Implications of Neutrinoless Double Beta Decay

Summary

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Vivek Kumar Nautiyal
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Under the Supervision of
Dr. Ramesh Chandra

Department of Applied Physics
School for Physical Sciences
Babasaheb Bhimrao Ambedkar University, Lucknow
U.P., (India) – 226025
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CONTENTS

Chapter 1

Introduction

The discovery of radioactivity by Henri Becquerel in 1896 opened a new dimension in the field of nuclear physics. Alpha, beta and gamma rays are the relics of radioactive decays, and their emissions are governed via strong, weak and electromagnetic interactions respectively. In 1914, James Chadwick reported the continuous energy spectrum of beta radiation. In order to explain this continuous spectrum, Wolfgang Pauli proposed the emission of a chargeless and massless fermion along with beta particle. After the discovery of neutron by J. Chadwick, the particle proposed by Pauli was called *neutrino* (little neutron) by Enrico Fermi. The experimental detection of antineutrino was done by F. Reines and C. W. Cowan during 1954-1956 [Cowan *et al.* (1956)]. The mass as well as nature of neutrinos is not completely known till date.

1.1 Neutrino Mass

1.1.1 Dirac and Majorana neutrino masses

Dirac was the first person who gave the theory of antiparticles in order to explain the negative energy problem. Unlike positron (e^+), antiparticle of a electron, neutral pion (π^0) has its own antiparticle whereas neutral kaon (K^0) has distinct antiparticle. Both pion and kaon are composed of quarks and antiquarks. In order to resolve the discrepancy E. Majorana (1937) gave the concept of particle which is its own antiparticle called as

Majorana particle.

In order to understand the difference between Majorana and Dirac neutrino, first we have to assume that there exist a left handed massive neutrino ν_L. If CPT invariance is valid then there exists a right handed antineutrino $\overline{\nu}_R$. Since the speed of massive particles is less than the speed of light, there is always a choice to select the frame of reference which moves faster than the reference frame in which ν_L exists. Thus, the Lorentz transformation alters the direction of the momentum and keeps its spin unaltered and ν_L turns into right handed ν_R which may or may not be same as $\overline{\nu}_R$. If ν_R is not same, then ν_R has its own particle $\overline{\nu}_L$. Thus, there exist four states (quadruplet state) at a time known as Dirac neutrino ν^D. On the other side, if ν_R is the same particle as ν_L then, there exist only one pair of state. This pair of state is known as Majorana neutrino ν^M.

1.1.2 Neutrino Oscillation

The neutrino oscillation can be studied at accelerators (LSND, KEK and K2K) as well as reactors (KamLAND, CHOOZ and Palo verde). The extra-terrestrial sources, which provide information regarding neutrino oscillation are atmospheric neutrinos, solar neutrinos, neutrinos from supernova explosion. However, the neutrino oscillation data provide only mass squared difference and the actual neutrino mass cannot be extracted. On the other hand, the study of tritium single β decay and $\beta\beta$ decay together can provide sharpest limits on the mass and nature of the electron neutrino.

1.1.3 Experimental aspects of Neutrino oscillations

In 1998, the experimental detection of neutrino oscillations came forward with the observation of up-down asymmetry in atmospheric neutrino muon events through the detection of cherenkov radiation in Super Kamiokande. In 2002, SNO experiments gave the indication of vanishing of solar neutrino ν_e through the observation of the CC and NC reactions. In 2004, the KamLAND reactor neutrino experiment was observed the distortion in $\overline{\nu}_e$ spectrum. The accelerator long-baseline K2K and MINOS experiments were confirmed the results of neutrino oscillations obtained from the atmospheric Super-Kamiokande experiment. Neutrino oscillations were not found in the CHOOZ and Palo Verde reactor experiments. The MiniBooNE experiment doesn't confirm accelerator short-baseline LSND indication. The result provided by LSND needs an existence of sterile neutrinos. All these experiments provides the large data to support the neutrino having masses and mixing among the ν_e, ν_μ and ν_τ neutrinos.

1.2 Double Beta decay

The nuclear $\beta\beta$ decay, which is a rare and remarkable second order process of weak interaction in nature, is expected to proceed through two different modes, namely two neutrino double beta ($2\nu\beta\beta$) decay and neutrinoless double beta ($0\nu\beta\beta$) decay. The half-life for the two neutrino double beta ($2\nu\beta^-\beta^-$) decay was calculated by Maria Goepert-Mayer (1935) at the suggestion of Eugene P. Wigner, in which a nucleus decays as follows

$$ {}^A_Z X \rightarrow {}^A_{Z+2} Y + 2e^- + 2\overline{\nu}_e \qquad (1.1) $$

In 1939, W. Furry conjectured that if neutrino is a Majorana particle [Majorana (1937) and Racah (1937)], the neutrinoless double beta ($0\nu\beta^-\beta^-$) decay could take place. This process takes place as follows

$$ {}^A_Z X \rightarrow {}^A_{Z+2} Y + 2e^- \qquad (1.2) $$

In $0\nu\beta^-\beta^-$ decay, there is no emission of neutrinos, although conservation of energy and momentum required the emission of a neutrino in single β decay. The energy and momentum could be conserved in this decay mode releasing two electrons only. Hence, the $2\nu\beta^-\beta^-$ and $0\nu\beta^-\beta^-$ decay modes are competing processes.

1.3 Nuclear structure aspect of $\beta^-\beta^-$ decay

The $0\nu\beta^-\beta^-$ decay has not been observed so far. Hence, the models predict the half-lives assuming certain value for the neutrino mass or conversely extract various gauge parameters from the observed half-life limits of the $0\nu\beta^-\beta^-$ decay. The reliability of predictions can be judged a priori only from the success of a nuclear model in explaining various observed physical properties of nuclei. The common practice is to calculate the $M_{2\nu}$ to start with, and compare them with the experimentally observed value as the two decay modes involve the same set of initial and final nuclear wave functions.

Over the past few years, several nuclear models have been employed to calculate the $2\nu\beta^-\beta^-$ and $0\nu\beta^-\beta^-$ decay rates in two-nucleon (2n) mechanism. They can be broadly classified into three types, namely shell model and its variants, quasiparticle random phase approximation (QRPA) and its extensions and alternative models. The details about these models have been discussed by Suhonen *et al.* (1998) and Faessler *et al.* (1998).

1.4 Experimental search for $\beta^-\beta^-$ decay

The experimental aspects of $0\nu\beta^-\beta^-$ decay has been reviewed over the years to update its experimental status [Barabash *et al.* (2018), Dell'Oro *et al.* (2015)]. The $0\nu\beta^-\beta^-$ decay has not been experimentally observed till date and aim of all the present experimental activities is to observe this particular decay mode. A few of the promising experimental projects running world wide are CUORE, GERDA, KamLAND-Zen, EXO, SuperNEMO etc.. Other experiments and R&D projects are in the pipeline.

1.5 Objective of the thesis

All the nuclei undergoing $\beta\beta$ decay are even-even type, in which the pairing degrees of freedom play an important role. Hence, it is desirable to have a model which incorporates the pairing and deformation degrees of freedom on equal footing in its formalism. For this purpose, the PHFB model is one of the most natural choices. To examine the explicit role of deformation degrees of freedom vis-à-vis the suppression, the pairing plus quadrupole-quadrupole interaction (PQQ) will be the most appropriate choice.

The PHFB model in conjunction with a pairing plus multipole type of effective two-body interaction has been successfully applied to study $2\nu\beta^-\beta^-$ and $0\nu\beta^-\beta^-$ decay modes.

In the present work the single particle energies used were derived from the Woods-Saxon potential as most of the present nuclear models are using Woods-Saxon single particle energies. In Chapter 2, we have calculated the spectroscopic properties of the nuclei participating in $\beta^-\beta^-$ decay and compared with the available experimental data as a test of the reliability of the wave functions. Subsequently, the PHFB wave functions are employed to study $2\nu\beta^-\beta^-$ decay in Chapter 3. In Chapter 4, we have studied the $0\nu\beta^-\beta^-$ decay of above nuclei. We discuss a number of necessary improvements to be incorporated in the PHFB model for a more reliable study of the $\beta\beta$ decay in Chapter 5.

Chapter 2

Spectroscopic properties of some nuclei in mass range $A = 90 - 150$ participating in $\beta^-\beta^-$ decay

The present chapter is organized in the following Sections. In Section 2.1, a brief outline of the PHFB model has been given. The formalism to calculate the spectroscopic properties has been presented in Section 2.2. We have compared the calculated sub-shell occupation numbers, yrast spectra and electromagnetic properties with available experimental data for nuclei participating in $\beta^-\beta^-$ decay in Section 2.3. Finally, the conclusions are given in Section 2.4.

2.1 The PHFB model

A detailed derivation of the HFB method has been given by Baranger (1963), Villars (1966) and Goodman (1979). The projection technique in the context of HFB method was developed by Onishi and Yosida (1966). The nuclear many-body Hamiltonian is given by

$$H = \sum_{\alpha} \varepsilon_\alpha a_\alpha^+ a_\beta + \frac{1}{4} \sum_{\alpha\beta\gamma\delta} \langle\alpha\beta|\ V\ |\gamma\delta\rangle a_\alpha^+ a_\beta^+ a_\delta a_\gamma \tag{2.1}$$

The essential idea behind the HFB theory is to transform particle coordinates to quasiparticle coordinates through general Bogoliubov transformation such that the quasiparticles are relatively weakly interacting. Essentially, the Hamiltonian H is expressed as

$$H = E_0 + H_{qp} + H_{qp-int} \tag{2.2}$$

where E_0 is the energy of the quasiparticle vacuum, H_{qp} is the elementary quasiparticle excitations and H_{qp-int} is a weak interaction between the quasiparticles. In HFB theory, the interaction between the quasiparticles is usually neglected and the Hamiltonian H is approximated by an independent quasiparticle Hamiltonian. In time dependent HFB (TDHFB) or the quasiparticle random phase approximation (QRPA), some effects of quasiparticle interaction can be included.

2.2 Spectroscopic properties of yrast states

The axially symmetric HFB intrinsic state with $K = 0$ can be written as

$$|\Phi_0\rangle = \prod_{im} \left(u_{im} + v_{im} b^\dagger_{im} b^\dagger_{i\overline{m}} \right) |0\rangle \tag{2.3}$$

where the creation operators $b^\dagger_{im}$ and $b^\dagger_{i\overline{m}}$ are given by

$$b^\dagger_{im} = \sum_\alpha C_{i\alpha,m} a^\dagger_{\alpha m} \qquad and \qquad b^\dagger_{i\overline{m}} = \sum_\alpha (-1)^{l+j-m} C_{i\alpha,m} a^\dagger_{\alpha,-m} \tag{2.4}$$

The wave function $|\Phi_0\rangle$ can be recast into the form

$$|\Phi_0\rangle = N\ exp \left(\frac{1}{2} \sum_{\alpha\beta} f_{\alpha\beta} a^\dagger_\alpha a^\dagger_\beta |0\rangle \right) \tag{2.5}$$

with

$$f_{\alpha\beta} = \sum_i C_{im_\alpha, j_\alpha} C_{im_\beta, j_\beta} \left(\frac{v_{im_\alpha}}{u_{im_\alpha}} \right) \delta_{m_\alpha - m_\beta} \tag{2.6}$$

Here N is a normalization constant. Using the standard projection technique [Onishi *et al.* (1966)], a state with good angular momentum is obtained from the HFB intrinsic state through the following relation.

$$
\begin{aligned}
\left|\Psi_0^J\right\rangle &= P_{00}^J \left|\Phi_0\right\rangle \\
&= \left[\frac{(2J+1)}{8\pi^2}\right] \int D_{00}^J(\Omega) R(\Omega) \left|\Phi_0\right\rangle d\Omega
\end{aligned} \tag{2.7}
$$

We have given expressions to calculate the nuclear spectroscopic properties, namely subshell occupation numbers, yrast spectra, reduced transition probabilities for $B(E2{:}0^+ \rightarrow 2^+)$, deformation parameters β_2 and g-factors $g(2^+)$ in the subsections 2.2.1 – 2.2.4 in the thesis.

2.3 Results and discussions

In case of ^{94,96}Zr, ^{94,96,100}Mo, ^{100,104}Ru, ^{104,110}Pd and ^{110}Cd nuclei, we treat the doubly even ^{76}Sr ($N = Z = 38$) nucleus as an inert core with the valence space spanned by $1p_{1/2}$, $2s_{1/2}$, $1d_{3/2}$, $1d_{5/2}$, $0g_{7/2}$, $0g_{9/2}$ and $0h_{11/2}$ orbits for protons and neutrons. For ^{128,130}Te, ^{128,130}Xe, ^{150}Nd and ^{150}Sm nuclei, the doubly even ^{100}Sn ($N = Z = 50$) nucleus has been treated as an inert core with the valence space spanned by $2s_{1/2}$,$1d_{3/2}$, $1d_{5/2}$, $1f_{7/2}$, $0g_{7/2}$, $0h_{9/2}$ and $0h_{11/2}$ orbits for protons and neutrons. The single particle energies (SPEs) for ^{94,96}Zr, ^{94,96,100}Mo, ^{100,104}Ru, ^{104,110}Pd, ^{110}Cd, ^{128,130}Te, ^{128,130}Xe, ^{150}Nd and ^{150}Sm are derived from Woods-Saxon potential as proposed by Blomqvist and Wahlborn (1960).

We use a Hamiltonian with Pairing plus Quadrupole-Quadrupole plus Hexadecapole-Hexadecapole (*PQQHH)* type of effective two-body interaction. The Hamiltonian is explicitly written as

$$
H = H_{sp} + V(P) + \zeta_{qq}\left[V(QQ) + V(HH)\right] \tag{2.8}
$$

where ζ_{qq} is an arbitrary parameter and the final results are obtained by setting $\zeta_{qq} = 1$. The purpose of introducing ζ_{qq} is to study the role of deformation by varying the strength of *QQHH* interaction. Further, H_{sp}, $V(P)$, $V(QQ)$ and $V(HH)$ denote the single particle Hamiltonian, pairing part, quadrupole-quadrupole part and hexadecapole-hexadecapole of the effective two-body interaction.

The spectroscopic properties of nuclei under study have been calculated in PHFB model using four parametrizations of the effective two-body interaction, namely $PQQ1$,

$PQQHH1$, $PQQ2$ and $PQQHH2$ parametrizations and compared with the experimental data. An overall agreement has been found between theoretically calculated and experimentally observed values.

2.4 Conclusions

To summarize, we have calculated the yrast spectra, reduced $B(E2{:}0^+ \rightarrow 2^+)$ transition probabilities, deformation parameters β_2 and g-factors $g(2^+)$ of ^{94,96}Zr, ^{94,96,100}Mo, ^{100,104}Ru, ^{104,110}Pd, ^{110}Cd, ^{128,130}Te, ^{128,130}Xe, ^{150}Nd and ^{150}Sm isotopes, participating in $\beta^-\beta^-$ decay, and compared them with the available experimental data as a test of reliability of the wave functions.

Chapter 3

$2\nu\beta^-\beta^-$ decay of ^{94,96}Zr, ^{100}Mo, ^{104}Ru, ^{110}Pd, ^{128,130}Te and ^{150}Nd isotopes for the $0^+ \to 0^+$ transition

The present chapter is organized as follows. In Section 3.1, we present the theoretical formalism to calculate the half-life of the $2\nu\beta^-\beta^-$ decay. In Section 3.2, the results of $2\nu\beta^-\beta^-$ decay of ^{94,96}Zr, ^{100}Mo, ^{104}Ru, ^{110}Pd, ^{128,130}Te and ^{150}Nd nuclei for the $0^+ \to 0^+$ transition are given and discussed. In the same section, the effect of deformation on $M_{2\nu}$ is also investigated. Finally, the conclusions are given in Section 3.3.

3.1 Theoretical formalism

The theoretical formalism to calculate the half-life of the $2\nu\beta^-\beta^-$ decay has been given in a number of reviews [Haxton and Stephenson (1984), Doi *et al.* (1985), (1992), Tomoda (1991) and Suhonen and Civitarese (1998)].

The inverse half-life of the $2\nu\beta^-\beta^-$ decay for the $0^+ \to 0^+$ transition is given by

$$[T_{1/2}^{2\nu}(0^+ \to 0^+)]^{-1} = G_{2\nu}|M_{2\nu}|^2 \tag{3.1}$$

where the integrated kinematical factor $G_{2\nu}$ can be calculated with good accuracy [Doi *et al.* (1985), (1992) and Boehm and Vogel (1992)] and the NTME $M_{2\nu}$ is given by

$$\begin{aligned} M_{2\nu} &= \sum_N \frac{\langle 0_F^+ || \boldsymbol{\sigma}\tau^+ || 1_N^+ \rangle \langle 1_N^+ || \boldsymbol{\sigma}\tau^+ || 0_I^+ \rangle}{E_N - (E_I + E_F)/2} \\ &= \sum_N \frac{\langle 0_F^+ || \boldsymbol{\sigma}\tau^+ || 1_N^+ \rangle \langle 1_N^+ || \boldsymbol{\sigma}\tau^+ || 0_I^+ \rangle}{E_0 + E_N - E_I} \end{aligned} \tag{3.2}$$

with

$$E_0 = \frac{1}{2}(E_I - E_F) = \frac{1}{2} Q_{\beta\beta} + m_e \tag{3.3}$$

We have carried out the summation over intermediate states by using the *summation method* given by Civitarese and Suhonen (1993).

3.1.1 NTME in the PHFB model

The expression for the NTME $M_{2\nu}$ of $2\nu\beta^-\beta^-$ decay for the $0^+ \to 0^+$ transition in PHFB model is given as

$$\begin{aligned} M_{2\nu} &= \sum_{\pi,\nu} \frac{\langle \Psi_{00}^{J_f=0} || \boldsymbol{\sigma}\cdot\boldsymbol{\sigma}\tau^+\tau^+ || \Psi_{00}^{J_i=0} \rangle}{E_0 + \varepsilon(n_\pi, l_\pi, j_\pi) - \varepsilon(n_\nu, l_\nu, j_\nu)} \\ &= \left[n_{(Z,N)}^{Ji=0} n_{(Z+2,N-2)}^{J_f=0} \right]^{-1/2} \int_0^\pi n_{(Z,N),(Z+2,N-2)}(\theta) \sum_{\alpha\beta\gamma\delta} \frac{\langle \alpha\beta \,|\boldsymbol{\sigma}_1 . \boldsymbol{\sigma}_2 \tau^+\tau^+|\, \gamma\delta \rangle}{E_0 + \varepsilon_\alpha(n_\pi, l_\pi, j_\pi) - \varepsilon_\gamma(n_\nu, l_\nu, j_\nu)} \\ &\times \sum_{\varepsilon\eta} \frac{\left(f_{Z+2,N-2}^{(\pi)*} \right)_{\varepsilon\beta} \left(F_{Z,N}^{(\nu)*} \right)_{\eta\delta}}{\left[\left(1 + F_{Z,N}^{(\pi)}(\theta) f_{Z+2,N-2}^{(\pi)*} \right) \right]_{\varepsilon\alpha} \left[\left(1 + F_{Z,N}^{(\nu)}(\theta) f_{Z+2,N-2}^{(\nu)*} \right) \right]_{\gamma\eta}} sin\theta d\theta \end{aligned} \tag{3.4}$$

where

$$n^J = \int_0^\pi \left[det \left(1 + F^{(\pi)} f^{(\pi)\dagger} \right) \right]^{1/2} \left[det \left(1 + F^{(\nu)} f^{(\nu)\dagger} \right) \right]^{1/2} d_{00}^J(\theta) sin(\theta) d\theta \tag{3.5}$$

and

$$n_{(Z,N),(Z+2,N-2)}(\theta) = \left[det \left(1 + F_{Z,N}^{(\nu)} f_{Z+2,N-2}^{(\nu)\dagger} \right) \right]^{1/2} \times \left[det \left(1 + F_{Z,N}^{(\pi)} f_{Z+2,N-2}^{(\pi)\dagger} \right) \right]^{1/2} \tag{3.6}$$

The $\pi(\nu)$ represents the proton (neutron) of nuclei involved in the $2\nu\beta^-\beta^-$ decay process. The matrices $f_{Z,N}$ and $F_{Z,N}(\theta)$ are given by

$$f_{Z,N} = \sum_i C_{ij_\alpha,m_\alpha} C_{ij_\beta,m_\beta} \left(v_{im_\alpha}/u_{im_\alpha} \right) \delta_{m_\alpha,-m_\beta} \tag{3.7}$$

$$F_{Z,N}(\theta) = \sum_{m'_\alpha m'_\beta} d_{m_\alpha,m'_\alpha}^{j_\alpha}(\theta) d_{m_\beta,m'_\beta}^{j_\beta}(\theta) f_{j_\alpha m'_\alpha, j_\beta m'_\beta} \tag{3.8}$$

3.2 Results and discussions

Our aim is to study the $2\nu\beta^-\beta^-$ decay of ^{94,96}Zr, ^{100}Mo, ^{104}Ru, ^{110}Pd, ^{128,130}Te and ^{150}Nd isotopes for the $0^+ \to 0^+$ transition. The NTMEs $M_{2\nu}$ calculated with PHFB wave functions generated with four different parametrizations of effective two-body interactions, namely $PQQ1$, $PQQHH1$, $PQQ2$ and $PQQHH2$ along with the estimated average $\overline{M}_{2\nu}$, uncertainties $\Delta\overline{M}_{2\nu}$ and experimental $M_{2\nu}$ [Barabash (2010)] are presented in Table 3.1. In the average NTMEs $\overline{M}_{2\nu}$, the maximum uncertainty $\Delta\overline{M}_{2\nu}$ turns out to be about 45%, which shows that the NTMEs $M_{2\nu}$ are highly sensitive to the deformation content of the intrinsic wave functions. The calculated $\overline{M}_{2\nu}$ in the present work agrees very well with the experimental $M_{2\nu}$ for ^{96}Zr, ^{100}Mo and ^{128}Te isotopes within error bars. In case of ^{130}Te and ^{150}Nd, the calculated $\overline{M}_{2\nu}$ are about 2.5 times larger and 3.4 times smaller, respectively than the experimental $M_{2\nu}$.

3.2.1 Deformation effects

To quantify the deformation effect on $M_{2\nu}$, we define a quantity $D_{2\nu}$ as the ratio of $M_{2\nu}$ at zero deformation ($\zeta_{qq} = 0$) and full deformation ($\zeta_{qq} = 1$). The deformation effect ratio $D_{2\nu}$ is given by

$$D_{2\nu} = \frac{M_{2\nu}(\zeta_{qq} = 0)}{M_{2\nu}(\zeta_{qq} = 1)} \tag{3.9}$$

The values of $D_{2\nu}$ are 2.37, 3.80, 3.41, 25.85, 5.27, 3.50, 4.81 and 12.30 for ^{94,96}Zr, ^{100}Mo, ^{104}Ru, ^{110}Pd, ^{128,130}Te and ^{150}Nd nuclei, respectively, which suggest that the deformation play a crucial role in the evaluation of NTMEs $M_{2\nu}$.

3.3 Conclusions

Presently, a number of nuclear models have been employed to calculate $M_{2\nu}$ and the calculated $M_{2\nu}$ in these models vary substantially. The validity of these nuclear models can not be uniquely established due to large error bars in experimental results as well as uncertainty in g_A. In the present work, we have shown that the deformations of the intrinsic states play a crucial role in reproducing a realistic NTME.

Table 3.1: Theoretically calculated NTMEs $M_{2\nu}$ within the PHFB model with four different parametrizations and their average value $\overline{M}_{2\nu}$ along with experimental values [Barabash (2010)].

Nuclei	$M_{2\nu}$				$\overline{M}_{2\nu}$	$M_{2\nu}$(Exp.)
	$PQQ1$	$PQQHH1$	$PQQ2$	$PQQHH2$		
^{94}Zr	0.064	0.060	0.133	0.058	0.079±0.036	–
^{96}Zr	0.055	0.056	0.053	0.054	0.054±0.001	$0.048^{+0.002}_{-0.002}$
^{100}Mo	0.127	0.128	0.127	0.127	0.127±0.001	$0.123^{+0.004}_{-0.003}$
^{104}Ru	0.020	0.021	0.023	0.022	0.021±0.001	–
^{110}Pd	0.085	0.124	0.105	0.115	0.107±0.017	–
^{128}Te	0.026	0.022	0.027	0.021	0.024±0.003	$0.024^{+0.003}_{-0.002}$
^{130}Te	0.047	0.044	0.041	0.050	0.045±0.004	$0.017^{+0.002}_{-0.001}$
^{150}Nd	0.010	0.008	0.010	0.008	0.009±0.001	$0.031^{+0.002}_{-0.002}$

Chapter 4

$0\nu\beta^-\beta^-$ decay of ^{94,96}Zr, ^{100}Mo, ^{104}Ru, ^{110}Pd, ^{128,130}Te and ^{150}Nd nuclei in the left-right symmetric grand unified theory

The present chapter is organized as follows. The $0\nu\beta^-\beta^-$ decay has been already studied extensively in the light neutrino approximation [Haxton *et al.* (1984), Doi *et al.* (1985), Vergados [(1986), (2002)], Tomoda (1991), Suhonen *et al.* (1998), Faessler *et al.* (1998)]. In *LRSM*, the $0\nu\beta^-\beta^-$ decay has been studied by Mohapatra [(1986)], Doi *et al.* (1993) and Hirsch *et al.* (1996). Therefore, the theoretical formalism for $0\nu\beta^-\beta^-$ decay in *LRSM* is given briefly in Section 4.1. In Section 4.2 , results of our calculation are presented and discussed. The role of deformation on the NTMEs due to light Majorana neutrino exchange $M^{(0\nu)}$ as well as heavy Majorana neutrino exchange $M^{(0N)}$ is studied in the same Section. Finally, we present concluding remarks in Section 4.3.

4.1 Theoretical formalism

In the mass mechanism, the inverse half-life $T_{1/2}^{0\nu}$ of $0\nu\beta^-\beta^-$ decay for the $0^+ \to 0^+$ transition is given by

$$\left[T_{1/2}^{0\nu}(0^+ \to 0^+)\right]^{-1} = \left(\frac{\langle m_\nu \rangle}{m_e}\right)^2 C_{mm}^{LL} + \left(\frac{m_p}{\langle M_N \rangle}\right)^2 C_{mm}^{NN} + \left(\frac{\langle m_\nu \rangle}{m_e}\right)\left(\frac{m_p}{\langle M_N \rangle}\right) C_{mm}^{NL} \quad (4.1)$$

where

$$\langle m_\nu \rangle = \Sigma_i' U_{ei}^2 m_i \quad (4.2)$$

$$\langle M_N \rangle^{-1} = \Sigma_i'' U_{ei}^2 m_i^{-1} \quad (4.3)$$

and the nuclear factors-of-merit C_{mm} are defined by the following relations.

$$C_{mm}^{LL} = G_{01}\left|M^{(0\nu)}\right|^2 = G_{01}\left(-M_F^{(0\nu)} + M_{GT}^{(0\nu)} + M_T^{(0\nu)}\right)^2 \quad (4.4)$$

$$C_{mm}^{NN} = G_{01}\left|M^{(0N)}\right|^2 = G_{01}\left(-M_{Fh}^{(0N)} + M_{GTh}^{(0N)} + M_{Th}^{(0N)}\right)^2 \quad (4.5)$$

$$C_{mm}^{NL} = 2G_{01}\left(-M_F^{(0\nu)} + M_{GT}^{(0\nu)} + M_T^{(0\nu)}\right)\left(-M_{Fh}^{(0N)} + M_{GTh}^{(0N)} + M_{Th}^{(0N)}\right) \quad (4.6)$$

where the NTMEs $M_K^{(I)}$ ($K = F,\ GT,\ T,\ Fh,\ GTh,\ Th$ and $I = 0\nu,\ 0N$) are [Simkovic *et al.* (2001), (2008), (2009)]

$$M_K^{(I)} = \sum_{n,m} \left\langle 0_F^+ \left\| O_{K,nm} \tau_n^+ \tau_m^+ \right\| 0_I^+ \right\rangle \quad (4.7)$$

with

$$O_F = H_F(r_{nm}) \quad (4.8)$$

$$O_{GT} = \boldsymbol{\sigma}_n \cdot \boldsymbol{\sigma}_m H_{GT}(r_{nm}) \quad (4.9)$$

$$O_T = \left[3(\boldsymbol{\sigma}_n \cdot \widehat{\mathbf{r}}_{nm})(\boldsymbol{\sigma}_m \cdot \widehat{\mathbf{r}}_{nm}) - \boldsymbol{\sigma}_n \cdot \boldsymbol{\sigma}_m\right] H_T(r_{nm}) \quad (4.10)$$

$$O_{Fh} = H_{Fh}(r_{nm}) \quad (4.11)$$

$$O_{GTh} = \boldsymbol{\sigma}_n \cdot \boldsymbol{\sigma}_m H_{GTh}(r_{nm}) \quad (4.12)$$

$$O_{Th} = \left[3(\boldsymbol{\sigma}_n \cdot \widehat{\mathbf{r}}_{nm})(\boldsymbol{\sigma}_m \cdot \widehat{\mathbf{r}}_{nm}) - \boldsymbol{\sigma}_n \cdot \boldsymbol{\sigma}_m\right] H_{Th}(r_{nm}) \quad (4.13)$$

The neutrino potentials due to light Majorana neutrino exchange $H_\alpha(r_{nm})$ and heavy Majorana neutrino exchange $H_{\alpha h}(r_{nm})$ associated with Fermi, Gamow-Teller (GT) and tensor operators are given by

$$H_\alpha(r_{nm}) = \frac{2R}{\pi}\int \frac{f_\alpha(qr_{nm})}{\left(q+\overline{A}\right)} h_\alpha(q) q\, dq \quad (4.14)$$

$$H_{\alpha h}(r_{nm}) = \frac{2R}{(m_p m_e)\pi}\int f_\alpha(qr_{nm})\, h_\alpha(q) q^2 dq \quad (4.15)$$

where $f_\alpha\left(qr_{nm}\right) = j_0\left(qr_{nm}\right)$ for $\alpha = F$, GT and $f_T\left(qr_{nm}\right) = j_2\left(qr_{nm}\right)$.

4.2 Results and discussions

Employing the same PHFB wave functions generated with four parametrizaions of pairing plus multipolar type of effective two-body interaction, the required NTMEs due to light Majorana neutrino exchange $M^{(0\nu)}$ as well as heavy Majorana neutrino exchange $M^{(0N)}$ for ^{94,96}Zr, ^{100}Mo, ^{104}Ru, ^{110}Pd, ^{128,130}Te and ^{150}Nd isotopes are calculated with the consideration of three different parametrizations of the Jastrow type of SRC and their mean $\overline{M}^{(I)}$ and standard deviation $\Delta\overline{M}^{(I)}$ is estimated. It turns out that the uncertainties $\Delta\overline{M}^{(0\nu)}$ are of the order of 10%, but for ^{110}Pd for which $\Delta\overline{M}^{(0\nu)}$ is approximately 14%. For the case of NTMEs associated with heavy Majorana neutrino exchange, the maximum uncertainty $\Delta\overline{M}^{(0N)}$ is about 37%.

The limits on the effective mass of light Majorana neutrino $\langle m_\nu \rangle$ as well as heavy Majorana neutrino $\langle M_N \rangle$ are extracted from the most recent observed limits on half-lives $T_{1/2}^{0\nu}$ of $0\nu\beta^-\beta^-$ decay using the average NTMEs $\overline{M}^{(I)}$ $(I = 0\nu, 0N)$. The results are presented in Table 4.1 and 4.2. The extracted upper limits on $\langle m_\nu \rangle$ for ^{100}Mo and ^{130}Te nuclei are 0.45 eV and 0.17 eV, respectively. The predicted half-lives of $0\nu\beta^-\beta^-$ decay are also given for $\langle m_\nu \rangle = 50$ meV. The extracted lower limits on $\langle M_N \rangle$ are 3.27×10^7 GeV and 8.99×10^7 GeV, for ^{100}Mo and ^{130}Te nuclei, respectively.

In the Table 4.1, we present the predicted half-lives assuming $\langle m_\nu \rangle = 50$ meV. Nuclear sensitivities $\xi^{(I)}$ $(I = 0\nu, 0N)$, which are related to mass sensitivities, defined by Simkovic *et al.* (1999) are given as

$$\xi^{(I)} = 10^8 \sqrt{G_{01}} \left| M^{(I)} \right| \tag{4.16}$$

with an arbitrary normalization factor 10^8 so that the nuclear sensitivities turn out to be order of unity. They are presented in Table 4.1 and 4.2 for light and heavy exchange of Majorana neutrinos, respectively.

4.2.1 Deformation effect

The effect of deformation on $M^{(I)}$ is quantified by the quantity $D^{(I)}$ defined as the ratio of $M^{(I)}$ at zero deformation ($\zeta_{qq} = 0$) and full deformation ($\zeta_{qq} = 1$) [Chaturvedi *et al.* (2008)].

$$D^{(I)} = \frac{M^{(I)}(\zeta_{qq} = 0)}{M^{(I)}(\zeta_{qq} = 1)}, \tag{4.17}$$

The values of $D^{(I)}$ suggest that the NTMEs $M^{(I)}$ are suppressed by factor of about 2–11 in the mass range $A = 90 - 150$.

4.3 Conclusions

The required NTMEs to study the $0\nu\beta^-\beta^-$ decay of ^{94,96}Zr, ^{100}Mo, ^{104}Ru, ^{110}Pd, ^{128,130}Te and ^{150}Nd isotopes within mass mechanism are calculated within PHFB model using four different parametrization of pairing plus multipolar type of effective two body interaction with three different parametrizations of SRC. The effect due to FNS and SRC is investigated on NTMEs due to light and heavy neutrino exchange. The effects due to deformation are in between a factor of 2–11. We have also extracted limits on the effective neutrino mass $\langle m_\nu \rangle$ and $\langle M_N \rangle$ from the available limits on experimental half-lives $T_{1/2}^{0\nu}$ using average NTMEs $\overline{M}^{(I)}(I = 0\nu, 0N)$ calculated in the PHFB model. The extracted limits on $\langle m_\nu \rangle$ and $\langle M_N \rangle$ for ^{130}Te nuclei are very stringent.

Table 4.1: Average NTMEs $\overline{M}^{(0\nu)}$ and uncertainties $\Delta\overline{M}^{(0\nu)}$ due to light Majorana neutrino exchange along with effective neutrino mass $\langle m_\nu \rangle$ (in eV), predicted half-lives $T_{1/2}^{(0\nu)}$ (for $\langle m_\nu \rangle = 50$ meV) and nuclear sensitivities $\xi^{(0\nu)}$ for the $0\nu\beta^-\beta^-$ decay of ^{94,96}Zr, ^{100}Mo, ^{110}Pd, ^{128,130}Te and ^{150}Nd isotopes.

Nuclei	$\overline{M}^{(0\nu)}$	$\Delta\overline{M}^{(0\nu)}$	$\langle m_\nu \rangle$	$T_{1/2}^{(0\nu)}$	$\xi^{(0\nu)}$
^{94}Zr	3.467	0.399	8.08×10^{2}	4.96×10^{27}	14.50
^{96}Zr	2.396	0.206	9.67	3.44×10^{26}	55.07
^{100}Mo	5.379	0.448	0.45	8.84×10^{25}	108.7
^{110}Pd	4.565	0.648	1.29×10^{3}	4.02×10^{26}	50.97
^{128}Te	2.647	0.273	15.32	1.03×10^{28}	10.06
^{130}Te	3.980	0.350	0.17	1.80×10^{26}	76.24
^{150}Nd	1.388	0.152	6.48	3.36×10^{26}	55.75

Table 4.2: Average NTMEs $\overline{M}^{(0N)}$ and uncertainties $\Delta\overline{M}^{(0N)}$ due to heavy Majorana neutrino exchange along with effective neutrino mass $\langle M_N \rangle$ (in GeV) and nuclear sensitivities $\xi^{(0N)}$ for the $0\nu\beta^-\beta^-$ decay of ^{94,96}Zr, ^{100}Mo, ^{110}Pd, ^{128,130}Te and ^{150}Nd isotopes.

Nuclei	$\overline{M}^{(0N)}$	$\Delta\overline{M}^{(0N)}$	$\langle M_N \rangle$	$\xi^{(0N)}$
^{94}Zr	113.11	39.73	1.93×10^{4}	4.73×10^{2}
^{96}Zr	80.26	29.54	1.66×10^{6}	1.84×10^{3}
^{100}Mo	164.62	59.66	3.27×10^{7}	3.33×10^{3}
^{110}Pd	139.89	53.05	1.13×10^{4}	1.56×10^{3}
^{128}Te	94.33	32.51	1.12×10^{6}	3.58×10^{2}
^{130}Te	129.13	44.04	8.99×10^{7}	2.47×10^{3}
^{150}Nd	44.62	15.75	2.38×10^{6}	1.79×10^{3}

Chapter 5

Conclusions

The implications of present studies on nuclear $\beta\beta$ decay are far reaching in nature. The validity of different models employed for nuclear structure calculations can be tested by studying the $2\nu\beta^-\beta^-$ decay. It is observed that in all cases of the $2\nu\beta^-\beta^-$ decay, the NTMEs $M_{2\nu}$ are sufficiently quenched. The main motive of all the theoretical calculations is to understand the physical mechanism responsible for the suppression of $M_{2\nu}$. The $0\nu\beta^-\beta^-$ decay is a convenient tool to test the physics beyond the SM, and it has not been observed so far. Hence the models predict half-lives assuming certain value for the neutrino mass or conversely extract various parameters from the observed limits on half-lives $T_{1/2}^{0\nu}$ of the $0\nu\beta^-\beta^-$ decay. The reliability of predictions can be judged a priori only from the success of the nuclear model in explaining various observed physical properties of nuclei. The common practice is to calculate the $M_{2\nu}$ to start with and compare with the experimentally observed value as the two decay modes involve the same set of initial and final nuclear wave functions.

The PHFB model is one of the convenient many-body techniques to study the medium and heavy mass nuclei in which the pairing and deformation degrees of freedom are treated on equal footing. In the present work, our aim was to study the $0\nu\beta^-\beta^-$ decay of ^{94,96}Zr, ^{100}Mo, ^{104}Ru, ^{110}Pd, ^{128,130}Te and ^{150}Nd for the $0^+ \rightarrow 0^+$ transition within the PHFB model using pairing plus multipolar type of effective two-body interaction. Specifically, we use pairing plus quadrupole-quadrupole plus hexadecapole-hexadecapole interaction and derive four different parametrizations, namely $PQQ1$, $PQQHH1$, $PQQ2$ and $PQQHH2$.

The single particle energies used in the present work were derived from Woods-Saxon potential for the nuclei participating in $\beta^-\beta^-$ decay and adjusted to reproduce the experimentally observed sub-shell occupation numbers where ever available. We have tested the quality of wave functions by comparing the calculated values of a number of nuclear properties with the available experimental data. To be more specific, we have computed the yrast spectra, reduced $B(E2{:}0^+ \rightarrow 2^+)$ transition probabilities, deformation parameters β_2, g-factors $g(2^+)$ and NTMEs $M_{2\nu}$ of $2\nu\beta^-\beta^-$ decay for the $0^+ \rightarrow 0^+$ transition.

In case of $0\nu\beta^-\beta^-$ decay, we have studied the $0^+ \rightarrow 0^+$ transition of ^{94,96}Zr, ^{100}Mo, ^{104}Ru, ^{110}Pd, ^{128,130}Te and ^{150}Nd nuclei in the mass mechanism and extracted limits on effective mass of light neutrino $\langle m_\nu \rangle$ and heavy neutrinos $\langle M_N \rangle$ from the observed limits on half-lives $T^{0\nu}_{1/2}$ for the $0^+ \rightarrow 0^+$ transition. The best limit comes from the $0\nu\beta^-\beta^-$ decay of ^{130}Te nucleus studied experimentally under the 'CUORE' project [Alduino *et al.* (2018)]. The extracted limits on $\langle m_\nu \rangle$ and $\langle M_N \rangle$ for ^{130}Te nuclei is 0.17 eV and 8.99×10^7 GeV, respectively, from the observed limit on the half-live $T^{0\nu}_{1/2} > 1.5 \times 10^{25}$ yr of $0\nu\beta^-\beta^-$ decay [Alduino *et al.* (2018)]. The study of role of deformation on the NTMEs $M^{(I)}$ $(I = 0\nu, 0N)$ of $0\nu\beta^-\beta^-$ decay for the $0^+ \rightarrow 0^+$ transition suggests that the nuclear structure effects are also important in case of $0\nu\beta^-\beta^-$ decay. The uncertainties associated with the NTMEs $M^{(0\nu)}$ and $M^{(0N)}$ for $0\nu\beta^-\beta^-$ decay are estimated by preforming a statistical analysis. The mean $\overline{M}^{(I)}$ and their standard deviations $\Delta\overline{M}^{(I)}$ are calculated and it turns out that the uncertainties $\Delta\overline{M}^{(0\nu)}$ are of the order of 10%, but for ^{110}Pd for which $\Delta\overline{M}^{(0\nu)}$ is approximately 14%. For the case of heavy Majorana neutrino exchange, the maximum uncertainty $\Delta\overline{M}^{(0N)}$ is about 37%.

The phase space factors are calculated exactly and the main uncertainty in the calculation of decay rates comes from the NTMEs, which are model dependent. The PHFB method includes the pairing and deformation degrees of freedom on equal footing. However, the trend of results suggest that it is necessary to incorporate a number of improvements in our study of $\beta\beta$ decay in general and $2\nu\beta^-\beta^-$ decay in particular, which are discussed in the thesis.

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