAN INFERENTIAL PROCEDURES IN VARIOUS LIFE TESTING MODELS

ABSTRACT

of

THESIS

SUBMITTED TO

BABASAHEB BHIMRAO AMBEDKAR UNIVERSITY
(A CENTRAL UNIVERSITY)
LUCKNOW

BABASAHEB
BHIMRAO
AMBEDKAR
UNIVERSITY
LUCKNOW
प्रज्ञा शील करुणा
ESTABLISHED 1996

FOR THE AWARD OF DEGREE OF

Doctor of Philosophy

IN

APPLIED STATISTICS

Submitted by

Vaidehi Singh

Under the Supervision of

Dr. Surinder Kumar
Professor

DEPARTMENT OF APPLIED STATISTICS
SCHOOL FOR PHYSICAL SCIENCES
BABASAHEB BHIMRAO AMBEDKAR UNIVERSITY
(A CENTRAL UNIVERSITY)
VidyaVihar, Raebareli Road Lucknow-226025(U.P.), India

Enrolment No. - 440/14 **2020**

Abstract

The concept of **Sequential Analysis** which was introduced by Wald (1947)[17] is considered, the SPRT as well as its robustness is studied for various life-testing models. Epstein and Sobel (1955)[8], Johnson (1966)[9], Phatarford (1971)[13] and Joshi and Shah (1990)[10] dealt with the problem of testing simple as well as composite hypothesis for shape and scale parameters. In **Reliability theory**, the reliability $R(t)$ and stress-strength reliability R are obtained for various models[see Church and Harris (1970)[5] and Chaturvedi and Sharma (2010)[3]]. Transformation method is an important method of obtaining the MLE'S, UMVUE'S and confidence interval for complex models, it provides simpler way to deal with complex distributions, one may refer to Sathe and Shah (1981)[14] and Chaturvedi and Surinder (1999)[2]. In **Bayesian Inference**, the Bayes estimators are obtained for different loss functions under various informative and non-informative priors. Bayes estimators for pdf, cdf, $R(t)$ and R are also obtained. For a brief review one may refer to Zellner (1986) [18], Varian (1975)[16] and Chaturvedi and Pathak (2013)[4] etc.

In this study, I have worked on sequential testing analysis, reliability theory estimation and Bayesian estimation for various life testing models. The whole work is summarised chapterwise in the following chapters.

In **Chapter 1**, an introductory review on the theory of Sequential analysis, Reliability theory and Bayesian Inference is discussed. Some related significant terminologies are explained with the help of basic definitions and notations relevant to the thesis.

In **Chapter 2**, sequential testing procedures are developed for testing the hypotheses regarding the parameters of the New Weibull-Pareto Distribution (NWPD) proposed by Nasiru and Lugnterah (2015)[12]. Theoretical expression for the Operating Characteristics (OC) and Average Sample Number (ASN) functions are derived for the scale parameters of the distribution. The robustness of the SPRT'S in respect of OC and ASN functions is studied, when the distribution under study has undergone a change. The results are presented through Tables and Graphs, so that one can see the numerical evaluated departures in OC and ASN functions.

In **Chapter 3**, the sequential testing procedures are developed for testing the hypotheses regarding the shape and rate parameters of the Positive Exponential Family of Distribution(PEFD) proposed by Liang (2008)[11]. The robustness of the SPRT'S in respect of OC and ASN functions are studied, when the distribution under study has undergone a change. The acceptance and rejection regions for H_0 against H_1 are derived in case of rate parameter. The expressions of OC and ASN functions for the robustness of the SPRT in case of rate parameter, when the coefficient of variation is known are also derived and studied. Finally, the results are presented through Tables and Graphs, so that one can see the numerical evaluated departures in OC and ASN functions.

In **Chapter 4**, the problem of Sequential Probability Ratio Test (SPRT) is considered for Generalised Inverse Weibull Distribution (GIWD) proposed by Drapella (1993)[6]. The GIWD has hazard function which has a unimodal shape. Hence, the GIWD could be an appropriate model for fitting the data which has the convex and then the concave shape. Robustness of the SPRT is studied for the parameters involved in this model, under the conditions when these parameters have undergone a change.

In **Chapter 5**, the probability of disaster is studied when the strength of the items follows power function distribution and the stress of the manufactured items/devices follows OGE-G distribution[see El-Damcese et. al (2015)[7]]. In order to study the probability of disaster, a relationship between the parameters of OGE-G and power function distribution is established through the reliability measure $P = Pr(Y > X)$. Finally, through the relationship among the parameters involved in the model is used to get the optimum cost function when the cost function is linear in terms of parameters.

In **Chapter 6**, the estimation of $R(t)$, $R = P(Y > X)$ and $\alpha = P(X > Y)$ for the Positive Exponential Family of Distribution (PEFD) is considered. The UMVUE'S, MLE'S and Confidence Interval are derived. These estimators are derived through the method of Transformation. The $\alpha = P(X > \gamma)$, which is termed as probability of disaster is also derived when random stress X follows PEFD and finite strength follows Power function distribution.

In **Chapter 7**, the Bayes estimators for the scale parameter of a family of lifetime distributions proposed by Chaturvedi and Rani (1997)[1] are considered under the assumptions of non-informative and conjugate priors. The uniform and inverted gamma priors are observed to obtain the posterior distribution for the scale parameter of this family of lifetime distributions under different loss functions. Finally, the performance is compared by the values obtained through MCMC simulation techniques.

In **Chapter 8**, two parameter Generalized Rayleigh Distribution proposed by Surles and Padgett (2001)[15] is considered. In this chapter, firstly, the Bayes estimators for the positive and negative powers of the parameters are obtained. Then, through using these Bayes estimators, the estimates of the reliability function and stress-strength reliability under Type II censoring for two parameter Generalized Rayleigh Distribution are obtained. Estimation procedure is done for different priors under different loss functions.

Bibliography

[1] Chaturvedi, A. and Rani, U. (1998): Classical and Bayesian reliability estimation of the generalized Maxwell failure distribution. *Jour. Statist. Res.*, 32, 113-120.

[2] Chaturvedi, A. and Surinder, K. (1999): Further remarks on estimating the reliability function of exponential distribution under Type I and Type II censorings. *Brazilian Jour. Prob. Statist.*, 13, 29-39.

[3] Chaturvedi, A. and Sharma, V. (2010): A note on the estimation of $P(Y > X)$ in two parameter exponential distribution. *Statistica*, 44(1), 73-75.

[4] Chaturvedi, A. and Pathak, A. (2013): Bayesian estimation procedures for three parameter exponentiated Weibull distribution under entropy loss function and type II censoring. *InterStat*. interstat.statjournals.net/YEAR/2013/abstracts/1306001.php.

[5] Church, J. D. and Harris, B. (1970): The estimation of reliability from stress-strength relationships. *Technometrics*, 12, 49-54.

[6] Drapella, A. (1993): Complementary Weibull distribution: unknown or just forgotten. *Qual. Reliab. Eng. Int.*, 9, 383-385.

[7] El-Damcese, M. A., Mustafa, A., El-Desouky, B. S. and Mustafa, M. E. (2015): The Odd Generalized Exponential Gompertz Distribution. *Applied Mathematics*, 6, 2340-2353.

[8] Epstein, B. and Sobel, M. (1955): Sequential life tests in exponential case. *Ann. Math. Statist.*, 26, 82-93.

[9] Johnson, N. L. (1966): Cumulative sum control chart and the Weibull Distribution. *Technometrics*, 8, 481-491.

[10] Joshi, S. and Shah, M. (1990): Sequential Analysis applied to testing the mean of an inverse Gaussian distribution with known coefficient of variation. *Commun. Statist.-Theor. Meth.*, 19(4), 1457-1466.

[11] Liang, T. (2008): Empirical Bayes estimation of reliability in a positive exponential family. *Commun. Statist.-Theor. Meth.*, 37(13), 2052-2065.

[12] Nasiru, S. and Lugnterah, A. (2015): The New Weibull Pareto Distribution. *Pakistan Journal of Statistics and Operation Research*, 11(1), 101-112.

[13] Phatarfod, R. M. (1971): A sequential test for Gamma distribution. *Jour. Amer. Statist. Assoc.*, 66, 876-878.

[14] Sathe, Y. S. and Shah. S. P. (1981): On estimating $P(X < Y)$ for the exponential distribution. *Commun. Statist. -Theor. Meth.*, A10, 39-47.

[15] Surles, J. G. and Padgett, W. J. (2001): Inference for reliability and stress-strength for a scaled Burr Type X distribution. *Lifetime Data Analysis*, 7, 187-200.

[16] Varian, H. R. (1975): A Bayesian Approach to Real Estate Assessment. Studies in Bayesian Econometrics and Statistics, S. E. Fienberg and A. Zellner (eds.), *North-Holland Amsterdam*, 195-208.

[17] Wald, A. (1947): Sequential Analysis. *John Wiley and Sons*: New York.

[18] Zellner, A. (1986): Bayesian Methods and Prediction using Asymmetric Loss Function. *Jour. Amer. Statist. Assoc.*, 81, 446-451.