Mathematical modeling and analysis of emission and mitigation of global warming gases

ABSTRACT
OF
THESIS

Submitted to
Babasaheb Bhimrao Ambedkar University
(A Central University)
Lucknow

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प्रज्ञा शील करुणा
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for the Award of the Degree of

Doctor of Philosophy

in

MATHEMATICS

Under the Supervision of
Dr. MAITRI VERMA

Research Scholar
ALOK KUMAR VERMA
Enrollment No. 1330/18

DEPARTMENT OF MATHEMATICS
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ABSTRACT

Global warming is one of the most important social, economic, and environmental issues of the present time. It is a phenomenon in which the mean global surface temperature rises because of increased amounts of greenhouse gases in the Earth's atmosphere. Global warming has largely affected humans by affecting the food and water supply and through the increase in heat-related illness, vector-borne and water-borne diseases. The anthropogenic climate changes are slowly destroying the ecosystem diversity by increasing the vulnerability of rare species to get extinct. Global warming has affected the coastal ecosystem through the degradation of coastal habitats, saltwater intrusion into coastal aquifers, damage of the coastal infrastructure, increase in the coastal floods caused by storm surges associated with cyclones and tsunamis, etc. Since global warming is causing many irreversible alterations in the environment and climate, therefore it is necessary to design and implement policies for the mitigation of global warming. A comprehensive understanding of the effects of various crucial factors influencing the emission and control of greenhouse gases, particularly carbon dioxide (CO_2) and methane (CH_4), which are the most abundant greenhouse gases, is requisite. This thesis work aims to formulate mathematical models to explore the impact of various crucial factors on the emission of greenhouse gases and investigate the strategies for the control of atmospheric concentrations of greenhouse gases.

The research work done in this thesis is organized into seven chapters. **Chapter 1** is an introductory chapter that includes a brief introduction, a review of relevant literature, and the methodology used to analyze the models.

The energy sector is one of the prime anthropogenic sources of most abundant greenhouse gas, carbon dioxide. **Chapter 2** presents a mathematical model to examine the impact of mitigation options to control the CO_2 emission from the energy sector on the atmospheric level of CO_2. In the modeling process, two kinds of mitigation options, one that cut down the CO_2 emission rate from energy generation and another that cut down the energy consumption rate are considered. The quali-

tative analysis of the model is performed to grasp the system's long-term behaviour. The model parameters are fitted to actual data on CO_2 concentration, global energy consumption, and population. It is demonstrated that the equilibrium level of CO_2 decreases when the efficiencies of mitigation options to lower the energy consumption rate and CO_2 emission rate per unit energy use increase. The optimal control model is proposed to investigate the strategies which bring optimal reduction in CO_2 level at minimal mitigation cost by considering the efficiency of mitigation options to lower the CO_2 emission and energy consumption rates as time-dependent control variables. Numerical simulations are carried out to confirm the theoretical findings and determine the optimal control strategies under various settings of energy consumption rate, carbon dioxide emission rate, and maximum efficiencies of mitigation options to reduce CO_2 emissions. It is found that development and implementation of more efficient mitigation techniques, and the adoption of low-carbon energy sources lead to a reduction in the mitigation cost.

Chapter 3 presents a nonlinear mathematical model to study the impact of renewable energy technologies on the control of the atmospheric concentration of carbon dioxide. The model is comprised of a set of nonlinear differential equations that capture the dynamic relationship between the human population, carbon dioxide level, energy use, and renewable energy technologies. The mathematical analysis of the model is presented to explore the long-term impact of renewable energy technologies on the control of atmospheric CO_2 level. Model analysis shows that the increase in the rate of implementation of renewable energy technologies and the maximum efficiency of renewable energy technologies to cut down CO_2 emissions lead to a decline in the equilibrium level of CO_2 in the Earth's atmosphere. The implementation cost of renewable energy technologies is one of the main barriers to their large-scale implementation. This study identifies the strategies for the optimal reduction in CO_2 level while minimizing the cost of implementation of renewable energy technologies by using the optimal control theory. Sensitivity analysis is presented to depict the effect of changes in the key parameters over the system's dynamics. Numerical simulations are performed to complement the analytical findings.

The loss of global forest cover as a result of human activity has contributed considerably to the rise in carbon dioxide levels. Reforestation and afforestation are approaches to lower the atmospheric concentration of CO_2; nevertheless, large-scale plantations have several demographic, ecological, and economic constraints. In this scenario, plantation of genetically engineered trees that absorb more CO_2 from the atmosphere may help to meet the CO_2 mitigation goals. **Chapter 4** presents a mathematical model to study the impact of plantation of genetically modified trees on the reduction of the atmospheric level of carbon dioxide. A comprehensive qualitative analysis of the model is performed. It is found that an increase in the plantation efforts leads to declination in the atmospheric carbon dioxide level. The model is calibrated to fit the actual data of atmospheric carbon dioxide concentration, human population, and forest area. The influence of important factors on the dynamics of forest cover and atmospheric CO_2 gas is demonstrated using numerical simulations. The optimum control theory is used to investigate the mitigation policies to reduce the CO_2 concentration in the atmosphere while minimizing the implementation cost of plantation programmes.

Over the past few decades, fossil fuel consumption has increased drastically to fulfill the energy demand of the growing population and economy. The population pressure has not only contributed to the increase in fossil fuel consumption but also accelerated the deforestation for industrial, agricultural, and infrastructure expansion. **Chapter 5** presents a non-linear mathematical model to study the effect of an increase in fossil fuel use and deforestation due to population pressure on the atmospheric carbon dioxide concentration. Further, the effect of economic efforts applied to reduce the population pressure over the control of atmospheric CO_2 levels is explored. The model analysis shows an upsurge in the equilibrium CO_2 level with an increase in the fossil fuel consumption rate. Further, it is found that if the deforestation rate crosses a critical threshold, the interior equilibrium state of the system loses stability and periodic solutions arise via Hopf-bifurcation. It is shown that at high values of deforestation rate, an increase in the implementation rate of economic efforts applied to reduce the population pressure may cause declination in the amplitude of periodic solutions, and above a critical value of implementation rate of economic effort, the periodic oscillations may die out and CO_2 concentration gets stabilized.

The integrated rice-livestock farming system is an agricultural practice adopted in many developing nations to enhance rice and livestock production while optimizing the use of agricultural waste. However, rice and livestock farming is the prime contributor to the increase in atmospheric level of methane, the second most potential global warming gas after carbon dioxide. In this scenario, it is important to comprehend the role of methane mitigation options in the abatement of methane emissions from the integrated rice-livestock farming system. **Chapter 6** presents a nonlinear mathematical model to investigate the effect of mitigation options that curtail methane emissions from the rice-livestock farming system over the reduction of atmospheric level of methane. A qualitative analysis of the model system is carried out to explore its long-term behaviour. The conditions under which an increase in the implementation rate of mitigation options leads to the reduction in the atmospheric level of methane are derived. The model parameters are estimated using secondary data of the atmospheric methane concentration, rice yield, and livestock population. Sensitivity analysis is presented to evaluate the influence of perturbations in values of key parameters over the system's dynamics. Numerical simulations are performed to verify the theoretical findings.

The enhanced level of greenhouse gases in the atmosphere must be lowered to attain the climate change mitigation goal. For this purpose, many nations are developing and executing various mitigation policies to minimize greenhouse gas emissions. However, a time lag between the creation and implementation of these mitigation strategies can have an impact on their ability to reduce greenhouse gas levels in the atmosphere. **Chapter 7** examines the impact of mitigation strategies and the time lag associated with their deployment on the reduction of atmospheric level of greenhouse gases. In the modeling process, it is considered that mitigation strategies act in two ways: first, they lower the rate of greenhouse gas emissions from anthropogenic sources, and second, they enhance the rate of greenhouse gas removal from the atmosphere. To investigate the long-term behaviour of the suggested model system, a comprehensive stability analysis is performed. It is found that an increase in the implementation rate of mitigation options and their efficiencies to reduce greenhouse gas emissions from point sources and enhance the uptake rate of greenhouse gases lead to a decrease in equilibrium greenhouse gas concentration. It is found that if the delay in the application of mitigation strategies crosses a threshold value, the system enters into limit cycle oscillations via Hopf-bifurcation. The analytical expression for determining the critical value of delay parameter at which the system enters into limit cycle oscillations via Hopf-bifurcation is derived. To determine how changes in key parameters would affect the dynamics of the system, a sensitivity analysis is carried out.

At last, the thesis is concluded with the possible future scopes of the present work.