Convection in Fluid and Porous Medium under Modulation

THESIS

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PREFACE

The thesis entitled **"Convection in Fluid and Porous Medium under Modulation"** comprising of analytical/numerical solutions of some problems related with the topic, is an outcome of the research work carried out by me during the course of investigations under the supervision of Prof. B.S. Bhadauria, Professor, Department of Applied Mathematics, School for Physical Sciences, Babasaheb Bhimrao Ambedkar University, Lucknow.

Rayleigh-Bénard convection is an example of convective thermal instability, introduced by Chandrasekhar(1961) in ordinary fluid layers worldwide since last one century. The porous media corresponding to this problem is known as Horton-Rogers-Lapwood convection and it has many applications in various fields of engineering, thermal sciences and geophysics. Regulating the convective phenomenon in thermal sciences is of considerable importance due to its numerous application, such as in engineering problems, in industries, etc. Therefore, the relevant studies on regulation of heat and mass transfer in the following chapters is performed. For more detailed analysis on thermal instability, refer some excellent books: Ingham and Pop(2005), Nield and Bejan(2012) and vafai(2000).

The **first problem** deals with the thermorheological effect of temperature dependent viscous fluid in the presence of imposed time periodic gravity modulation for double diffusive convection. The temperature dependent viscosity fluid gives rise to variation in top and bottom structures and referred as a non-Boussinesq effect. A weakly non-linear analysis of thermal instability under gravity modulation, with temperature dependent viscosity, using the power series expansion in terms of the amplitude of gravity modulation, which is considered to be small, is carried out for double-diffusive convection in porous media. Nusselt number and Sherwood number are calculated numerically through the non-autonomous equation involving amplitude of convection using Ginzburg-Landau equation. By exploring the non-linear effect of solute Rayleigh number (Ra_s), Lewis number (L_e), Vadász number (Va), thermorheological parameter and amplitude of gravity modulation analytically. The curves for heat and mass transfer with respect to slow time variation are depicted graphically. Further, streamlines, isotherms and isohalines for different values of time are also drawn.

In the **second problem**, the combined effect of internal heating and time periodic gravity modulation on oscillatory convection in a viscoelastic fluid layer, using complex non-autonomous Ginzburg-Landau equation, is studied. A weakly non-linear stability analysis has been performed by using power series expansion in terms of the amplitude of gravity modulation. The Nusselt number has been obtained in terms of the amplitude for oscillatory mode of convection. The influence of relaxation (λ_1) and retardation (λ_2) time of viscoelastic fluid, on heat transfer, have been discussed. It is found that modulation has a destabilizing effect at low frequencies and stabilizing effect at high frequencies. Further, it is also found that overstability advances the onset of convection more with internal heating, hence increases heat transfer. The subcritical Hopf bifurcation and Pitchfork bifurcation are also studied.

The **third study** deals with the effect of various parameters on chaotic convection in an anisotropic porous medium, under gravity modulation, is investigated. For this, the problem is reduced into Lorenz system (nonautonomous) by employing truncated Galerkin expansion method. It is found that the system shows either chaotic or periodic nature for suitable values of scaled Rayleigh number (R). The influence of amplitude of modulation (δ) is to advance the chaotic nature in the system while that of frequency of modulation (Ω) has tendency to delay the chaotic behaviour. The effects of Péclet (P_e) number and anisotropic parameters on the chaotic system are also studied and found that they delay the chaotic nature. The phase portrait and time domain diagrams of the Lorenz system for suitable parametric values have been used to analyse the system.

The **fourth study** deals with the investigation on thermal instability in a couple-stress fluid saturated rotating porous medium under temperature modulation for oscillatory as well as chaotic convection by adopting complex Ginzburg-Landau equation and Lorenz system. Couple-stress fluid is a kind of non-Newtonian fluid having polar effects. To study oscillatory mode, a weakly non-linear stability analysis has been carried out in terms of the amplitude of temperature modulation(assumed to be small quantity), using power series expansion. Here, three cases of temperature modulation are considered; In-phase modulation, Out-phase modulation and Lower-boundary modulation for oscillatory convection. The Nusselt number has been obtained in terms of the small amplitude for oscillatory mode of convection to govern the heat transport in the system. On the other hand, for chaotic convection, the governing equations are reduced into a (non-autonomous) Lorenz system by using truncated Galerkin expansion method. The effects of Vadász number, Couple-stress parameter (C_1) and amplitude of modulation are found to have destabilizing effect, whereas the frequency of modulation and Taylor number (Ta) show a stabilizing effect on oscillatory convection. Further, it is found that the effect of scaled Rayleigh number R and amplitude of modulation δ are to advance the chaotic convection while scaled Taylor number (T_A) is to delay the chaotic convection.

In the **fifth problem**, the effect of temperature modulation on chaotic convection in a viscoelastic fluid saturated porous medium has been investigated. For this, the problem is reduced into Lorenz system by employing truncated Galerkin expansion method. The effect of scaled Rayleigh number R on the chaotic system is studied and found that it has periodic and chaotic both nature in a defined interval. Amplitude of modulation is to advance the chaotic nature in the system. The effects of scaled relaxation (Γ) parameter and retardation parameter (Λ') on the system are also studied. The phase portrait and time domain diagrams of the Lorenz system for suitable parametric values have been used to analyse the system.

In the **last problem**, a nonlinear convection in nanofluid saturated porous medium is studied in the presence of effective throughflow and internal heating under gravity modulation. Nanofluid is mixture of a base fluid and nanoparticles. A weakly nonlinear stability analysis has been carried out to obtain the Nusselt number, which is found to be the function of various parameters related to heat and mass transport across the porous medium. The time periodic vertical vibration is used to regulate the thermal instability in the system. The parameters arise due to nanofluid like Le, Na, Rn are destabilizing the system. The effects of remaining parameters are also studied thoroughly.