VARIOUS LOSS FUNCTIONS AND RELATED INFERENTIAL PROCEDURES

ABSTRACT
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ABSTRACT

The reliability field has a long history that dates back to the early 1930s. The $R(t) = Pr(X > t)$ reliability function is defined as the probability of failure-free operation until time 't'. During World War II, both Germany and the United States used basic reliability ideas to understand the need for equipment dependability enhancement. Between 1945 and 1950, it conducted several studies on electronic equipment failure, equipment maintenance and repair costs, and so on.

Another measure of reliability under the stress-strength setup is $R = Pr(Y > X)$, which indicates the reliability of an item subjected to random stress X and random strength Y in the stress-strength setup, is another measure of reliability. Birnbaum (1956) was one of the first scholars to deal with the stress-strength content model $Pr(X < Y)$. Later on, it was developed by Birnbaum and McCarty (1958). The term stress-strength was coined by Church and Harris in 1970. Stress-strength problems have been explored by Pugh (1963), Basu (1964), Johnson (1975), Sathe and Shah (1981), Chao (1982), Tyagi and Bhattacharya (1989), Chaturvedi and Rani (1997, 1998), Chaturvedi and Surinder (1999), Chaturvedi and Singh (2006, 2008). In Chapter 2, Chapter 3, and Chapter 4, we worked along the same lines.

The Bayesian problem has been studied by many researchers. However, in many applications, it becomes necessary to make decisions that are based on a fixed unknown quantity called the ‘true state of nature’ or the ‘parameter’. In the classical paradigm, such decision-making processes include elements such as conducting a random experiment to gather information about an unknown state of nature. The decision's efficacy is then assessed using a function that is frequently finalized according to the need of the decision-maker. The majority of the issues in the former category are based on symmetrical loss functions such as squared error loss functions, absolute loss functions, etc. An asymmetric loss function, on the other hand, should be employed when underestimation is more serious than overestimation or vice versa. In the Bayesian study, the Bayes estimators of scale parameters are obtained for various loss functions under various informative and non-informative priors. Varian (1975), Zellner (1986), Chaturvedi and Singh (2006), and others provide a good overview.

The thesis entitled "Inferences on Various loss functions and related inferential procedures" is divided into six chapters, the first of which, Chapter 1, presents an overview of the classical and the Bayesian paradigm and its important components like prior distribution and loss functions. It also includes Reliability and Stress-Strength models. Review of the literature and research topics are also covered.

In Chapter 2, we have considered stress-strength model $R = Pr(X < Y)$ when both stress and strength follows the inverse Rayleigh distribution and obtain the estimators of R. In order to obtain the maximum likelihood estimator (MLE), uniformly minimum variance unbiased estimator (UMVUE) and interval estimator by using transformation method. Also, the case, when Stress follows inverse Rayleigh distribution and Strength follows Power function distribution, is considered. We have studied the problem by establishing the relationship among the parameters of the distributions of Stress and Strength of the manufacturing items. Further, these results are explained with an example and are utilized to get the optimum cost of any item when the cost function is linear in terms of parameters.

In Chapter 3, we have considered a generalized family of distributions proposed by Chaturvedi *et al*. (2009). The problem of estimating stress-strength reliability function $R = Pr(X < Y)$ is considered. In order to obtain the maximum likelihood estimator (MLE), uniformly minimum variance unbiased estimator (UMVUE), and interval estimator for the considered model, the technique of transformation method is used.

In Chapter 4, we proposed a new generalized family of inverse lifetime distribution. The problem of estimating stress-strength reliability function $R = Pr(X < Y)$ is considered. In order to obtain the maximum likelihood estimator (MLE), uniformly minimum variance unbiased estimator (UMVUE), and interval estimator for the considered model, the technique of transformation method is used.

In Chapter 5, we considered the generalized family of distributions proposed by Chaturvedi *et al*. (2009). Bayes procedures are used to estimate the scale parameter of the considered family of distributions. Expressions for posterior distributions, for various prior distributions (non-informative, informative, and conjugate) of scale parameter, are derived. Bayes estimators and posterior risk are evaluated for comparison under various priors. For the comparison based on loss functions: Squared Error Loss Function (SELF), Weighted Squared Error Loss Function (WSELF), Precautionary Loss Functions (PLF), Weighted Loss Function (WLF), Modified Squared Error Loss Function (MSELF), Stein's Loss Function (SLF), Entropy loss function (ELF), Logarithmic loss function (LLF) and K-loss function (KLF) are considered. The performance of the estimator is assessed based on its relative posterior risk.

In Chapter 6, we considered the generalized family of distributions proposed in Chapter 4 by Kumar and Ankit (2020). Bayes procedures are applied and expressions for posterior distributions of scale parameters for various non-informative, informative, and conjugate prior distributions are derived. Expressions for Bayes estimators and posterior risk are also derived under various loss functions. For the comparison, Squared Error Loss Function (SELF), Weighted Squared Error Loss Function (WSELF), Precautionary Loss Functions (PLF), Weighted Loss Function (WLF), Modified Squared Error Loss Function (MSELF), Stein's Loss Function (SLF), Entropy loss function (ELF), Logarithmic loss function (LLF) and K-loss function (KLF) are considered. The performance of the estimator is assessed based on its relative posterior risk.

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