ON A NEW STOCHASTIC ORDER, ITS PROPERTIES, AND APPLICATIONS

ABSTRACT

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Abstract

To compare random variables, one may usually compare their mean, median, dispersion or variance. However, in some applied problems, these measures do not provide fruitful information because once in a while, they may have the same values or do not exist. Stochastic orders are introduced to overcome these drawbacks. A Stochastic order is a partial order which evaluates the concept that one random variable is "larger" than the other with the use of underlying distribution functions in more complex way. Particularly, they compare random variables with the use of different functions such as likelihood ratio, hazard rate function, residual life function, etc that tell us about the "location" or "magnitude," "variability" or "dispersion" of random phenomena. Stochastic orders and their properties have given a wide range of meaningful applications over the last five decades in areas like reliability, risk theory, survival analysis, economics, management science (see, for references, Arcones, Kvam, and Samaniego (2002), Balakrishnan and Zhao (2011), Boland, Singh, and Cukic (2004), Hazra and Misra (2020), Li and Li (2013), Singh and Misra (1994), and Zhang and Balakrishnan (2016)).

There are several stochastic orders exist in the literature that compare "location" or "magnitude," "variability" or "dispersion" of random variables. These stochastic orders are introduced by the use of marginal distribution of random variables and known as univariate stochastic orders. For a discussion of several univariate stochastic orders, their properties, and applications, we refer the readers to Chapter 1 of Müller and Stoyan (2002), Chapter $(1,2,4)$ of Shaked and Shanthikumar (2007), and Chapter 2 of Belzunce, Martínez-Riquelme, and Mulero (2015).

Univariate stochastic orders do not care about the mutual relationship between the ordering of random variables except the stochastic precedence order which is based on the joint distribution of random variables and introduced by Boland, Singh, and Cukic (2004). In various practical situations, it is easy to avoid dependence structure between ordering of random variables but in a few cases it becomes difficult to not consider dependence between them. To resolve this problem, several authors introduced some well-defined bivariate versions of univariate stochastic orders by considering the dependence between ordering of random variables and named as "the joint stochastic orders." They also demonstrate their properties, implications, and applications in various areas (see, for references, Aly and Kochar (1993), Balakrishnan, Barmalzan, and Kosari (2017), Belzunce, Martínez-Riquelme, Pellerey, and Zalzadeh (2016), Belzunce, Ortega, Pellerey, and Ruiz (2007), Li and You (2015), Shanthikumar and Yao (1991), and Zhang and Cheung (2020)).

This thesis targets to add some new joint stochastic orders and illustrate their usefulness, properties, implications, and applications in the research field. These are:

(i) joint hazard rate order ($\leq_{\mathrm{jhr}}$)

(ii) joint reversed hazard rate order ($\leq_{\mathrm{jrh}}$)

(iii) residual stochastic precedence order ($\leq_{\mathrm{rsp}}$)

(iv) inactivity stochastic precedence order ($\leq_{\mathrm{isp}}$)

In Chapter 1, we provide an introductory review on existence of stochastic orders. Some relevant and important discussion on the bivariate stochastic orders are mentioned in this chapter. This chapter also consists of the basic notations and definitions relevant to this thesis.

In Chapter 2, we introduce a new joint stochastic order based on the residual lifetimes of two nonnegative dependent random variables and the stochastic precedence order. We name this order as "residual stochastic precedence order." This order describes the bivariate version of the residual probability order which was introduced by Zardasht and

Asadi (2010). Section 2.2 is based on the relationships between the residual stochastic precedence order and the other existing well-known stochastic orders. We also develop some characterization results of this new stochastic order. In Section 2.3, we discuss some preservation properties of this order and provide some results. In addition, we study one of its possible applications in reliability theory, which is given in Section 2.4. For this, we compare the lifetimes of the two series systems, each consisting of two components, and having the lifetimes $\wedge\{X_1, X_2\}$ and $\wedge\{X_1, X_3\}$, respectively with respect to the residual stochastic precedence order.

Chapter 3 is devoted to the extension of inactivity probability order, defined by Abouelmagd, Hamed, Ebraheim, and Afify (2018), to the case of nonindependent random variables. It is done by defining a new joint stochastic order based on the inactivity times of two nonnegative dependent random variables and the stochastic precedence order. We name it "inactivity stochastic precedence order." In Section 3.2, we recall some useful definitions and implications of univariate stochastic orders as well as bivariate stochastic orders used in this chapter. Aim of Section 3.3 is to consider the relationships between this new stochastic order and the other existing well known stochastic orders. Some characterization results of this order have also been discussed in this section. In Section 3.4, we develop preservation properties of the new stochastic order and give the results. Section 3.5 is devoted to discuss this order using real data set as well as simulated data with the help of few examples.

Motivated by the importance of joint stochastic orders, we consider two new joint stochastic orders named as "joint hazard rate order" and "joint reversed hazard rate order," which are defined by Misra, Gupta, and Misra (2020) and Misra, Gupta, and Chanchal (2020), respectively, in Chapter 4. In Section 4.2, we examine the relationship of two new stochastic orders with the other existing well-defined stochastic orders, and also we discuss an important result with the help of two newly defined stochastic orders which are the residual stochastic precedence order and the inactivity stochastic precedence order. Some implication results have also been discussed.

In Chapter 5, we try to discuss applications of the residual stochastic precedence order and the inactivity stochastic precedence order, which are defined by Misra, Gupta, and Misra (2020) and Misra, Gupta, and Chanchal (2020), respectively, on Covid-19 data. We consider the Covid-19 data over different countries in the world and try to find out the COVID-19 effect on males and females by using these two stochastic orders. Section 5.2 is based upon the description of data which is obtained from the website `https://globalhealth5050.org/covid19/sex-disaggregated-data-tracker/` and this data is updated as on October 20, 2020. Also, it is used to perform the analysis and statistical methods. In Section 5.3, we calculate the descriptive statistics of the Covid-19 data and obtain some results by using R-software, and compare the results from same data which is till May, 2020. We also discuss the interpretations of these results.

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