

# HEAT TRANSFER IN NANOFUIDS UNDER VARIOUS CONFIGURATIONS

## ABSTRACT OF THESIS

Submitted to  
Babasaheb Bhimrao Ambedkar University  
(A Central University)

Lucknow



for the Award of the Degree of

**Doctor of Philosophy**

in

**MATHEMATICS**

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**2024**

# ABSTRACT

Everything in the galaxy that has some mass and occupies space is known as matter. Matter is categorized into four major parts, solid, fluid, plasma and made matter. Our focus in this thesis is only on Fluids and its properties. Heat transfer describes the flow of heat (thermal energy) due to temperature differences and the subsequent temperature distribution and changes. The study of transport phenomena concerns the exchange of momentum, energy, and mass in the form of conduction, convection, and radiation. Representation of heat transfer is shown in Fig. (1).

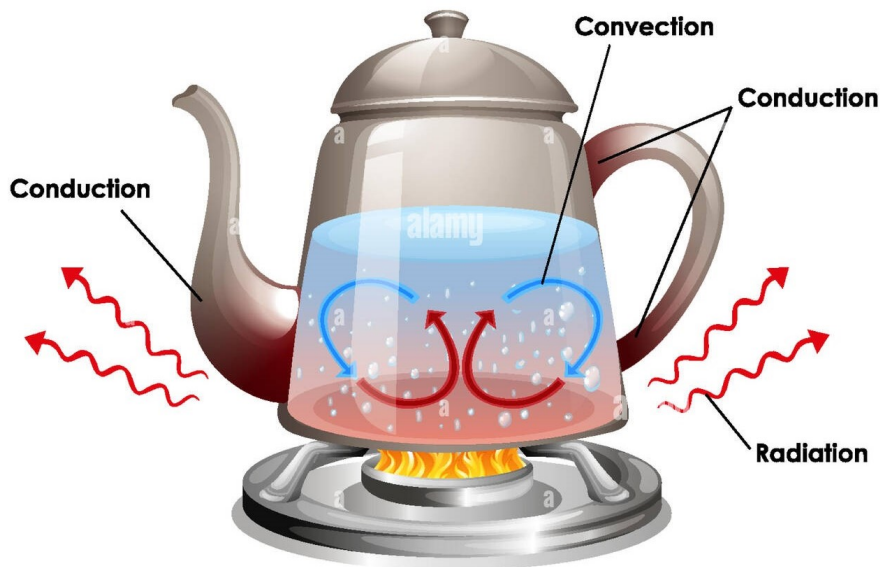


Figure 1: Representation of heat transfer

- **Conduction:** Conduction is a way of heat transfer in which heat is transferred from the hot end of the body to the cool end of the body by direct contact due to temperature differences, whether the body may or may not move. Examples of conduction are heating rod, melting chocolate etc.
- **Convection:** Convection is a way of heat transfer in which heat is transferred from one place to another by the movement of the fluid particle due to their density difference. Examples of convection are ocean currents, boiling water etc.

- **Radiation:** Radiation is the way of heat transfer in which heat is transformed from one spot to another without a medium. In the radiation, heat is transformed in the form of wave through space. Examples of radiation are electromagnetic radiation, particle radiation, gravitational radiation, acoustic radiation etc.

The objective of this thesis entitled “**HEAT TRANSFER IN NANOFUIDS UNDER VARIOUS CONFIGURATIONS**” is to investigate the heat transfer as well as mass transfer in nanofluids which are the suspension of nanoparticles into some basefluid (Choi, 1995), under the various configurations. Both, onset of convection, and transport of heat and mass have been analysed to get a better insight of the considered problems using Galerkin’s method, truncated Fourier series method, and RKF-45 method. The Lorenz model has been used for the investigation. The effects of throughflow, gravity modulation, magneto field, etc., over convective instability is also examined. The considered problems can have various industrial, engineering, and medical applications such as in automobile industry, energy savings, electronic devices, etc. The thesis comprises seven chapters which are described as follows:

**Chapter 1** consists of several basic definitions and concepts which are used in the entire thesis. The basic idea of nanofluids, mathematical models, Hele-Shaw cell, throughflow, various analytical and numerical methods used to solve the problems, applications, literature survey, etc. are covered in this chapter.

In **Chapter 2**, the effect of gravity modulation and throughflow on thermal instability in a Hele-Shaw cell filled with Oldroyd-B nanofluid is investigated using both linear and nonlinear analyses and plotted streamlines, isotherms and isohaline to visualize the conduction, convection, and steady states. Both linear and nonlinear stability analyses have been presented to depict the onset of convection and heat and mass transfer. It has been determined that the effects of throughflow, gravity modulation, Deborah number, retardation parameter are essential to the mass and heat transport in the system. Results of both Oldroyd-B nanofluid and ordinary nanofluid

are similar in linear analysis. Both heat and mass transfer increase while increasing the values of throughflow and Deborah number. On increasing the value of Hele-Shaw number both heat and mass transfer decrease in the system. Increasing the retardation parameter decreases the heat and mass transport in the system.

In **Chapter 3**, a weakly nonlinear stability analysis has been done to know about heat and mass transport in the system for unsteady case using thermal difference. Nonlinear stability analysis is carried out using the truncated Fourier series method. Here, temperature has been considered as constant and nanoparticles flux as zero at the upper and lower boundaries of the system. The effect of Hele-Shaw number, Deborah number, retardation parameter, interphase heat transfer coefficient, modified thermal capacity ratio, thermal diffusivity ratio, amplitude and frequency of gravity modulation on the heat and mass transfer are depicted graphically. Mathematica NDSolve and the Runge-Kutta-Fehlberg (RKF-45) methods has been used to compare the results of heat and mass transfer in the system. Heat and mass transport for both (liquid and particle phase) square waveform gravity modulation is found to be more than under sine waveform and triangular waveform gravity modulation.

In **Chapter 4**, examines the impact of throughflow with G-jitter on thermal instability in a rotating layer of nano-liquid with magnetic field. Nonlinear stability analysis is carried out using the truncated Fourier series method, whereas linear stability analysis is carried out using the normal mode approach. Mathematica NDSolve and the Runge-Kutta-Fehlberg (RKF-45) methods are used to investigate the heat and mass transfer in the system. Throughflow has a dual impact on the system. The magnetic Chandrasekhar number and Taylor number both have stabilizing effects. In a nonlinear study, the influence of throughflow, magnetic Chandrasekhar number, Taylor number, the frequency of G-jitter, and the amplitude of G-jitter on heat and mass transport are investigated. Heat and mass transport increase on increasing throughflow, which decrease on increasing the value of Taylor number.

In **Chapter 5**, heat and mass transfer in various enclosures filled with Oldroyd-B nanofluid has been investigated under various types of gravity modulation. Normal mode technique is used to study linear stability while truncated Fourier series method

has been used to study nonlinear stability. It is found that heat transfer is faster in tall enclosure than in square and shallow enclosures. Effects of three types of gravity modulation has been investigated in this chapter. Deborah number, retardation parameter, gravity modulation, porous parameter, porosity, and aspect ratio all play important role in heat and mass transfer. Porous parameter and porosity delayed the rate of heat and mass transportation. On increasing retardation parameter, heat and mass transport decreases. Heat transfer as well as mass transfer in the system are enhanced by raising the value of the Deborah number. Further, heat and mass transfer are increased in the system by the aspect ratio.

In **Chapter 6**, the impact of three different types of gravity modulation (sinusoidal, saw-tooth and day-night function) on the instability of micro-polar nanofluid filled within Hele-Shaw cell has been investigated. Nonlinear stability analysis uses truncated Fourier series method, while for linear stability analysis normal mode method has been used. The micro-polar parameter and Hele-Shaw number have stabilizing effects, whereas Lewis number and nano-particle concentration Rayleigh-number have destabilizing effects. In nonlinear stability analysis, the impact of micro-polar parameter, Prandtl number, the frequency of gravity modulation, the amplitude of gravity modulation, and other parameters has been examined on heat and mass transfer. Both heat and mass transport decreases in the region while increasing the micro-polar parameter and Hele-Shaw number. On increasing frequency of gravity modulation, heat/mass transport increases. The Runge-Kutta-Fehlberg (RKF-45) and Mathematica NDSolve methods are used to compare the results on heat and mass transport. The day night function has more heat and mass transport than sinusoidal and saw-tooth functions.

In **Chapter 7**, heat and mass transfer in a micro-polar nanofluid with through-flow under several types of gravity modulation (sine, square and triangular waveform) has been investigated using both linear and nonlinear analyses. While nonlinear stability analysis is carried out using the truncated Fourier series method, linear stability analysis has been done using the normal mode method. Plots have been drawn for streamlines, isotherms and isohaline to visualize the conduction, convection, and

steady states. The system is stabilized by the effects of the micro-polar parameter and the Hele-Shaw number. More heat and mass transfer has been observed for square waveform than in the sine waveform and triangular waveform. Effect of several parameters, such as micro-polar parameter, Prandtl number, amplitude of G-jitter, Hele-Shaw number, etc. has been investigated on heat and mass transfer. Heat and mass transfer is increased on increasing the amplitude of modulation while decreases on increasing the Hele-Shaw number.

In the end the thesis is concluded with the possible future scopes of the current work.

The list of publications is as follows:

1. B.S. Bhadauria, **Anish Kumar**, Awanish Kumar and S.N. Rai, “The combined effect of gravity modulation and throughflow on thermal instability in the Hele-Shaw cell filled with Oldroyd-B nanofluid”, published in the *Journal of Nanofluids*, vol. 12, 1681-1691, 2023. (Chapter 2)
2. **Anish Kumar**, B.S. Bhadauria, Awanish Kumar and S.N. Rai, “Effect of different types of gravity modulation on the instability of micro-polar nano-fluid of finite extent in horizontal directions”, published in the *Chinese Journal of Physics*, vol. 84, 312-329, 2023. (Chapter 6)
3. **Anish Kumar**, B.S. Bhadauria, “Nonlinear exploration of Oldroyd-B nano-liquid filled in Hele-Shaw cell under several types of gravity modulation with a thermal difference”, Published in the *AIP Conference Proceedings, Scopus Indexed*, vol. 3025, 030008, 2024. (Chapter 3)
4. **Anish Kumar**, B.S. Bhadauria, Brijesh Kumar Singh, and Awanish Kumar, “The combinatorial impact of through-flow and G-jitter on instability exploration of a rotating layer of nano-liquid with the magnetic field”, Published in the *International Conference on Mathematical Modelling, Applied Analysis and Computation, Cham: Springer Nature Switzerland*, 343-367, 2024. (Chapter 4)

## Under Revision

1. **Anish Kumar**, B.S. Bhadauria, Ismail, S.N. Rai, “Thermal instability on heat and mass transfer of Oldroyd-B nanofluid with several types of gravity modulation in various enclosures”, Under Revision in the *Chinese Journal of Physics*, 2024. (Chapter 5)

## Communicated

1. **Anish Kumar**, B.S. Bhadauria, Ismail, “Study of heat and mass transfer of several types of G-jitter and throughflow on the micropolar nanofluid under the Hele-Shaw cell”. (Chapter 7)

## References

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