A STUDY OF VARIOUS LIFE TESTING MODELS AND THEIR INFERENTIAL PROCEDURES

ABSTRACT

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Abstract

Reliability is the probability of the systems/devices to perform its stated purpose adequately without any failure. Reliability can be measure by two method, one as a function of 't' i.e. $P_r(X > t)$ [see Church and Harris (1970) [12] and Basu (1964) [3]] and another is $P = P_r(Y > X)$ [see Awad et al.(1981) [2], Chao (1982) [5] and Kumar et al. (2019) [15]]. The concept of probability of disaster is focused by Alam and Roohi (2003) [1], where the stress increases over the strength. The problem of estimating the MLE'S, UMVUE'S and confidence interval for the reliability function [see McCool (1991) [18] and Chaturvedi and Surinder (1999)[9]] are further revisited through the methof of transformation. Wald (1947) [21] introduced the concept of sequential analysis for testing the simple null hypothesis H_0 vs simple alternative H_1. The SPRT'S and robustness of SPRT'S is studied by various author. For a brief review one may refer to Oakland (1950) [20], Chaturvedi et al. (2000) [11] Harter and Moore (1976) [13], Chaturvedi et al. (1998) [10], and Kumar et al. (2018) [14]. We have also developed the SPRT and studied its robustness under different situations. In Bayesian inference, Bayes estimator and posterior risk is obtained for the comparison on the basis of different priors (i.e. informative and non-informative) as well as for different loss function. Bayes estimator for pdf, cdf, $R(t)$ and R are also obtained by several authors for reference see, Bhttacharya (1967) [4], Chatruvedi and Singh (2006) [7] and Chatruvedi and Pathak (2013) [6].

Chapter 1 of the thesis provides an introductory review of the research topics covered and also states of significance as well as their applications.

In the **Chapter 2**, we assume the case when stress X follows the Exponentiated Weibull distribution and the strength Y follows the Power function distribution. Through establishing the relationship among the parameters and on assigning the different possible values for the parameters under study, the probability of disaster is studied and their behaviour is also studied by obtaining the numerical values.

In **Chapter 3**, we considered the pdf proposed by Chaturvedi and Singh (2008) [8]. The problem of estimating $R(t) = P_r(X > t)$, which is defined as the probability that a system survives until time 't' and $R = P_r(Y > X)$, which reprsents the stress-strength model are revisited. In order to obatain the maximum likelihood estimators (MLE'S), uniformly minimum variance unbiased estimators (UMVUS'S), interval estimators and the Bayes estimators for the considered model, the technique of Transformation Method is used.

In the **Chapter 4**, the sequential testing procedures (SPRT) and robustness of SPRT in respect of OC and ASN functions when the distribution under study has undergone a change are derived for the parameters (shape and rate) of Erlang distribution under the two simple hypotheses. The acceptance and rejection regions for simple hypotheses vs simple alternative are derived for rate parameter of the distribution. The mathematical expressions for the robustness of the SPRT of OC and ASN functions for the rate parameter of distribution, when the coefficient of variation is known are also studied. These results are presented through the Tables and Graphs, so that one may see the numerical evaluated departures in respect of OC and ASN functions.

In the **Chapter 5**, we are considering a family of lifetime distributions given by Liang (2008) [17]. For the scale parameter of such distribution, Bayes estimators and posterior risk are evaluated for the comparison under the informative priors and non-informative priors. For the comparison on the basis of loss functions: Squared Error Loss Function (SELF), Quadratic Loss Functions (QLF) and Precautionary Loss Functions (PLF) are considered. The performance of the estimator is assessed on the basis of its relative posterior risk. Markov Chain Monte Carlo (MCMC) are used for Simulation to compare the performance of these estimators.

In the **Chapter 6**, risk, posterior risk and Bayes risk is obtained for the parameters of the three parameter Generalised Rayleigh Distribution under type II censoring. Here we are considering the positive and negative powers of the parameters. Bayes estimators of the reliability function $R(t) = P_r(X > t)$ and the stress-strength model $P = P_r(X > Y)$ is also obtained with the help of Bayes estimators. Numerical findings are presented through the Tables and Figures.

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