Uncertainties in the nuclear transition matrix elements of neutrinoless double beta decay

Summary

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Chapter 1

Introduction

Neutrinos are quite fascinating particles due to their weird nature. They came into existence during the first three minutes of the Big Bang, in which the universe was created 13.7 ± 0.2 Gyr ago. Although the neutrinos permeate most of the space all around us but they hardly interact with anything. In 1930, Wolfgang Pauli postulated the existence of electron neutrino ν_e - a chargeless, massless and spin 1/2 particle - to explain the continuous nature of the beta spectrum. In 1956, Cowan and Reines experimentally detected neutrinos at the Savannah river reactor site. In the theory of Standard Model (SM) the neutrinos are assumed to be massless particles. The confirmation of neutrino oscillations in solar, atmospheric, accelerator and reactor neutrino sources has established the fact that neutrinos have mass. The mass and exact nature of neutrinos -Dirac or Majorana character- are not completely known till today.

The study of tritium single β decay and $\beta^-\beta^-$ decay together has the potential to extract sharpest limits on the neutrino mass and explain the nature of the electron neutrinos. Further, these determining features of the elusive neutrinos through the observation of $0\nu\beta^-\beta^-$ decay could provide a deep insight into the nature of the universe during the earliest moments of the Big Bang.

1.1 The Nuclear $\beta\beta$ decay

The nuclear $\beta\beta$ decay is a rare second order weak transition between two isobars having even Z-even N configuration and differing in nuclear charge by two units. The $\beta\beta$ decay candidates are stable against single β decay either due to energy conservation or angular momentum mismatch. There are two modes of $\beta\beta$ decay, one involving the emission of two neutrinos ($2\nu\beta\beta$ decay) and the other neutrinoless double beta ($0\nu\beta\beta$) decay. The former conserves the lepton number L exactly and is an allowed process within SM. In the latter one, the lepton number is violated by two units and it has the potential to probe physics beyond the SM. The $2\nu\beta\beta$ decay can be regarded as a simultaneous transformation of two neutrons into two protons, which leads to the final state with emission of two electrons and two antielectron-neutrinos ($2\nu\beta^-\beta^-$ decay) i.e.,

$$ {}^{A}_{Z}X \rightarrow {}^{A}_{Z+2}Y + 2e^- + 2\overline{\nu}_e \tag{1.1}$$

The $0\nu\beta^-\beta^-$ decay, one of the most interesting case of $\beta\beta$ decay, is given as

$$ {}^{A}_{Z}X \rightarrow {}^{A}_{Z+2}Y + 2e^- \tag{1.2}$$

and was considered first by Racah [Racah (1937)] and W. Furry [Furry (1939)] as a tool to distinguish whether the neutrino is of Majorana or Dirac nature.

1.1.1 $2\nu\beta^-\beta^-$ decay and validity of nuclear models

The inverse half-life of the $2\nu\beta^-\beta^-$ decay for the $0^+ \rightarrow 0^+$ transition is given by

$$[T^{2\nu}_{1/2}(0^+ \rightarrow 0^+)]^{-1} = G_{2\nu} \left| M_{2\nu} \right|^2 \tag{1.3}$$

where $G_{2\nu}$ is the integrated kinematical factor and can be calculated with good accuracy. The nuclear transition matrix element (NTME) $M_{2\nu}$ is a model dependent quantity. The half-life $T^{2\nu}_{1/2}$ of $2\nu\beta^-\beta^-$ decay has been already measured for eleven nuclei out of 35 possible candidates. Using the experimental half-life $T^{2\nu}_{1/2}$ and accurately known integrated kinematical factor $G_{2\nu}$, the values of $M_{2\nu}$ can be extracted directly from Eq.(1.3). It is observed that in all cases of $2\nu\beta^-\beta^-$ decay, the NTMEs $M_{2\nu}$ are sufficiently quenched. The main motive of all theoretical calculations is to understand the physical mechanism responsible for the observed suppression of $M_{2\nu}$. Hence, the validity of different nuclear models can be tested through the calculation of $M_{2\nu}$.

1.1.2 $0\nu\beta^-\beta^-$ decay and physics beyond the SM

In the SM, the neutrinos are massless due to the absence of the right-handed neutrinos and the existence of an exact global $B-L$ symmetry. The gauge theories beyond the SM fulfill the above mentioned requirements. The possible ways for breaking the $B-L$ symmetry are (i) explicit breaking of $B-L$ symmetry where Lagrangian contains terms that break $B-L$ symmetry, (ii) spontaneous breaking of local $B-L$ symmetry and (iii) spontaneous breaking of global $B-L$ symmetry. Thus, the study of $\beta\beta$ decay in general and $0\nu\beta^-\beta^-$ decay in particular is a convenient tool to test the following important ramifications vis-a-vis constraints on parameters of various gauge theoretical models beyond the SM, namely (i) lepton number violation, (ii) mass and charge conjugation properties of the electron-neutrino and (iii) possible right handed admixtures in the weak leptonic current.

1.2 Present motivation

Our present aim is to study the $0\nu\beta^-\beta^-$ decay of some nuclei in the mass range $90 \leq A \leq 150$ for the $0^+ \rightarrow 0^+$ transition to extract various gauge-theoretical parameters, namely effective light neutrino mass $< m_\nu >$, the effective weak coupling constants $< \lambda >$ and $< \eta >$ for coupling of right-handed leptonic current with right-handed and left-handed nucleonic currents.

The present thesis is organized in following five chapters. In Chapter 1, we give introduction of the present work and related literature survey is presented. In Chapter 2, we test the reliability of PHFB wave functions by comparing the calculated yrast spectra, reduced $B(E2{:}0^+ \rightarrow 2^+)$ transition probabilities and deformation parameters β_2 of ^{94,96}Zr, ^{94,96,100}Mo, ^{100,104}Ru, ^{104,110}Pd, ^{110}Cd, ^{128,130}Te, ^{128,130}Xe, ^{150}Nd and ^{150}Sm nuclei participating in the $0\nu\beta^-\beta^-$ decay, with the available experimental data. Subsequently, the PHFB wave functions are employed to study the $2\nu\beta^-\beta^-$ decay for the $0^+ \rightarrow J^+$ transition in Chapter 3. In Chapter 4, we study the $0\nu\beta^-\beta^-$ decay of same nuclei in the mass range $90 \leq A \leq 150$ for the $0^+ \rightarrow 0^+$ transition and extract limits on effective gauge theoretical parameters $\langle m_\nu \rangle$, $\langle \lambda \rangle$ and $\langle \eta \rangle$. In Chapter 5, we intend to suggest and discuss a number of necessary improvements that need to be incorporated into the PHFB model for a more reliable study of the $\beta\beta$ decay.

Chapter 2

Reliability of PHFB wave functions of some nuclei in the mass range $90 \leq A \leq 150$ participating in $\beta^-\beta^-$ decay

The present chapter is organized as follows. A detailed derivation of the HFB method has been given by Baranger (1963), Villars (1966) and Goodman (1979). The projection technique in the context of HFB method was developed by Onishi and Yosida (1966). In Section 2.1, we present a brief outline of the PHFB model in order to define various quantities and make the discussion self-contained. In Section 2.2, the formalism to calculate the spectroscopic properties has been presented [Dixit *et al.* (2002)]. We compare the calculated E_{2^+} and electromagnetic properties with available experimental data for ^{94,96}Zr, ^{94,96,100}Mo, ^{100,104}Ru, ^{104,110}Pd, ^{110}Cd, ^{128,130}Te, ^{128,130}Xe, ^{150}Nd and ^{150}Sm nuclei in Section 2.3. Finally, the conclusions are given in Section 2.4.

2.1 The PHFB model

The axially symmetric HFB intrinsic state with $K = 0$ can be written as

$$|\Phi_0\rangle = \prod_{im} \left(u_{im} + v_{im} b^\dagger_{im} b^\dagger_{i\overline{m}} \right) |0\rangle \tag{2.1}$$

where the creation operators $b^\dagger_{im}$ and $b^\dagger_{i\overline{m}}$ are given by

$$b^\dagger_{im} = \sum_\alpha C_{i\alpha,m} a^\dagger_{\alpha m} \qquad and \qquad b^\dagger_{i\overline{m}} = \sum_\alpha (-1)^{l+j-m} C_{i\alpha,m} a^\dagger_{\alpha,-m} \tag{2.2}$$

The wave function $|\Phi_0\rangle$ can be recast into the form

$$|\Phi_0\rangle = N\ exp\left(\frac{1}{2}\sum_{\alpha\beta} f_{\alpha\beta} a^\dagger_\alpha a^\dagger_\beta |0\rangle\right) \tag{2.3}$$

with

$$f_{\alpha\beta} = \sum_i C_{im_\alpha,j_\alpha} C_{im_\beta,j_\beta} \left(\frac{v_{im_\alpha}}{u_{im_\alpha}}\right) \delta_{m_\alpha - m_\beta} \tag{2.4}$$

where N is a normalization constant. A state with good angular momentum **J** is obtained from the HFB intrinsic state using the standard projection technique [Onishi *et al.* (1966)] through the following relation.

$$\begin{aligned} \left|\Psi^J_0\right\rangle &= P^J_{00}\left|\Phi_0\right\rangle \\ &= \left[\frac{(2J+1)}{8\pi^2}\right] \int D^J_{00}(\Omega) R(\Omega) \left|\Phi_0\right\rangle d\Omega \end{aligned} \tag{2.5}$$

2.2 Spectroscopic properties of yrast states

In the below, we have presented expressions to calculate various nuclear spectroscopic properties, namely yrast energy spectra, reduced $B(E2{:}0^+ \rightarrow 2^+)$ transition probabilities and deformation parameters β_2 [Dixit *et al.* (2002)].

2.3 Results and discussions

In this section, we present the results for the theoretically calculated sub-shell occupation numbers, excited energies E_{2^+} of yrast 2^+ state, reduced $B(E2{:}0^+ \rightarrow 2^+)$ transition probabilities and deformation parameters β_2 for ^{94,96}Zr, ^{94,96,100}Mo, ^{100,104}Ru, ^{104,110}Pd, ^{110}Cd, ^{128,130}Te, ^{128,130}Xe, ^{150}Nd and ^{150}Sm nuclei and compare them with the available observed experimental data.

2.3.1 The yrast spectra and electromagnetic properties

The spectroscopic properties of ^{94,96}Zr, ^{94,96,100}Mo, ^{100,104}Ru, ^{104,110}Pd, ^{110}Cd, ^{128,130}Te, ^{128,130}Xe, ^{150}Nd and ^{150}Sm nuclei have been calculated in PHFB model using four parametrizations of effective two-body interaction, namely PQQ1, PQQHH1, PQQ2 and PQQHH2.

2.4 Conclusions

As a test of the reliability of the wave functions, we have calculated the sub-shell occupation numbers, excitation energy of the 2^+ state E_{2^+} , reduced $B(E2:0^+ \rightarrow 2^+)$ transition probabilities and deformation parameters β_2 of some nuclei undergoing $\beta^-\beta^-$ decay in the mass range A=90 – 150 and compared with the available experimental data. The overall agreement between the theoretically calculated and experimentally observed values makes us confident to apply the same PHFB wave functions to study the nuclear 2ν $\beta^-\beta^-$ decay of the nuclei under consideration.

Chapter 3

Two neutrino double beta decay of ^{94,96}Zr, ^{100}Mo, ^{104}Ru, ^{110}Pd, ^{128,130}Te and ^{150}Nd isotopes for the $0^+ \to J^+$ transition

The present chapter has been organized in following three sections. In Section 3.1, the theoretical formalism to calculate the half life $T_{1/2}^{2\nu}(J^+)$ of $2\nu\beta^-\beta^-$ decay is given. In Section 3.2, the results of $2\nu\beta^-\beta^-$ decay of ^{94,96}Zr, ^{100}Mo, ^{104}Ru, ^{110}Pd, ^{128,130}Te and ^{150}Nd nuclei are presented and discussed. The conclusions are given in Section 3.3.

3.1 Theoretical framework

The half life for the $0^+ \to J^+$ transition of $2\nu\beta^-\beta^-$ decay $T_{1/2}^{2\nu}(J^+)$ in 2n mechanism is given by

$$\left[T_{1/2}^{2\nu}(J^+)\right]^{-1} = G_{2\nu}(J^+)\left|M_{2\nu}(J^+)\right|^2 \tag{3.1}$$

where the integrated kinematical factor $G_{2\nu}(J^+)$ has been calculated with good accuracy [Stoica and Mirea (2013), Pahomi *et al.* (2014)]. The model dependent NTME $M_{2\nu}(J^+)$ is given by

$$M_{2\nu}(J^+) = \sqrt{\frac{1}{s}}\sum_N \frac{\left\langle 2_F^+ \left\|\sigma\tau^+\right\| 1_N^+\right\rangle \left\langle 1_N^+ \left\|\sigma\tau^+\right\| 0_I^+\right\rangle}{\left[E_0 + E_N - E_I\right]^s} \tag{3.2}$$

where $s = \{1 + 2\delta_{J2}\}$ and

$$E_0 = \frac{1}{2}(E_I - E_F) = \frac{1}{2}Q_{\beta\beta} + m_e \tag{3.3}$$

Presently, the summation over the intermediate states is performed using the summation method [Civitarese and Suhonen (1993)].

3.2 Results and discussions

As discussed earlier that the $0^+ \rightarrow 2^+$ transition of $2\nu\beta^-\beta^-$ decay has not been observed experimentally and only half-life limits are available. As a test of reliability of the wave function, in Table 3.1, the NTMEs $M_{2\nu}(0^+)$ calculated with four different parametrization of effective two-body interactions, namely PQQ1, PQQHH1, PQQ2, PQQHH2, are presented along with experimental values for comparison. It is observed that theoretically calculated NTMEs agree well with the experimental values within error bar except for the case ^{130}Te and ^{150}Nd. The estimated average (mean) NTMEs $\overline{M}_{2\nu}(J^+)$ and uncertainties (standard deviation) $\Delta\overline{M}_{2\nu}(J^+)$ has been calculated as follows:

$$\overline{M}_i = \frac{\sum_{k=1}^{N} M_i^k}{N} \tag{3.4}$$

and

$$\Delta\overline{M}_i = \frac{1}{\sqrt{N-1}}\left[\sum_{k=1}^{N}\left(\overline{M}_i - M_i^k\right)^2\right]^{1/2} \tag{3.5}$$

The Eqs. (3.4) and (3.5) define the best estimate of the mean and standard deviation for a Gaussian distribution. In Table 3.2, we present the presently calculated results of $M_{2\nu}(2^+)$ along with average values and uncertainties.

It is observed that there is a remarkable spread in the calculated NTMEs $M_{2\nu}(2^+)$ within different nuclear models which has been presented in the thesis.

3.3 Conclusions

Sets of four NTMEs $M_{2\nu}(2^+)$ have been calculated with in PHFB model using PQQ1, PQQHH1, PQQ2 and PQQHH2 parametrizations to study the $2\nu\beta^-\beta^-$ decay of ^{94,96}Zr, ^{100}Mo, ^{104}Ru, ^{110}Pd, ^{128,130}Te and ^{150}Nd isotopes for the $0^+ \rightarrow 2^+$ transition. The observation of Raduta *et al.* (2007) that the inclusion of deformation in the mean field can reduce the NTMEs $M_{2\nu}(2^+)$ calculated within pnQRPA up to a factor of 341, motivated us to study the $0^+ \rightarrow 2^+$ transition of $2\nu\beta^-\beta^-$ decay within PHFB approach treating the pairing and deformation degrees of freedom simultaneously on equal footing. It is noticed that with respect to NTMEs $M_{2\nu}(2^+)$ of Raduta *et al.* (2007), the average NTMEs $\overline{M}_{2\nu}(2^+)$ calculated using the PHFB approach are further suppressed by a factor between 1 – 450 approximately corresponding to ^{96}Zr and ^{128}Te isotopes, respectively.

Table 3.1: Theoretically calculated NTMEs $M_{2\nu}$ within the PHFB model with four different parametrizations and their average value $\overline{M}_{2\nu}$ along with experimental values [Barabash (2010)].

Nuclei	$M_{2\nu}(0^+)$				$\overline{M}_{2\nu}(0^+)$	$\Delta\overline{M}_{2\nu}(0^+)$	$M_{2\nu}(0^+)$(Exp.)
	PQQ1	PQQHH1	PQQ2	PQQHH2			
^{94}Zr	0.064	0.060	0.133	0.058	0.079	0.036	–
^{96}Zr	0.055	0.056	0.053	0.054	0.054	0.001	$0.048^{+0.002}_{-0.002}$
^{100}Mo	0.127	0.128	0.127	0.127	0.127	0.001	$0.123^{+0.004}_{-0.003}$
^{104}Ru	0.020	0.021	0.023	0.022	0.021	0.001	–
^{110}Pd	0.085	0.124	0.105	0.115	0.107	0.017	–
^{128}Te	0.026	0.022	0.027	0.021	0.024	0.003	$0.024^{+0.003}_{-0.002}$
^{130}Te	0.047	0.044	0.041	0.050	0.045	0.004	$0.017^{+0.002}_{-0.001}$
^{150}Nd	0.010	0.008	0.010	0.008	0.009	0.001	$0.031^{+0.002}_{-0.002}$

Table 3.2: Calculated NTMEs $M_{2\nu}(2^+)$ within the PHFB model and their average $\overline{M}_{2\nu}(2^+)$ along with standard deviation $\Delta\overline{M}_{2\nu}(2^+)$.

Nuclei	$M_{2\nu}(2^+)$				$\overline{M}_{2\nu}(2^+)$	$\Delta\overline{M}_{2\nu}(2^+)$
	PQQ1	PQQHH1	PQQ2	PQQHH2		
^{94}Zr	1.49×10^{-4}	1.49×10^{-4}	1.01×10^{-4}	1.45×10^{-4}	1.360×10^{-4}	0.234×10^{-4}
^{96}Zr	1.23×10^{-4}	1.34×10^{-4}	1.18×10^{-4}	1.27×10^{-4}	1.255×10^{-4}	0.067×10^{-4}
^{100}Mo	1.57×10^{-5}	1.47×10^{-5}	1.80×10^{-5}	1.14×10^{-5}	1.495×10^{-5}	0.274×10^{-5}
^{104}Ru	8.18×10^{-6}	8.66×10^{-6}	8.24×10^{-6}	8.85×10^{-6}	8.483×10^{-6}	0.325×10^{-6}
^{110}Pd	1.31×10^{-4}	2.03×10^{-4}	1.48×10^{-4}	2.01×10^{-4}	1.708×10^{-4}	0.367×10^{-4}
^{128}Te	1.82×10^{-6}	2.62×10^{-6}	1.90×10^{-6}	2.84×10^{-6}	2.295×10^{-6}	0.511×10^{-6}
^{130}Te	5.39×10^{-6}	6.01×10^{-6}	5.01×10^{-6}	6.06×10^{-6}	5.617×10^{-6}	0.506×10^{-6}
^{150}Nd	1.43×10^{-7}	1.71×10^{-7}	1.13×10^{-7}	1.37×10^{-7}	1.410×10^{-7}	0.238×10^{-7}

Chapter 4

Neutrinoless double beta decay of ^{94,96}Zr, ^{100}Mo, ^{110}Pd, ^{128,130}Te and ^{150}Nd isotopes for the $0^+ \to 0^+$ transition

We intent to employ the same set of initial and final nuclear wave functions as used in Chapter 2 and Chapter 3 to study the $0\nu\beta^-\beta^-$ decay of ^{94,96}Zr, ^{100}Mo, ^{110}Pd, ^{128,130}Te and ^{150}Nd nuclei in the present chapter.

In Section 4.1, the theoretical formalism to calculate the decay rate of $0\nu\beta^-\beta^-$ decay has been derived in 2n mechanism for light neutrinos. The results of our calculation are presented and discussed in Section 4.2. We have calculated NTMEs in PHFB model for above mentioned nuclei. Further, the gauge-theoretical parameters, namely effective electron-neutrino mass $\langle m_\nu \rangle$ and effective weak coupling constants $\langle \lambda \rangle$ and $\langle \eta \rangle$ for the coupling of right-handed leptonic current with right-handed and left-handed hadronic currents, respectively, are extracted from the observed limits on half-lives $T_{1/2}^{0\nu}$ of the $0\nu\beta^-\beta^-$ decay. Finally, we present some concluding remarks in Section 4.3.

4.1 Theoretical formalism

Using the standard approximations of Doi *et al.* (1985), with CP conservation, the rate for the $0^+ \to 0^+$ transition of $0\nu\beta^-\beta^-$ decay is given by

$$\begin{aligned} \left[T_{1/2}^{0\nu}\right]^{-1} &= \frac{\left|\langle m_\nu \rangle\right|^2}{m_e} C_{mm} + \frac{\left|\langle m_\nu \rangle\right|}{m_e} \langle \lambda \rangle C_{m\lambda} + \frac{\left|\langle m_\nu \rangle\right|}{m_e} \langle \eta \rangle C_{m\eta} \\ &+ \langle \lambda \rangle^2 C_{\lambda\lambda} + \langle \eta \rangle^2 C_{\eta\eta} + \langle \lambda \rangle \langle \eta \rangle C_{\lambda\eta} \end{aligned} \tag{4.1}$$

where

$$\langle m_\nu \rangle = \sum_i' U_{ei}^2 m_i \tag{4.2}$$

$$\langle \lambda \rangle = \lambda \left| \sum_i' \left(\frac{g_V'}{g_V} \right) U_{ei} V_{ei} \right| \tag{4.3}$$

$$\langle \eta \rangle = \eta \left| \sum_i' U_{ei} V_{ei} \right| \tag{4.4}$$

and the nuclear structure factors C_{xy} are written as

$$C_{mm} = G_{01} \left| M^{(0\nu)} \right|^2 \tag{4.5}$$

$$C_{m\lambda} = M^{(0\nu)} \left(G_{04} M_{1+} - G_{03} M_{2-} \right) \tag{4.6}$$

$$C_{m\eta} = M^{(0\nu)} (G_{03} M_{2+} - G_{04} M_{1-} - G_{05} M_P + G_{06} M_R) \tag{4.7}$$

$$C_{\lambda\lambda} = G_{02} \left| M_{2-} \right|^2 - \frac{2}{9} G_{03} (M_{1+} M_{2-}) + \frac{1}{9} G_{04} \left| M_{1+} \right|^2 \tag{4.8}$$

$$\begin{aligned} C_{\eta\eta} &= G_{02} \left| M_{2+} \right|^2 - \frac{2}{9} G_{03} (M_{1-} M_{2+}) + \frac{1}{9} G_{04} \left| M_{1-} \right|^2 \\ &\quad - G_{07} (M_P M_R) + G_{08} \left| M_P \right|^2 + G_{09} \left| M_R \right|^2 \end{aligned} \tag{4.9}$$

$$\begin{aligned} C_{\lambda\eta} &= -2 G_{02} (M_{2+} M_{2-}) + \frac{2}{9} G_{03} (M_{2+} M_{1+} + M_{2-} M_{1-}) \\ &\quad - \frac{2}{9} G_{04} (M_{1-} M_{1+}) \end{aligned} \tag{4.10}$$

In addition, the combinations of NTMEs $M^{(0\nu)}$ and $M_{i\pm}$ $(i = 1, 2)$ are defined as

$$M^{(0\nu)} = M_{GT} - M_F + M_T \tag{4.11}$$

$$M_{1\pm} = M_{qGT} - 6 M_{qT} \pm 3 M_{qF} \tag{4.12}$$

$$M_{2\pm} = M_{\omega GT} \pm M_{\omega F} - \frac{1}{9} M_{1\mp} \tag{4.13}$$

Employing the generally agreed closure approximation in conjunction with the HFB wave functions, the NTMEs M_α ($\alpha = F, GT, T, \omega F, \omega GT, qF, qGT, qT, P$ and R) appearing in the expressions of nuclear structure factors C_{xy} are calculated by using the following expression [Rath *et al.* (2010)].

$$\begin{aligned} M_\alpha &= \left\langle 0_f^+ \left\| O_\alpha(\mathbf{r}, \boldsymbol{\sigma}) \right\| 0_i^+ \right\rangle \\ &= \left[n^{J_f=0} n^{Ji=0} \right]^{-1/2} \int_0^\pi d\theta sin\theta n_{(Z,N),(Z+2,N-2)}(\theta) \times \sum_{\alpha\beta\gamma\delta} \left\langle \alpha\beta \left| O_\alpha(\mathbf{r}, \boldsymbol{\sigma}) \right| \gamma\delta \right\rangle \\ &\quad \times \sum_{\varepsilon\eta} \frac{\left(f_{Z+2,N-2}^{(\pi)*} \right)_{\varepsilon\beta}}{\left[\left(1 + F_{Z,N}^{(\pi)}(\theta) f_{Z+2,N-2}^{(\pi)*} \right) \right]_{\varepsilon\alpha}} \times \frac{\left(F_{Z,N}^{(\nu)*} \right)_{\eta\delta}}{\left[\left(1 + F_{Z,N}^{(\nu)}(\theta) f_{Z+2,N-2}^{(\nu)*} \right) \right]_{\gamma\eta}} \end{aligned} \tag{4.14}$$

where

$$n^J = \int_0^\pi \left[det\left(1 + F^{(\pi)} f^{(\pi)\dagger}\right)\right]^{1/2} \left[det\left(1 + F^{(\nu)} f^{(\nu)\dagger}\right)\right]^{1/2} d^J_{00}(\theta) sin(\theta) d\theta \qquad (4.15)$$

and

$$\begin{aligned} n_{(Z,N),(Z+2,N-2)}(\theta) &= \left[det\left(1 + F^{(\nu)}_{Z,N} f^{(\nu)\dagger}_{Z+2,N-2}\right)\right]^{1/2} \\ &\times \left[det\left(1 + F^{(\pi)}_{Z,N} f^{(\pi)\dagger}_{Z+2,N-2}\right)\right]^{1/2} \qquad (4.16) \end{aligned}$$

4.2 Results and discussions

In the present work, we employ the same PHFB wave-function used in Chapter 2 and Chapter 3, i.e. wave functions generated with four different parametrizations of the effective two-body interaction, namely PQQ1, PQQHH1, PQQ2 and PQQHH2. Further, we use three different parametrizations of the SRC due to Miller-Spencer parametrization, Argonne NN and CD-Bonn potentials and call them SRC1, SRC2 and SRC3, respectively. The required NTMEs, namely M_F, $M_{\omega F}$, M_{qF}, M_{GT}, $M_{\omega GT}$, M_{qGT}, M_T, M_{qT}, M_P and M_R are calculated for ^{94,96}Zr, ^{100}Mo, ^{104}Ru,^{110}Pd, ^{128,130}Te and ^{150}Nd nuclei for the $0^+ \rightarrow 0^+$ transition within the approximations of point nucleons (P), nucleons having finite size (FNS) and also with SRC (F+SRC).

Each NTME has a set of twelve values due to four parametrizations of two-body interaction and three parametrizations of SRC. We have calculated their averages and standard deviations. In present work, we use the phase space factors calculated by Štefánik *et al.* (2015) and reevaluate them at $g_A = 1.2701$. Sets of twelve nuclear structure factors C_{mm}, $C_{m\lambda}$, $C_{m\eta}$, $C_{\lambda\lambda}$, $C_{\eta\eta}$ and $C_{\lambda\eta}$ are computed for ^{96}Zr, ^{100}Mo, ^{110}Pd, ^{130}Te and ^{150}Nd isotopes. The averages of these nuclear structure factors have been calculated.

Using the average nuclear structure factors $\overline{C}_{mm}$, $\overline{C}_{\lambda\lambda}$, $\overline{C}_{\eta\eta}$, on-axis limits on the effective mass of light neutrino $\langle m_\nu \rangle$, the effective weak coupling of right-handed leptonic current with right-handed hadronic current $< \lambda >$, and the effective weak coupling of right-handed leptonic current with left-handed hadronic current $< \eta >$ are extracted from the largest observed limits on half-lives $T^{0\nu}_{1/2}$ of $0\nu\beta^-\beta^-$decay and the results are given in Table 4.1.

4.3 CONCLUSIONS

To summarize, sets of twelve NTMEs, namely $M_{\omega F,qF}$, $M_{\omega GT,qGT}$, M_{qT}, M_P, and M_R are calculated using PHFB wave functions generated with four different parametrization of pairing plus multipolar type of effective two-body interaction, and three different parametrizations of SRC to study the $0\nu\beta^-\beta^-$ decay of ^{94,96}Zr, ^{100}Mo, ^{110}Pd, ^{128,130}Te and ^{150}Nd isotopes within mechanisms involving the light Majorana neutrino and right handed $V+A$ current. The maximum effect due to FNS is about 57% for $M_{T,P}$. Due to SRC1, SRC2 and SRC3, the maximum change in M_R is about 56%, 29% and 11%, respectively.

The maximum uncertainty in $\overline{M}_{F,\omega F,qF}$, $\overline{M}_{GT,\omega GT,qGT}$ and $\overline{M}_P$ is about15% but for ^{150}Nd, in which the standard deviation of $\overline{M}_P$ is about 28%. The maximum uncertainty in $\overline{M}_R$ is about 30%. The NTMEs $M_{T,qT}$ are quite uncertain. Using the average nuclear structure factors $\overline{C}_{mm}$, $\overline{C}_{\lambda\lambda}$, and $\overline{C}_{\eta\eta}$, the most stringent on-axis extracted limits on $\langle m_\nu \rangle$, $\langle \lambda \rangle$, and $\langle \eta \rangle$ from the most recent observed limits on half-lives $T_{1/2}^{0\nu}$ of ^{130}Te isotope are 0.17 eV, 2.41×10^{-7} and 2.55×10^{-9}, respectively.

Table 4.1: Experimental limits on half-lives $T^{0\nu}_{1/2}$ and the extracted on-axis limits on the effective mass of light neutrino $\langle m_\nu \rangle$, $\langle \lambda \rangle$, and $\langle \eta \rangle$ for the $0\nu\beta^-\beta^-$ decay of ^{96}Zr, ^{100}Mo, ^{110}Pd, ^{130}Te and ^{150}Nd isotopes. Predicted half-lives $T^{0\nu}_{1/2}$ of $0\nu\beta^-\beta^-$decay for two sets of parameters (i) $\langle m_\nu \rangle = 50$ meV (Case I) and (ii) $\langle m_\nu \rangle = 50$ meV, $\langle \lambda \rangle = 10^{-7}$ and $\langle \eta \rangle = 10^{-9}$ (Case II).

Nuclei	$T^{(0\nu)}_{1/2}$(Ex)	Ref.	$< m_\nu >$	$< \lambda >$	$< \eta >$	$T^{0\nu}_{1/2}$(I)	$T^{0\nu}_{1/2}$(II)
^{96}Zr	9.2×10^{21}	[1]	9.56	1.02×10^{-5}	1.21×10^{-7}	3.37×10^{26}	4.44×10^{25}
^{100}Mo	1.1×10^{24}	[2]	0.44	5.62×10^{-7}	6.09×10^{-9}	8.64×10^{25}	1.32×10^{25}
^{110}Pd	6.0×10^{17}	[3]	1.28×10^{3}	2.28×10^{-3}	1.71×10^{-5}	3.91×10^{26}	7.07×10^{25}
^{130}Te	1.5×10^{25}	[4]	0.17	2.41×10^{-7}	2.55×10^{-9}	1.76×10^{26}	3.24×10^{25}
^{150}Nd	2.0×10^{22}	[5]	6.39	6.53×10^{-6}	9.97×10^{-8}	3.26×10^{26}	4.84×10^{25}

References:

[1] Barabash *et al.* (2011) [2] Arnold *et al.* (2015) [3] Winter (1952)
[4] Alduino *et al.* (2018) [5] Arnold *et al.* (2016)

Chapter 5

Conclusions

In the present work, our aim was to study the $2\nu\beta^-\beta^-$ decay for the $0^+ \rightarrow J^+$ transition and the $0\nu\beta^-\beta^-$ decay for the $0^+ \rightarrow 0^+$ transition within mechanisms involving light Majorana neutrino mass and right handed current of some nuclei in the mass range $90 \leq A \leq 150$ in PHFB model. We used a pairing plus multipole type of effective two-body interaction and derive its four parametrizations, namely PQQ1, PQQHH1, PQQ2, PQQHH2. The single particle energies used were derived from Woods-Saxon potential. Further, we used three parametrizations of the SRC due to Miller-Spencer parametrization (SRC1), Argonne NN (SRC2) and CD-Bonn potentials (SRC3). In the absence of experimental observation of $0\nu\beta^-\beta^-$, the nuclear models predict half-lives $T_{1/2}^{0\nu}$ of $0\nu\beta^-\beta^-$ decay assuming certain value of neutrino mass or conversely, limits on various lepton number violating gauge-theoretical parameters are extracted from the observed experimental half-life limits by calculating the appropriate NTMEs. The reliability of the wave functions were checked by calculating nuclear spectroscopic properties, namely the yrast spectra, reduced $B(E2{:}0^+ \rightarrow 2^+)$ transition probabilities and β_2 parameters of the nuclei under going $\beta^-\beta^-$ decay and comparing them with the experimental values. After getting an overall agreement between theoretically calculated and experimentally observed values, the NTMEs $M_{2\nu}(J^+)$ and half-lives $T_{1/2}^{2\nu}(2^+)$ of $2\nu\beta^-\beta^-$ decay for the $0^+ \rightarrow J^+$ transition have been calculated for ^{94,96}Zr, ^{100}Mo, ^{104}Ru, ^{110}Pd, ^{128,130}Te and ^{150}Nd nuclei. It was noticed that the validity of different nuclear models employed for nuclear structure calculations can not be uniquely established in the absence of experimental results as well as uncertainty in g_A. Further work is necessary both in the experimental as well as theoretical front to judge the relative applicability, success and failure of these nuclear models.

Subsequently, the NTMEs of $0\nu\beta^-\beta^-$ decay for the $0^+ \rightarrow 0^+$ transition were calculated within mechanisms involving the light Majorana neutrino, and right handed $V + A$

currents. The effect of finite size of nucleons and short range correlations on NTMEs has been studied. We have also esimated mean and standard deviation of claculated NTMEs. Further, the limits on effective light Majorana neutrino mass $\langle m_\nu \rangle$, effective weak coupling of right-handed leptonic current with right-handed hadronic current $\langle \lambda \rangle$ and with left-handed hadronic current $\langle \eta \rangle$ were extracted from the most recent observed half-life limits and appropriate NTMEs. The most stringent limit comes from the $0\nu\beta^-\beta^-$ decay of ^{130}Te isotope. The extracted limits on $\langle m_\nu \rangle$, $\langle \lambda \rangle$, and $\langle \eta \rangle$ are 0.17 eV, 2.41×10^{-7} and 2.55×10^{-9}, respectively.

Uncertainty in NTMEs

The main uncertainty in the calculation of decay rates or extraction of gauge theoretical parameters lies with in the NTMEs. The NTMEs are purely model dependent quantities and one needs reliable calculation of NTMEs for better theoretical prediction. On the one hand, there is a large uncertainty in the NTMEs calculated in different nuclear models. The uncertainty in NTMEs is mainly due to different approaches used in the theoretical calculations [Engel (2015)]. In the nuclear structure calculation different nuclear models use different model spaces and different effective interactions. Further, there is no general guideline or clear cut prescription to fix the single particle energies and parameters of the two-body interactions. Simkovic *et al.* (2009) have shown that the choice of SRC and axial vector coupling constant g_A is also the source of uncertainty in the NTMEs.

The trend of results in PHFB model suggests that it is necessary to incorporate a number of improvements, which are discussed in the thesis.

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