

# **NONLINEAR THERMAL INSTABILITY UNDER VARIOUS PHYSICAL CONFIGURATIONS**

**THESIS**

**SUBMITTED TO  
BABASAHEB BHIMRAO AMBEDKAR UNIVERSITY  
(A CENTRAL UNIVERSITY)  
LUCKNOW**

**BABASAHEB  
BHIMRAO  
AMBEDKAR  
UNIVERSITY**



• LUCKNOW •  
प्रज्ञा शील करुणा  
ESTABLISHED 1996

**FOR THE DEGREE OF  
Doctor of Philosophy  
IN  
APPLIED MATHEMATICS**

Submitted by

**PALLE KIRAN**

M.sc., M.Phil

ENROLMENT NUMBER 586/12

**SUPERVISOR**

**Prof. B.S. BHADAURIA**

Faculty of Sciences  
Department of Mathematics  
Banaras Hindu University  
Varanasi, 221005, India.

**CO-SUPERVISOR**

**Prof. VIPIN SAXENA**

Co-ordinator  
Department of Applied Mathematics  
Babasaheb Bhimrao Ambedkar University  
Lucknow, 226025, India.

**2014**

# Summery

The thesis entitled “**Nonlinear Thermal Instability under Various Physical Configurations**” comprising of analytical/numerical solutions of some problems related with The topic, is an outcome of the research work carried out by me during the course of Investigations under the guidance of Dr. B.S. Bhadauria, Professor, Department of Applied Mathematics, School for Physical Sciences, Babasaheb Bhimrao Ambedkar University, Lucknow. Rayleigh-Bénard convection is a paradigmatic example of convective thermal instability in Ordinary fluid layers. The porous media analogue of this problem is known as Horton-Rogers-Lapwood convection, and it is of paramount interest due to its applications in various Fields of engineering, thermal sciences and geophysics. Regulating the convective Phenomenon in thermal sciences is of considerable importance due to its numerous Applications in many engineering problems. Keeping mind the regulations of heat and mass transfer we have presented our results in the following chapters in which some of the work has been published.

The **chapter 1** is of introductory part. The key features of the discipline are stated in this Chapter. We describe the governing equation of dynamical systems. This chapter also consists Of the basic definitions, relevant to the thesis topic. The literature survey of thermal convection in different hydrodynamic configurations and different kinds of modulations has been explained.

In **chapter 2**, we have presented thermal instability in anisotropic horizontal porous medium saturated with

temperature dependant viscous fluid with time periodic temperature modulation. A weak non-linear stability analysis has been performed for the stationary mode of convection, and heat transport in terms of the Nusselt number is calculated. The effects of thermo rheological parameter, amplitude and frequency of modulation, thermo-mechanical anisotropies and Vadasz number on heat transport have been analyzed and depicted graphically. It is also found that, the heat transport can be controlled effectively by a mechanism that is external to the system.

In **chapter 3**, using complex non-autonomous Ginzburg-Landau equation, we have investigated nonlinear oscillatory convection in fluid layer (section 3.1) and porous medium (section 3.2) under Gravity modulation, considering viscoelastic fluids in the layer. The influence of (stress) relaxation and (strain) retardation times of viscoelastic fluid on heat transfer has been discussed. The study establishes that the heat transport can be controlled effectively by a mechanism that is external to the system. Modulation has a destabilizing effect at low frequencies and a stabilizing effect at high frequencies, which increases with increasing the amplitude of modulation. We also found that overstability advances the onset of convection, hence increases heat transfer.

In **chapter 4**, a nonlinear oscillatory convection in viscoelastic fluid saturated porous medium (section 4.1) and double diffusive convection in viscoelastic fluid layer (section 4.2) under temperature modulation has been investigated. The time periodic temperature profile on the boundaries has been considered and its effect on the system has been investigated. The effect of relaxation and retardation times of viscoelastic fluid on heat transfer and mass transfer has been discussed. The average value

of Nusselt number is obtained numerically while using the value of Nusselt number and found the good approximation (or combination) of frequency and phase angle where heat and mass transfer is enhances or diminishes.

In **chapter 5**, the influence of sinusoidally varying magnetic field and rotational speed effects on Rayleigh-Bénard convection is carried out. In section 5.1, we have developed an analytic study of heat transport in an electrically conducting fluid layer under nonuniform time dependent magnetic field. The applied vertical magnetic field consists of two parts; constant part, and a time dependent periodic part, which varies sinusoidally with time. Using weakly nonlinear theory, the Ginzburg-Landau equation is solved through NDSolve Mathematica 8, and the results are verified using Runge-Kutta Fehlberg method. The Nusselt number is obtained in terms of various system parameters and the effect of each parameter on heat transport is reported in detail. The effect of magnetic Prandtl number  $Pm$  amplitude of modulation  $\delta$  is to enhance the heat transfer. The Chandrasekhar number  $Q$ , modulation frequency  $\Omega$  is to stabilize the system. Further, it is found that magnetic modulation can effectively be used in either enhancing the heat transfer or diminishing it. In section 5.2, a theoretical investigation has been carried out to study the combined effect of rotation speed modulation and internal heating on thermal instability in a temperature dependent viscous horizontal fluid layer. Using Ginzburg-Landau analysis it is found that, the modulated rotation speed has a stabilizing effect for different values of modulation frequency. Further, internal heating and thermo-rheological parameter is found to destabilize the system. In **chapter 6**, in the light of earlier work proposed by Johnathan et al. (2014) motivated us to make a chaotic mode of convection

under temperature modulation. The analysis of buoyancy driven convection for moderate Prandtl number in a fluid saturated porous layer heated from below and subject to thermal modulation is presented. It's been investigated a better combination of values of  $\Omega$ ,  $\delta$  and scaled Rayleigh number  $R$  provides a way to chaos. It is also found that temperature modulation of the boundaries is to enhance the behaviour of the chaotic motions. In **chapter 7**, thermal instability has been investigated in non-Newtonian fluids. In section

7.1, we study nonlinear convection in a porous medium saturated with nanofluid under gravity modulation, and calculate heat and mass transport across the porous medium. The nonuniform vertical vibrations of the system, which can be realized by oscillating the system vertically, are considered to vary sinusoidally with time. A nonlinear stability analysis has been performed to obtain the Nusselt number, which is found to be the function of thermal Rayleigh number, concentration Rayleigh number, Lewis number, modified diffusivity ratio, amplitude and frequency of modulation. The effects of various physical parameters have been investigated on heat and mass transfer. It is found that gravity modulation can be used effectively to regulate the stability of the system. In section 7.2, the effect of vertical throughflow on oscillatory convection in a viscoelastic fluid saturated porous medium has been investigated. The heat transport is investigated in terms of both the Nusselt and average Nusselt numbers, governed by the non-autonomous complex Ginzburg-Landau equation using weak nonlinear stability analysis. The effect of vertical throughflow is found to stabilize the system irrespective of the direction of throughflow. The viscoelastic parameters have both destabilizing effect and stabilizing effect on the system. Further,

it is also found that heat transfer is more in the oscillatory mode of convection rather than stationary. The thesis contains a summary in English, and every chapter has its individual list of References. Altogether, the thesis contains 250 pages.