Numerical Computation of Various Fluid Flow Problems

THESIS

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Abstract

Partial differential equations (PDEs)are the basis tools of various mathematical models occurred not only in physical, chemical and biological phenomena but also in the other fields such as in economics, financial forecasting, image processing and many more. Notice that physical phenomena consisting of certain hereditary properties can't be explain via a mathematical model involving only classical differential/integral operators as they are local in nature. The fractional differential/integral operators, introduced by a great mathematician Leibniz (1965), are non-local in nature (i.e., the next state of a system depends not only upon its current state but also upon all of its previous states) and have memory effects as well as embedded capability to explain the physical phenomena which are not explained accurately in terms of the classical operators, and so, the fractional operators are more realistic and become very popular in modeling certain complex systems arising in fluid mechanics, viscoelasticity, mathematical biology, life sciences, electrochemistry, physics, economics, control theory and many more. The above finding invoked the researchers to develop the techniques to study the behavior of such type of the PDEs model. In the past years, various methods have been developed to study these models of PDEs, among them, finite difference method, compact finite difference method (Lele, 1992), differential quadrature method (Shu, 2000) and collocation methods are employed very broadly to solve classical PDEs.

The main aim of this thesis is to present a numerical study of some fluid flow problems occurred in the form of time dependent partial differential equations (PDEs). We consider both classical and fractional model of PDEs. Specially, in classical order PDEs, we consider Burgers' equations in one and two dimensions and *Kuramoto-Sivashinsky* equation in one dimension whereas in fractional model of PDE, we consider coupled viscous Burgers

equations, Navier-Stokes equations and fractional model of PDEs with proportional delay. These classical model of PDEs have been studied numerically using Bellman's differential quadrature method (DQM) (Bellman et al, 1975) with different set of base functions and Lele's compact finite difference schemes (Lele, 1992). The fractional model of PDEs have been studied by using fractional variational technique, reduced differential transform method and perturbation techniques.

Chapter 1 deals with introductory information on the work done. Besides some basic definitions and preliminaries, used throughout the work, we present a brief introduction to differential quadrature method, reduced differential transform method and perturbation techniques and their existing literature review.

In **Chapter 2**, a novel approach: *modified extended cubic B-spline differential quadrature (mECDQ) method* in space discretization has been developed with time integration algorithm for numerical simulation of initial values system of nonlinear Burgers' equation in $(1+1)$ dimension and coupled Burgers equation in $(n+1)$ dimension $(n = 1, 2)$ with appropriate Boundary conditions. The mECDQ method, DQM with modified extended cubic B-splines as base functions, is used to convert the initial boundary value system of the Burgers' equation into a initial value system of ordinary differential equations (ODEs), in time. We prefer an optimal five stage four order strong stability preserving Runge-Kutta method (SSP-RK54) to solve this resulting system of ODEs. Six test problems are considered to test the accuracy and efficiency of mECDQ method. The proposed results are compared with the exact solutions in terms of L_2 and L_∞ errors and the existing results. The mECDQ scheme is shown conditionally stable Burgers' equations.

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Chapter 3 concerns with new method *modified trigonometric cubic B-spline differential quadrature method (MTB-DQM)* in space discretization with SSP-RK54 algorithm for solving the time dependent PDEs. Specially, the proposed algorithm has been implemented for nonlinear Burgers' equations. First, MTB-DQM (DQM with modified trigonometric cubic B-splines as base functions) is used to convert the initial boundary value system of Burgers' equation into initial value system of first order ODEs, in time, thereafter SSP-RK54 algorithm has been employed for solving the resulting system of ODEs. Four test problems are considered to illustrate the accuracy/efficiency of the method in terms of L_2 and L_∞ error norms and their comparisons with existing results. Moreover, MTB-DQM is shown conditionally stable for various grid points, and computed presented results are better than the results obtained by almost all the existing schemes.

Chapter 4 adopted a *compact finite difference scheme* (Lele, 1992) in the space discretizations while optimal four-stage, order three strong stability-preserving time-stepping Runge-Kutta scheme in the time, for the numerical simulation of one dimensional Kuramoto-Sivashinsky equation "$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \beta\frac{\partial^2 u}{\partial x^2} + \gamma\frac{\partial^4 u}{\partial x^4}$", arises in the study of flame front propagation, phase turbulence in reaction-diffusion system and in many other biological and chemical processes. The efficiency of proposed scheme is confirmed by six test problems with known exact solutions. The numerical results demonstrate the reliability and efficiency of the algorithm developed.

Chapter 5 deals with an analytical study of time-fractional Navier-Stokes equation, considering fractional derivative of caputo type:

$$\mathcal{D}_t^\alpha U + (U \cdot \nabla)U = \nu\nabla^2 U - \tfrac{1}{\rho}\nabla p, \quad \text{on } \Omega \times (0, T)$$

The approximate analytical solutions are obtained by adopting two reliable methods: Fractional reduced differential transform method and the a new integral projected differential transform method. The accuracy/efficiency of these methods is illustrated by three test problems of the time fractional Navier-Stokes equation. The scheme is found to be very reliable, effective and efficient powerful technique to solve wide range of problems arising in engineering and sciences. The small size of computation FRDTM contrary to the other schemes, is its strength.

Chapter 6 deals with an approximate analytical solution of multi-dimensional, time-fractional coupled viscous Burgers' (TFCB) equation obtained by employing "homotopy perturbation method". The validity and efficiency of homotopy perturbation method has been illustrate the efficiency by considering three different examples of TFCB equation. The results are also depicted in graphically for different values of fractional order α and Reynolds number. It is found that the proposed series solutions converges rapidly for large Reynolds numbers (Re$\geq$ 100).

In **Chapter 7**, at first some properties of $(n+1)$-dimensional Extended FRDTM for delayed TFPDEs are presented. Approximate analytic solutions of $(1+1)$ dimensional TF-PDEs with proportional delay and generalized Burgers' equations with proportional delay are obtained by two reliable methods: 1) fractional variation iteration method (FVIM), and 2) Extended fractional reduced differential transform method (Extended FRDTM). The approximate solutions from the either method are obtained in series form that converges to the exact solution behaviors very fast. The efficiency/validity of these methods is illustrated by three test problems of TFPDEs with proportional delay. The finding shows that Extended FRDTM is easy to implement as compared to FVIM. The small size of computation of Extended FRDTM is its strength.

In **Chapter 8**, the *homotopy perturbation transform method* (HPTM) (i.e., hybrid of homotopy perturbation technique & Laplace transform) has been implemented for solving initial value autonomous system of time-fractional partial differential equations (TFPDEs) with proportional delay, including generalized Burgers' equations with proportional delay. The numerical study of three examples of TFPDEs with proportional delay are presented to test the efficiency and validity of proposed HPTM. The obtained solutions are in series form, converges very fast. The HPTM seems very reliable, effective and efficient powerful technique for various the study of many physical models arising in various branches of sciences and engineering.