INSTABILITY IN NANOFLUIDS UNDER LTE AND LTNE

ABSTRACT
of
THESIS

Submitted to
Babasaheb Bhimrao Ambedkar University
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Lucknow

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Under the Supervision of
Prof. B. S. BHADAURIA

Research Scholar
ANURAG SRIVASTAVA
Enrollment No. 1057/17

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SCHOOL FOR PHYSICAL SCIENCES
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VIDYA VIHAR, RAEBARELI ROAD, LUCKNOW-226 025
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ABSTRACT

Hydrodynamic systems have an innate characteristic that they are not always able to sustain themselves in their initial state against the small disturbances in the parameters defining them. This characteristic gives rise to the concept of hydrodynamic instability.

Thermal instability appears when the temperature (T) of the system crosses a particular critical value. In Fig. 1, two infinitely extended horizontal parallel plates, filled with a fluid, are considered, 'd' distance apart. The temperature of the lower plate is higher than the upper plate. When the temperature across the boundaries is below the critical temperature (T_c), heat transport is due to conduction and the system is said to be *stable*. At the critical temperature, the process of convection sets

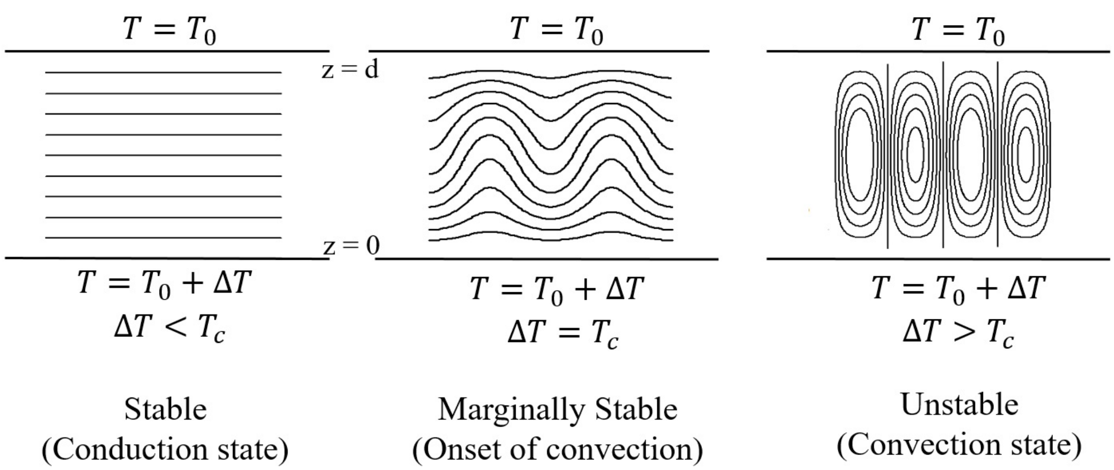

Figure 1: Thermal instability

in, and the system is said to be *marginally stable*. Beyond the critical temperature, fully developed convection cells can be observed and the system is said to be *unstable* (Chandrasekhar, 1981).

The objective of this thesis entitled "**INSTABILITY IN NANOFLUIDS UNDER LTE AND LTNE**" is to investigate the thermal instability in nanofluids which are the suspension of nanoparticles into some basefluid (Choi, 1995), under the influence of local thermal equilibrium (LTE) and local thermal non-equilibrium (LTNE) among the different phases viz fluid phase, nanoparticle phase, and solid-matrix (porous media) phase. Both, onset of instability (convection), and transport of heat and mass have been analysed to get a better insight of the considered problems using normal mode technique, Galerkin's method, truncated Fourier series method, and RKF-45 method. The impact of various other parameters, like through-flow, internal heating, modulated gravity, temperature dependent viscosity, rate of rotation, etc., over convective instability is also examined. The considered problems can have various industrial, engineering, and medical applications such as in automobile industry, electronic devices, energy savings, human blood circulatory system, etc., because of the enhanced thermal conductivity of nanofluids, very small size of nanoparticles, and the choice of different types of nanoparticles and basefluids. The thesis comprises seven chapters which are described as follows:

Chapter 1 consists of various fundamental definitions and concepts used in the entire thesis. The basic idea of nanofluids, their preparation methods, mathematical models, applications, porous medium, various analytical and numerical methods used to solve the problems, literature survey, etc. are included in this chapter.

In **Chapter 2**, the combined effect of internal heating and through-flow in a porous medium layer saturated by nanofluid is studied under local thermal non-equilibrium. Free-Free isothermal boundaries have been considered for the analysis. The Brinkman model is used for the porous medium. The nanofluid is assumed to be incorporated with the effect of thermophoresis and Brownian motion. The particles, fluid, and solid-matrix are at different temperatures i.e. a three temperature model has been used for LTNE (Local Thermal Non-Equilibrium). In order to investigate the onset of convection and heat/mass transfer both linear and nonlinear stability analysis have been featured. It is concluded that effect of internal heating and through-flow play a vital role in the heat and mass transfer inside the system.

In **Chapter 3**, the regular and chaotic convection in top-heavy and bottom-heavy Boussinesq nanofluid confined between two horizontal layers heated underneath is investigated. A five-dimensional Lorenz like nonlinear model is obtained using Galerkin technique. This 5-D model possesses two major characteristics viz reflection symmetry and dissipation. The bifurcation diagrams and the phase portraits are used to analyse the characteristics of the dynamical system. The onset of chaos in ordinary fluid, nanofluid bottom-heavy, and nanofluid top-heavy configurations take place at scaled Rayleigh number (R) equals 24.73684211, 26.98908141, and 32.24788322 respectively. The case of ordinary fluids has also been deduced from the five-dimensional model. Weakly nonlinear analysis is also done in order to understand the heat and mass transport inside the system with the variation of scaled Rayleigh number. It is observed that the convection starts earlier in case of top-heavy configuration of nanofluid while gets delayed in bottom-heavy configuration as compared to an ordinary fluid. Moreover onset of chaos gets delayed in case of nanofluids. Another major finding of the study is that the system turns chaotic for higher values of scaled Rayleigh number in the top-heavy case as compared to that of in the bottom-heavy case of nanofluid.

In **Chapter 4**, linear as well as weakly nonlinear analyses have been done to understand the onset of convection and heat and mass transport in a composite nanofluid horizontal layer heated from below under LTNE effect. Two different types of nanoparticles are assumed to be suspended in the basefluid. Both the nanoparticles and the basefluid are taken to be at different temperature and therefore three temperature model is used for LTNE. Thermal Rayleigh number is evaluated analytically using Galerkin's approach while nonlinear analysis is done numerically. The effect of both top-heavy and bottom-heavy configurations of nanoparticles over convective instability is examined. It is found that the system is more stable in case of bottom heavy configuration as compared to that of top heavy case. Moreover, the effect of LTNE depends upon the concentration of nanoparticles significantly. A comparison between streamlines, isotherms, and isohalines for both LTE and LTNE cases is also presented.

Chapter 5 deals with the effect of gravity modulation and variable viscosity on Darcy-Bénard convection in a porous medium saturated with a nanofluid under LTNE with free-free and isothermal boundaries. Both linear as well as nonlinear stability analyses have been carried out. Non-linear analysis provides Nusselt numbers for fluid, nanoparticle, solid matrix and nanoparticle concentration as function of time.

Local thermal non-equilibrium among different phases is governed by three separate temperature equations. Effect of various parameters on the onset of convection has been investigated. Effects of frequency and amplitude of gravity modulation on heat and mass transfer has been studied and depicted graphically.

In **Chapter 6** Rayleigh-Bénard Convection is studied with water-copper (W-Cu) nanofluid using local thermal non-equilibrium approach for different temperature of baseliquid and nanoparticles under the influence of temperature dependent viscosity and g-jitter. LTNE is comprising of two temperature model. Linear analysis is done using normal mode method and Galerkin's technique while nonlinear analysis is carried out numerically using truncated Fourier series method. The numerical solutions obtained from NDSolve of Mathematica are verified by Runge-Kutta-Fehlberg (RKF-45) method. Both unsteady (time-dependent) and steady (time-independent) results are presented in the nonlinear part. The actual values of the non-dimensional parameters for W-Cu nanofluid are used for the calculation. The condition of zero nanoparticle flux is taken at the boundaries. It is observed that transport of heat in the LTNE case is mainly because of the particle phase and not because of the fluid phase. It is also found that modified thermal diffusivity ratio has a notable role over onset of convection. Moreover, the thermorheological parameter of viscosity and modulated gravity are more effective in the case of LTNE as compared to the LTE case.

In **Chapter 7**, the convective instability of blood-copper (B-Cu) nanoliquid filled in a porous medium is investigated using Casson model. Because of the antibacterial characteristics of copper, the thermal instability of B-Cu nanoliquid plays a crucial role in the field of medical science specifically in cardiovascular system. A thermal non-equilibrium condition is taken for the nanoliquid and porous matrix. The Brinkman model is used because of the high porosity of the material. The influence of rotation and through-flow (both positive and negative) over the instability has also been examined. The amount of through-flow and rate of rotation is respectively governed by Péclet number and Taylor number. A comparison between the impacts of through-flow and rate of rotation is done and it is found that through-flow is more effective than the rate of rotation, to delay the onset of convection. Both steady and unsteady weakly nonlinear analyses are performed to understand the heat transport in the system. It is concluded that Casson nanofluid parameter has both stabilizing and destabilizing impact depending upon the rate of rotation and therefore the current work can be possibly utilized in both the places where heat removal and heat conservation is required. One more important thing is observed that the effect of LTNE can also be regulated using Casson nanofluid parameter.

At last the thesis is concluded with the possible future scopes of the present work.

The list of publications is as follows:

Journals

1. B. S. Bhadauria, **Anurag Srivastava**, "Combined Effect of Internal Heating and Through-flow in a Nanofluid Saturated Porous Medium under Local Thermal Non-Equilibrium". *Journal of Porous Media* 25(2), 75-95, 2022.

2. **Anurag Srivastava**, B. S. Bhadauria, "Heat and Mass Transfer, and Chaotic Convection in Nanofluids". *Journal of Nanofluids* 12, 1-15, 2023.

3. **Anurag Srivastava**, B. S. Bhadauria, "Darcy-Bénard Convection in a Nanofluid Saturated Porous Medium under Local Thermal Non-Equilibrium, Variable Viscosity, and Gravity Modulation". *GANITA* 72(1), 233-247, 2022.

4. **Anurag Srivastava**, B. S. Bhadauria, "Thermal Instability of Blood-Copper Casson Nanoliquid Saturated Porous Medium under LTNE, Rotation, and Through-flow". *Journal of Porous Media.* (Accepted)

Conference Proceedings

1. **Anurag Srivastava**, B. S. Bhadauria, "Convective Instability in a Composite Nanofluid Layer under Local Thermal Non-Equilibrium" in, Frontiers in Industrial and Applied Mathematics, *Springer Proceedings in Mathematics & Statistics* 410 (Book Series), DOI: https://doi.org/10.1007/978-981-19-7272-0_9, 2023.

Communicated

1. **Anurag Srivastava**, B. S. Bhadauria, “Rayleigh-Bénard Convection in Water-Copper (W-Cu) Nanofluid under Local Thermal Non-Equilibrium, Temperature Dependent Viscosity, and Modulated Gravity: The Modified Results”.

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