INTERFACIAL STABILITIES IN CONFINED GEOMETRIES

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ABSTRACT

Instability phenomena appear in various disciplines including astrophysics, applied mathematics, geophysics, biology, physics, and oceanography. The transformation of fluid flow from laminar to turbulent is important, which finds applications in both engineering and natural sciences. The stability of a system under disturbance refers to its ability to return to its initial state, which indicates stability. Conversely, if the disturbance increases, causing the system properties to change, the system is considered to be unstable in response to the disturbance.

Hydrodynamic stability examines the stability and instability of fluid motions. A stable flow is consistent and observable, while an unstable flow represents a transitional state that may lead to another flow pattern or turbulence. Therefore, hydrodynamic stability is a critical area of study within fluid mechanics. The transition of fluid flows from laminar to turbulent has significant applications in engineering and natural environments. Instability phenomena are also prevalent in fields such as astrophysics, applied mathematics, geophysics, biology, physics, and oceanography. A fluid in motion or at rest can become unstable due to some perturbation, causing its flow characteristics to deviate from the original state. A system is considered stable with respect to a perturbation if the applied disturbance decays and the system returns to its original position. Conversely, if the perturbation amplifies and alters the system's characteristics, the system is deemed unstable. A fluid flow system is regarded as stable if it remains stable under all possible disturbances, while it is considered unstable if there exists at least one mode of disturbance that induces instability.

The hydrodynamic stability theory examines how a laminar flow reacts to a disturbance with modest or moderate amplitude. The flow is considered stable if it returns to its initial laminar state, but unstable if the disturbance intensifies and changes the laminar flow into a new state. In that circumstance, the flow is deemed unstable. Although instabilities frequently cause turbulent fluid motion, they can also change the flow into a different laminar state, which is typically more complex. The mathematical examination of the evaluation of disturbances superposed on a laminar base flow is the subject of stability theory.

In real-world applications, a perturbation is applied to the field variables before they are re-inserted into the governing equations to assess the stability of a particular flow field concerning minor changes in the system's physical parameters. In the case of linear stability analysis, the perturbations are chosen so that nonlinear term combinations are disregarded from the governing equations, and a subsequent linearization technique reduces the field equations to a mathematical issue. Some key concepts in hydrodynamic stability are given as follows;

LINEAR STABILITY ANALYSIS: This approach involves studying small disturbances or perturbations around a base flow to determine whether they grow or decay over time. Linear stability analysis provides insights into the initial stages of instability and is often used to derive stability criteria for different flow configurations.

NONLINEAR STABILITY ANALYSIS: Unlike linear stability analysis, nonlinear stability analysis considers larger disturbances that may lead to nonlinear effects such as turbulence. It focuses on understanding the evolution of disturbances beyond the linear regime and is crucial for predicting the long-term behavior of fluid flows.

TRANSITION TO TURBULENCE: Understanding the transition from laminar to turbulent flow is a fundamental problem in hydrodynamic stability. Turbulence can arise due to various mechanisms, including shear instabilities, centrifugal instabilities, and boundary layer separation. Investigating the conditions under which turbulence emerges is essential for improving our ability to control and predict fluid flows in practical applications.

APPLICATIONS: Hydrodynamic stability theory finds applications in diverse areas such as aerospace engineering, oceanography, atmospheric science, and industrial processes. For example, it is used to design more efficient aircraft wings, optimize the performance of marine vessels, and enhance mixing processes in chemical reactors.

INTERFACIAL INSTABILITY

Interfacial instability occurs when the boundary between two distinct phases or fluids becomes unstable due to various factors like differences in density, velocity, temperature, or concentration. This instability can lead to complex patterns, mixing, and turbulence at the interface. It is a critical concept in fluid dynamics and material science, impacting processes in natural phenomena, industrial applications, and technological developments. Understanding and managing interfacial instability is essential for optimizing the performance and efficiency of systems involving multi-phase flows or interfaces.

There are various types of instabilities that occur at interfaces. Some of them are discussed as follows;

Rayleigh-Taylor Instability

The Rayleigh-Taylor instability (RTI) is the instability of an interface involving two fluids having distinct densities, which arise when the heavy fluid lies on the top of the lighter fluid. The Rayleigh-Taylor instability plays an important role in many natural processes ranging from coastal upwelling, which helps to renew the nutrients near the surface of the sea.

Capillary instability

Capillary instability develops when surface tension exerts a force against a liquid cylinder in an infinite fluid. Film boiling, Liquid dispensers, and inkjet printers depend on the capillary instabilities of the two fluids' interface. Boiling and condensation operations, as well as several chemical and metallurgical processes, are examples of gas-liquid interactions in industrial applications.

Potential flow of incompressible fluids refers to a flow regime governed by Laplace's equation and characterized by irrotational velocity fields. It serves as a fundamental approximation in fluid dynamics, widely used due to its simplicity and applicability in various engineering and scientific contexts. According to the Helmholtz decomposition theorem, any solution of the Navier-Stokes equations can be decomposed into a rotational component and an irrotational component that individually satisfy Laplace’s equation. The theory of purely irrotational flows in viscous fluids is an effective approximation, particularly suitable for describing gas-liquid flows with high-viscosity liquids at low Reynolds numbers. This theory competes favorably with the traditional theory of irrotational flows in inviscid fluids.

In viscous potential flow analysis, a free surface considers viscous stresses through normal stress balance at the surface, neglecting tangential stresses. The present research work focuses on “**INTERFACIAL STABILITIES IN CONFINED GEOMETRIES**” using viscous potential flow theory, normal mode technique, and MATLAB-based computational programming. The study investigates the stability of interfaces in viscous-viscoelastic fluids across planar, cylindrical, and spherical geometries. The Newton-Raphson method is employed to solve the dispersion relation for critical wave numbers, and various graphs are generated to illustrate the behavior of flow variables concerning perturbation growth rates and wave numbers.

Chapter 1 presents a brief introduction to the general stability theory, Rayleigh-Taylor instability, and capillary instability along with some definitions and basic equations related to stability analysis. Various related studies are described by various authors in this field and a summary of the thesis is provided.

In **Chapter 2**, the focus is on the dynamics of interfaces between viscous-Rivlin-Ericksen fluids using linear stability theory, emphasizing mass and heat transfer across interfaces. The upper region comprises Rivlin-Ericksen fluid contrasted with a lower viscous fluid, exhibiting instability influenced by gravitational acceleration due to a top-heavy configuration, primarily governed by Rayleigh-Taylor instability. This study explores two-dimensional interfaces, applying viscous potential flow theory to establish relationships between perturbation growth and wave number. The critical stability condition derived involves the heat transport coefficient and wave number, highlighting how heat and mass transfer processes stabilize the interface. The viscoelastic coefficient enhances stability, while increased thickness of the viscoelastic fluid induces instability, with destabilizing influences observed in Atwood and Weber numbers.

Chapter 3 investigates interface dynamics between a viscous fluid and Walter's B viscoelastic fluid in planar configurations, employing irrotational flow theory. This interface facilitates heat and mass transfer, with linear stability theory establishing a direct relationship between perturbation growth and wave number. An implicit stability criterion is derived and analyzed using the Newton-Raphson numerical scheme, depicting interface behavior across non-dimensional parameters like Atwood, Weber, Froude, and Reynolds numbers. Increased heat transport delays instability, with the Newtonian fluid interface proving more stable than the Walter's B fluid interface.

Chapter 4 applies linear stability principles to analyze Rayleigh-Taylor instability at interfaces between power-law viscoelastic fluids and inviscid gases. The power-law viscoelastic fluid, above the gas, allows heat transfer between phases. An implicit dispersion relationship, expressed in terms of growth rate parameters, is solved using the Newton-Raphson method. Various plots illustrate how flow variables influence interface stability, where increased heat transfer delays instability onset. The power-law fluid interface exhibits greater stability than the inviscid fluid interface but less than its Newtonian counterpart. Higher power-law indices enhance stability, while denser power-law fluids diminish it, with both consistency coefficient and viscosity contributing to interface stabilization.

In **Chapter 5**, capillary instability theory is applied to interfaces between power-law viscoelastic liquids and inviscid gases within an annular region bounded by concentric cylinders. Linear stability analysis yields a dispersion relation, elucidating how flow parameters affect interface stability through plotted results. Enhanced heat transport stabilizes the system, while interfaces with power-law liquids prove more unstable compared to Newtonian fluids.

Chapter 6 conducts a linear instability analysis of capillary instability at interfaces between Oldroyd B viscoelastic liquids and viscous compressible fluids within an annular region defined by rigid cylinders. Potential flow theory models the Oldroyd-B liquid within a saturated porous medium, deriving a third-order expression for growth rate parameters, numerically evaluated to reveal that porous media decelerate disturbance growth. Increased porosity, however, destabilizes interfaces, while heightened heat and mass transport promote stability.

Chapter 7 investigates the stability of spherical interfaces formed by combinations of viscous and Oldroyd B viscoelastic fluids using linear stability analysis. The spherical geometry comprises a viscous fluid in the inner sphere and viscoelastic fluid in the outer sphere, modeled using viscous-viscoelastic irrotational theory. Equations are derived and solved mathematically, resulting in a polynomial equation describing perturbation growth, numerically evaluated to show faster growth at viscous fluid interfaces compared to viscoelastic-fluid-medium interfaces. Higher Weissenberg numbers increase interface instability, reflecting fluid viscoelasticity.

The **last Chapter** of the thesis is related to the conclusions and future scope of the work. In the present work, the stability results are achieved utilizing the linear theory of stability analysis. The nonlinear analysis of interfacial stability problems is very important because the governing equations describing these flows are nonlinear in nature. The same problems can be studied through the nonlinear analysis of stability theory.