THERMAL INSTABILITY IN NANOFLUIDS UNDER MODULATION WITHIN HELE-SHAW CELL

ABSTRACT

of

THESIS

Submitted to
Babasaheb Bhimrao Ambedkar University
(A Central University)
Lucknow

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LUCKNOW
प्रज्ञा शील करुणा
ESTABLISHED 1996

for the Award of the Degree of

Doctor of Philosophy

in

MATHEMATICS

Under the Supervision of
Prof. B. S. BHADAURIA

Research Scholar
SHOBH NATH RAI
(M.Sc., CSIR-UGC JRF/SRF)
Enrollment No. 604/14

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ABSTRACT

It is an intrinsic characteristic of hydrodynamic systems that they cannot always maintain their original state in the face of small variations in the parameters that define them. The term *hydrodynamic instability* is derived from this property. The hydrodynamic system can always be referred to in several ways depending according to how it reacts to these minor disturbances.

- **Stable:** If the disruptions are resolved gradually, the system is considered to be stable.
- **Unstable:** If the amplitude of the disruptions increases and the system never returns to its initial condition, the system is considered to be unstable.
- **Neutrally stable:** If the disruptions continue to have a comparable magnitude, the system is considered to be neutrally stable. The *marginal stable state* is another name for it.

Thermal instability occurs in a hydrodynamic system when there is a substantial temperature gradient throughout the boundaries. The system is referred to as stable whenever the temperature gradient is less than a critical level and the heat transfer is caused by a stationary conduction mode. When the temperature gradient between the boundaries approaches a certain magnitude, the conduction mode begins to lose stability, and the system eventually approaches a marginally stable state. Further than the critical temperature gradient, heat transport occurs as a consequence of the system's convective motion, which is presently considered to be unstable.

The objective of this thesis entitled "**THERMAL INSTABILITY IN NANOFLUIDS UNDER MODULATION WITHIN HELE-SHAW CELL**" is to investigate the thermal instability in nanofluids which are the suspension of nanoparticles into some basefluid (Choi, 1995), under the influence of modulation within Hele-Shaw cell. Both, onset of instability (convection), and transport of heat and mass have been analysed to get a better insight of the considered problems us-

ing Galerkin's method, truncated Fourier series method, and RKF-45 method. Both Lorenz model and Ginzburg-Landau (GBL) model have been used for the investigation. The impact of various other parameters, like through-flow, modulations, magneto-convection, etc., over convective instability is also examined. The considered problems can have various industrial, engineering, and medical applications such as in automobile industry, electronic devices, energy savings, etc., because of the enhanced thermal conductivity of nanofluids, very small size of nanoparticles, and basefluids. The thesis comprises seven chapters which are described as follows:

Chapter 1 consists of various fundamental definitions and concepts used in the entire thesis. The basic idea of nanofluids, mathematical models, Hele-Shaw cell, through-flow, various analytical and numerical methods used to solve the problems, applications, literature survey, etc. are included in this chapter.

In **Chapter 2**, the influence of trigonometric cosine, square, sawtooth, and triangular wave types of magnetic-field modulation in nanoliquid within Hele-Shaw cell is studied utilizing linear/nonlinear explorations. The solvability condition to the third-order solution of the referred model equation has been imposed to get the cubic Ginzburg-Landau equation (GBL-equation) which is utilized to measure the rate of heat (or mass) transfer. In the sequel, the influence of the non-dimensional parameters is discussed graphically in detail. It is demonstrated that Prandtl number(P_r)/magnetic Prandtl number(P_{rm})/Lewis-number (L_e)/redefined diffusivity-ratio (N_A)/ concentration Rayleigh-number (R_n) and magnitude of the magnetic-modulation (ϵ_1) destabilize the system, that is, the heat/mass transfer increases. On the other hand, nanoliquid magnetic-number (Q), Hele-Shaw number (Hs) as well as modulating-frequency (ω_1) stabilize the system. The outcomes demonstrate that the magnetic-field modulation can be imposed significantly to increase or decrease the heat/mass transfer.

Chapter 3 includes both linear and local nonlinear exploration to estimate the onset of instability and heat/mass transportation in nanoliquid (Walter-B viscoelastic) within the Hele-Shaw (HS) cell under the impact of thermal-modulation using cubic Ginzburg-Landau (GBL) equation. Three types of thermal-modulation have been considered viz symmetric thermal-modulation, asymmetric thermal-modulation, and lower-boundary thermal-modulation. It is described that elastic-parameter (H), nanoliquid Prandtl number (P_r), nanoliquid Lewis-number (L_e), modified diffusivity-ratio (N_A), concentration Rayleigh-number (R_n), and amplitude of the thermal-modulation (ϵ_2) destabilize the system, that is, the heat/mass transportation increases. On the other hand, HS-number (Hs) and modulating-frequency (ω_2) stabilize the system. Moreover, it is found that in all three types of thermal-modulation, maximum heat/mass transportation can be observed in the case of asymmetric modulation. Walter-B nanoliquid can be used to enhance the heat/mass transportation as compared to a normal nanoliquid.

In **Chapter 4**, thermal instability of magneto-convection in an electrically conducting nanoliquid confined within Hele-Shaw cell has been investigated subject to an applied time-periodic boundary thermal (ATBT) or gravitational modulation (ATGM), and surrounded by a constant vertical magnetic field. A steady portion and a time-dependent oscillatory portion constitute the temperature gradient seen between liquid layer's wall in the context of ATBT. In this scenario, both walls' temperatures are modulated. The liquid layer oscillation can be used to realise the externally applied time periodic component of the gravity field that is present in the ATGM problem. The perturbation is described in terms of the power series of the assumed-small convective amplitude. The impact of modulations on heat/mass transfer are examined utilising Ginzburg-Landau (GBL) approach. The impact of different parameters on the transportation of mass and heat is also explored. Additionally, it is observed that gravitational modulation is very much effective than thermal modulation. Lewis-number, modified-diffusivity ratio and concentration Rayleigh-number increase heat and mass transport in the system.

Chapter 5, uses linear/nonlinear studies to examine the effects of sine, square, and triangular waveforms of magnetic-field modulation in Walter-B nanoliquid (electrically-conducting) filled in a Hele-Shaw cell with through-flow. The applied magnetic field has periodic components that are both constant and time-dependent and changes sinusoidally over time. In order to examine heat and mass transfer in the liquid layer, a local nonlinear concept has been used. Formulation of the autonomous simultaneous ordinary-differential equations for the convection amplitude leads to the investigation of the heat/mass transportation factor. This convectional amplitude is calculated using Mathematica's built-in NDSolve tool, and the results have been validated using the Runge-Kutta-Fehlberg (RKF-45) technique. The Nusselt-number is found in terms of numerous system parameters, and each parameter's consequence on heat/mass transportation is described in detail. Hele-Shaw number, nanoliquid magnetic number, and frequency of magnetic modulation have a stabilizing impact on the system, whereas elastic parameter, magnetic Prandtl-number, nanoliquid Prandtl-number, and amplitude of magnetic modulation have destabilizing impact on the system. Additionally, it has been found that magnetic modulation can be utilised to efficiently regulate the heat and mass transport. The system's basic (conduction) state temperature profile and transportation of mass highly depend on through-flow.

In **Chapter 6**, the impact of sine, square and triangular wave-forms of g-Jitter (gravity-modulation) in Walter-B nanoliquid (electrically-conducting) filled in Hele-Shaw cell with magnetic-field effect is considered. The applied gravity field contains constant and time-dependent periodic parts, which change sinusoidally over time. A local nonlinear idea has been utilized to study heat/mass transportation in the liquid layer. The method for determining the heat/mass transportation factor involves developing a set of independent simultaneous ODEs for the convective magnitude. The Runge-Kutta-Fehlberg approach is used to confirm the findings of this convective magnitude calculation, which was done with the use of Mathematica's built-in tool NDSolve. The Nusselt-number is found in terms of numerous system parameters, and every parameter's consequence on heat/mass transportation is described in detail. The influence of elastic parameter, nanoliquid Prandtl-number, and amplitude of g-Jitter is to increase heat/mass transportation while the Hele-Shaw number, nanoliquid magnetic number, and frequency of g-Jitter have stabilizing influence on the system. Moreover, it is found that g-Jitter can be utilised effectively to increase or decrease heat/mass transportation.

In **Chapter 7**, due to the great significance of rotational-speed modulation in various areas of science and technology as well as in day-to-day life, aim in this study is to perform linear/nonlinear studies to examine the impacts of sinusoidal like sine waveform and non-sinusoidal like square and triangular waveforms of rotational-speed modulation in nanoliquid (electrically-conducting) filled within a Hele-Shaw cell with through-flow and magnetic-field. The applied rotational-speed modulation has periodic components that are both constant and time-dependent and changes sinusoidally over time. The linear analysis is done using Galerkin's approach while the nonlinear analysis is done using truncated Fourier series method. The results of Mathematica's built-in NDSolve tool are validated using the Runge-Kutta-Fehlberg technique. Hele-Shaw number, Taylor-number, nanoliquid magnetic number and frequency of rotational-speed modulation have a stabilizing impact on the system, whereas through-flow, magnetic Prandtl-number, nanoliquid Prandtl-number and amplitude of rotational-speed modulation have destabilizing impact on the system. Additionally, it has been found that rotational-speed modulation can be utilised to efficiently increase or decrease heat and mass movement. Square-waveform is most efficient for both heat and mass transport in comparison of sinusoidal and triangular waveforms.

In the end the thesis is concluded with the possible future scopes of the current work.

The list of publications is as follows:

Journals

1. **S.N.Rai**, B.S. Bhadauria, A. Kumar, B.K. Singh, "Thermal instability in nanoliquid under four types of magnetic-field modulation within Hele-Shaw cell", published in the *ASME Journal of Heat and Mass Transfer*, vol. 145(7), 072501-072513, 2023. (Chapter 2)

2. **S.N.Rai**, B.S. Bhadauria, A. Srivastava, "Study of thermal-instability in nanoliquid (Walter-B viscoelastic) within Hele-Shaw cell under thermal-modulation using cubic Ginzburg-Landau (GBL) equation", accepted in the *Journal of Nanofluids*, Article ID: JON:18R1, 2023. (Chapter 3)

3. **S.N.Rai**, B.S. Bhadauria, A. Srivastava, A. Kumar, "Thermal instability in Walter-B nanoliquid filled in Hele-Shaw cell under 3-types of magnetic-field modulation with through-flow", accepted in the *Special Topics & Reviews in Porous Media - An International Journal (Begell House)*, Article ID: STRPM-47492, 2023. (Chapter 5)

4. **S.N.Rai**, B.S. Bhadauria, "Thermal instability in electrically conducting nanoliquid filled in Hele-Shaw cell under 3-types of rotational-speed modulation with impact of through-flow and magnetic-field", published in the *Chinese Journal of Physics*,
doi.org/10.1016/j.cjph.2023.05.018, 2023. (Chapter 7)

Conference Proceedings

1. **S.N. Rai**, B.S. Bhadauria, "Heat/Mass transport in Walter-B nanoliquid filled in Hele-Shaw cell under 3-types of g-Jitters with magnetic field", accepted in AIP Conference Proceedings *Scopus indexed.* (Chapter 6)

Communicated

1. **S.N.Rai**, B.S. Bhadauria, A. Kumar, A. Kumar, "Study of thermal instability by magneto-convection in nanoliquid confined within Hele-Shaw cell under three types of thermal/gravity modulation". (Chapter 4)

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