SOME PROBLEMS ON INTERFACIAL INSTABILITIES

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ABSTRACT

Hydrodynamic stability concerns the stability and instability of the motions of fluids. A stable flow is observable because an unstable flow is only a state of transition to another flow, or possibly to turbulence and therefore, hydrodynamic stability is an important domain to study in fluid mechanics. The instability of fluid flows and their transition from laminar flow to turbulent flow has numerous applications in engineering and the natural environment. The instability phenomenon also occurs in the area of astrophysics, applied mathematics, geophysics, biology, physics, oceanography, etc. A fluid in motion or at rest may become unstable corresponding to some distortion and its flow characteristics may be different from the original one. A system is said to be stable with respect to a perturbation if the perturbation applied to a system decays and the system takes its original position. If the perturbation grows and the flow system changes its characteristics, the system is said to be unstable with respect to the given disturbance. A fluid flow system is to be considered stable if it is stable for all possible disturbances applied to the system; meanwhile, a fluid flow system is considered to be unstable if there exists at least one mode of disturbance for which the system is unstable.

The hydrodynamic stability theory examines how a laminar flow reacts to a disturbance with modest or moderate amplitude. The flow is considered stable if it returns to its initial laminar state, but unstable if the disturbance intensifies and changes the laminar flow into a new state. In that circumstance, the flow is deemed unstable. Although instabilities frequently cause turbulent fluid motion, they can also change the flow into a different laminar state, which is typically more complex. The

mathematical examination of the evaluation of disturbances superposed on a laminar base flow is the subject of stability theory.

In real-world applications, a perturbation is applied to the field variables before they are re-inserted into the governing equations to assess the stability of a particular flow field concerning minor changes in the system's physical parameters. In the case of linear stability analysis, the perturbations are chosen so that nonlinear term combinations are disregarded from the governing equations, and a subsequent linearization technique reduces the field equations to a mathematical issue.

INTERFACIAL INSTABILITY

Instabilities occur at liquid-liquid, liquid-gas, or gas-gas interfaces. Problems related to instability can be classified into two types; temporal and spatial. There are many industrial uses for interfacial instability, including designing cars, predicting the weather, studying blood flow, and designing packed bed reactors. The various types of interfacial instabilities occur in nature.

Rayleigh-Taylor Instability

The Rayleigh-Taylor instability (RTI) is the instability of an interface involving two fluids having distinct densities, which arise when the heavy fluid lies on the top of the lighter fluid. The Rayleigh-Taylor instability plays an important role in many natural processes ranging from coastal upwelling, which helps to renew the nutrients near the surface of the sea.

Kelvin-Helmholtz Instability

Instabilities of the plane interface between two superimposed fluid layers with a relative horizontal velocity and different physical parameters are known as Kelvin-

Helmholtz instability. Several circumstances cause Kelvin-Helmholtz instability, including wind blowing over oceans, cloud formations on Earth, and oil exploration.

Capillary instability

Capillary instability develops when surface tension exerts a force against a liquid cylinder in an infinite fluid. Film boiling, Liquid dispensers, and inkjet printers depend on the capillary instabilities of the two fluids' interface. Boiling and condensation operations, as well as several chemical and metallurgical processes, are examples of gas-liquid interactions in industrial applications.

Potential flows of incompressible fluids are solutions of the Navier-Stokes equations which satisfy Laplace's equations. The Helmholtz decomposition says that every solution of the Navier-Stokes equations can be decomposed into a rotational part and an irrotational part satisfying Laplace's equation. The theory of purely irrotational flows of a viscous fluid is an approximate theory that works well especially in gas-liquid flows of liquids of high viscosity, at low Reynolds numbers. The theory of purely irrotational flows of a viscous fluid can be seen as a very successful competitor to the theory of purely irrotational flows of an inviscid fluid. A free surface in viscous potential flow analysis takes into account viscous stresses through normal stress balance at the surface, but tangential stresses are not considered. The present research work is the study on **"SOME PROBLEMS ON INTERFACIAL INSTABILITIES"** through viscous potential flow theory, normal mode technique, and computer-based MATLAB programming. The stability of the interface of viscous-viscous incompressible and viscous-nanofluid fluids has been investigated theoretically in the cylindrical and spherical geometries. The Newton-Raphson approach is used to solve the dispersion relation for the critical wave number and the various graphs are plotted to depict the behavior of flow variables on the perturbations growth rate and wave number.

Chapter 1 presents a brief introduction to the general stability theory, Rayleigh-Taylor instability, capillary instability, and Kelvin-Helmholtz instability along with some definitions and basic equations related to stability analysis. Various related studies are described by various authors in this field and a summary of the thesis is provided.

The irrotational flow theory is utilized to investigate the stability of the liquid-vapor interface between two concentric rotating cylinders in **Chapter 2**. The annular region bounded by the outer and inner cylinders is filled with incompressible and viscous fluids. The heat/mass transportation is considered through the interface. Both the outer and inner cylinders are rotating with distinct angular velocities. To calculate the growth of disturbances, the normal mode technique is utilized. A second-order differential equation is achieved to get the maximum growth of disturbances. It is found that the asymmetric disturbances have a destabilizing character at the interface but the rotation of the outer cylinder has a stabilizing effect. It is observed that the arrangement gets destabilized on rotating of the inner cylinder but the rotation of the outer cylinder induces stability, and the most stable case is found when the inner cylinder is stationary and the outer cylinder is rotating

The linear instability analysis of the vapor–liquid interface is carried out in the region enclosed by two cylinders in **Chapter 3.** The annular region is taken as porous and both the cylinders are rotating with different angular velocities. The vapor is taken in the inner part, the liquid lies in the outer region and the transfer of heat and mass is allowed at the interface. It is also assumed that both the fluids are viscous and incompressible. Here, the normal mode technique is used to detect the growth of perturbations, and the potential flow theory of viscous fluids is employed. The quadratic relationship is achieved by using the Darcy-Brinkman model. It is noticed that when the inner cylinder is fixed and the outer cylinder is rotating, the system moves towards stability, but when the outer cylinder is fixed and the inner cylinder is rotating, the system gets destabilized. The vapor–liquid interface is more stable in a porous medium.

An unstable cylindrical jet interface is analyzed using viscous potential flow in **Chapter 4**. The jet interface experiences Rayleigh-Taylor instability when it moves radially. A radially moving cylindrical jet has been considered unstable without considering the viscosity effect in previous studies. The fluids taken inside and outside the cylinder are incompressible and viscous while the internal fluid moves radially. A second-order ordinary differential equation is achieved to establish the instability/stability criterion. The radial velocity and acceleration both have a significant impact on the stability of the jet. It is found that the viscous effects enhance the disturbances rapidly. The acquired stability criterion is applied to the cylindrical jets in HYLIFE-II which is basically an Inertial Confinement Fusion reactor.

In **Chapter 5**, the stability of a spherical bubble surrounded by a viscous-incompressible fluid is examined through the normal mode procedure and viscous potential flow analysis. A dispersion relationship between the growth rate parameter and wave number is achieved. The viscous, incompressible fluids are confined in the spherical shell and the interface distortion is expressed in terms of spherical harmonic functions. The highest growth of perturbations and corresponding mode is computed. The Plane configuration results are recovered when the mode of perturbations is very

large. Viscosity is found to stabilize the interface while the Weber number and Reynolds number have destabilizing nature.

The linear instability analysis of a spherical interface between a viscous fluid and Al_2O_3 – water nanofluid is performed in **Chapter 6**. The viscous fluid lies inside the sphere while the outside region contains nanofluid. In this model, the viscosity of the nanofluid is considered as a function of the base fluid viscosity, nanoparticles volume fraction, fractal aggregates, and nanoparticles shape. It is assumed that the perturbed flow is irrotational, and the linear perturbation equations are solved using viscous irrotational theory. In order to compute the perturbation growth rate, a 2-degree polynomial is derived. The nanofluid interface is found more stable than the viscous fluid interface. The density of nanofluid raises the amplitude of disturbances while the nano fluid's viscosity has a reverse effect.

The impact of porous media is investigated on the Kelvin-Helmholtz (K-H) instability under the cylindrical configuration in **Chapter 7.** The liquid-vapor interface is examined in the annular porous region enclosed by rigid cylinders and these cylinders are rotating with different angular velocities. The interface is transporting heat and mass from the liquid to the vapor phase and vice versa. The system is considered in such a way that the liquid lies above its vapor. The mathematical equations which govern the flow are worked out through viscous potential flow theory. The well-established normal mode procedure is used to calculate the critical value of relative velocity. Various plots indicating the effect of Rossby number, permeability, porosity, heat transfer, etc. are included in this paper. The presence of rotation delays the instability while heat transfer makes the interface more unstable. The interface is more unstable in a viscous medium than in a porous medium.

The **last Chapter** of the thesis is related to the conclusions and future scope of the work. In the present work, the stability results are achieved utilizing the linear theory of stability analysis. The nonlinear analysis of interfacial stability problems is very important because the governing equations describing these flows are nonlinear in nature. The same problems can be studied through the nonlinear analysis of stability theory.