ON THE EFFECT OF MEASUREMENT ERRORS ON ESTIMATION OF PARAMETERS IN FINITE POPULATION

THESIS

SUBMITTED TO

BABASAHEB BHIMRAO AMBEDKAR UNIVERSITY

(A CENTRAL UNIVERSITY)

LUCKNOW

BABASAHEB
BHIMRAO
AMBEDKAR
UNIVERSITY

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प्रज्ञा शील करूणा

ESTABLISHED 1996

FOR THE DEGREE OF

Doctor of Philosophy

IN

APPLIED STATISTICS

Submitted by

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2016

Dedicated to
My Beloved Parents

DECLARATION

I hereby declare that the research work embodied in this thesis entitled "**On the effect of measurement errors on estimation of parameters in finite population**" carried out by me under the supervision of **Dr. Shashi Bhushan, Associate Professor, Department of Mathematics and Statistics, Dr. Shakuntala Misra National Rehabilitation University, Lucknow** and the co-supervision of **Dr. Surinder Kumar, Associate Professor, Department of Applied Statistics, Babasaheb Bhimrao Ambedkar University, Lucknow** is an original work and does not contain any work submitted for the award of any degree in this university or any other University.

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CERTIFICATE

This is to certify that the thesis of titled "**On the effect of measurement errors on estimation of parameters in finite population**" submitted by **Mr. Arun Kumar** is an original research work and has not been previously submitted in part or full for the award of any other degree or diploma to this or any other university.

The thesis submitted to Babasaheb Bhimrao Ambedkar University, Lucknow satisfies all the requirements as stipulated in the Doctor of Philosophy (Ph.D.) regulations -1999 as amended in 2010 and it is fit for submission and evaluation for the award of the degree of Doctor of Philosophy of the University.

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ACKNOWLEDGEMENT

First and foremost I would like to express the deepest thank to my supervisor **Dr. Shashi Bhushan**, Associate Professor and Head, Department of Mathematics and Statistics, Dr. Shakuntala Misra National Rehabilitation University, Lucknow, for supporting me during these past four years. It has been an honour to be his Ph.D. student. He has taught me, both consciously and un-consciously, how good Statistics is. As my supervisor, he has constantly forced me to remain focused on achieving my goal. I appreciate all his contributions of time and ideas to make my Ph.D. experience productive and stimulating. The joy and enthusiasm he has for his research was contagious and motivational for me, even during tough time in the Ph.D. pursuit. I am also thankful for the excellent examples he has provided as a successful mentor and a great soul.

I would like to express appreciation to **Dr. Surinder Kumar**, Associate Professor and Head, Department of Applied Statistics, Babasaheb Bhimrao Ambedkar University, Lucknow, who is also my **Co-supervisor**, for his scientific advice and knowledge. I thank him for providing me with an opportunity to work with talented professors and researchers of our department. I also have to thank the members of my Ph.D. committee, for their helpful career advice and valuable suggestions in general.

I am also very grateful to Prof. R. C. Sobti, Vice Chancellor, Babasaheb Bhimrao Ambedkar University, Lucknow, for providing good research facilities and for building a healthy and motivational environment for research.

Lastly, I would like to thank my family and friends for all their love, support and encouragement that I needed. I specially thank my parents, brother and sisters. My hard-working parents have sacrificed their lives for my sibling and myself and provided unconditional love and care. I love so much to my family, and I would not have made it this far without them. My brother has been best friend all my life and I love him so much and thanks him for his support.

ARUN KUMAR

JUNE, 2016

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CHAPTER 1

INTRODUCTION

All along in sampling from finite population we implicitly assume that the study variable and the auxiliary variables are measured without any errors and that the data on these variables are "accurate". Here by accurate we mean that the observations are free from any observational or measurement error and they are not guess estimates, extrapolated, interpolated, or rounded of in any systematic manner, such as to the nearest hundredth dollar, and so on. Unfortunately, this ideal is not met in practice for a variety of reasons. Such measurement error is a potentially troublesome problem viz – a – viz the estimation of parameters in case of finite population.

The errors of measurement in the study variable still give unbiased estimates of the parameters and their variances. The estimated variances are now larger than in the case where there are no such errors of measurement.

In sampling theory it is well known that the efficiency of the estimators of unknown population parameters of the study variable y can be increased by use of known information on an auxiliary variable x. For improving the precision in estimating the unknown mean of study variable y by using the auxiliary variable x, which may be positively or negatively correlated with y with known; the single supplementary variable is used by Bhal and Tuteja (1991) for the exponential ratio and product type estimators. Shabbir and Gupta (2007) and Singh et al.

(2007, 09) have proposed several modified exponential estimators. Butt and Shahbaz (2009) have proposed some estimator for population mean in the case of two phase sampling using the prior information. A family of unbiased estimators has been proposed by Bhushan (2012a) using Jack – Knife technique. On similar lines, estimators for population mean using auxiliary variables under different situations have been discussed by Diana and Perrie (2007), Kalidar and Cingi (2004, 2005, 2006), Yadav and Kadilar (2013, 2014) among the other authors.

An important source of measurement errors in survey data is the nature of variables. The nature of the variable arising from the definition may often be such that exact measurement on it, is not available. This may happen mainly due to three reasons (see Shalabh (1997)). First is that the variable is clearly defined but it is hard to take correct observation at least with the currently available technique or because of the other types of practical difficulties. Second is that the variable is conceptually well defined but observation can be obtained only on some closely related substitute known as proxies and surrogates. Third is that the variable is fully comprehensible and well understood but it is not intrinsically defined and properly quantified.

Shalabh (1997) considered the estimation of population mean using ratio method and analyzed its properties in the presence of measurement error. Manisha and Singh (2001) studied the measurement error on new estimators obtained as combination of ratio and mean per unit estimators. Srivastava and Shalabh (2001) studied the effect of measurement errors on the regression method of estimation in survey sampling. Manisha and Singh (2002) further studied the role of regression estimator involving measurement errors in case of finite populations. Allen et al. (2003) and Singh and Karpe (2008, 09) studied property of some estimators of population mean and population variance under measurement error. Bhushan et al (2012b, 2012c) considered some important estimators of population mean under measurement errors.

Among the other types of non-sampling errors, a particular problem is that of non-response which is very common in surveys, especially involving human populations, as the data cannot always be collected for all the units selected in the sample. Two major types of non-response exist, the first being the unit non-response (referring to lack of completion of any part of the survey) and the other being item non-response (submission or participation in survey but failing to complete one or more components/questions of the survey). This problem of non-response leads to often misleading and unreliable estimates.

One of the major effects of non-response is that it reduces the sample size. This does not lead to wrong conclusions. But due to the smaller sample size, the precision of estimates will be smaller and the margins of error will be larger. Yet, another serious effect of non-response is that it can be selective. This occurs if due to non-response, specific groups are under-or-over represented in the survey. If these groups behave differently with respect to survey variables, this causes the estimators to be biased. Furthermore non-response also causes the variation of the estimates to increase. Thus it violates the unbiasedness and the reliability of the estimates obtained.

In the study of non-response it is convenient to think of the population as divided into two strata, the first stratum consisting of all the units for which the measurement would be obtained if the units happened to fall in the sample and the second stratum consisting of the units for which no measurements would be obtained. The composition of the two strata depends intimately on the methods used to find the units and obtain the data. There are different ways and means to control non-response errors present in any selected sample.

The problem of non-response in the sample surveys was first dealt by Hansen and Hurwitz (1946). Later it was reconsidered by Politz and Simmons (1949, 1950), El-Bardy (1956) and Srinath (1971) etc. Hansen and Hurwitz considered the problem of non-response in mail surveys, where the questionnaire were sent to the units (individuals) selected in the sample and once the deadline to return the completely filled questionnaire was over, the selected sample was divided into two groups: the first one is the response group of size n_1 consisting of all those units belonging to the sample of size n which responded to the questionnaire and the other being the non-response group of size n_2 consisting of all those units which choose not to respond to the questionnaires.

Now once the units in the non-responding group are identified, a sub-sample of size $r\left(r=\frac{n_2}{k}, k>1\right)$ is selected from the non-responding group and the data is obtained from these units by making some extra effort and collecting the information through direct interview. Then an estimator is constructed based on the responding units and the sub-sampled units from the non-responding group for estimating the population mean of the variable under study. This estimator for estimating the population mean in presence of non-response is defined as

$$\bar{y}^* = \frac{n_1}{n}\bar{y}_{(1)} + \frac{n_2}{n}\bar{y}^*_{(2)}$$

where $\left(\bar{y}_{(1)}, \bar{y}^*_{(2)}\right)$ are the sample means of $\bar{Y}$ based on n_1 units and r units respectively.

Even though by utilizing the Hansen and Hurwitz (1946) approach the non-response error present within a selected sample could be reduced and henceforth leading to lesser biased and more efficient estimates. Still the cost incurred to deal with the problem of non-response especially involving auxiliary information based on one or more auxiliary variables in two-phase sampling scheme remains a very important factor which needs to be paid a lot of attention during the planning of any survey. Therefore, along with the efficiency of the estimates the cost incurred for conducting the survey should also be kept in mind and emphasis should be paid to both percent relative efficiency and to the cost efficiency equally. After all the purpose of conducting any sample survey is to obtain precise estimates using less cost.

Cochran (1977), pursuing the Hansen and Hurwitz (1946) approach proposed the ratio and regression method of estimation while assuming that the sample units provide the auxiliary information but fail to provide the complete information on the variable under study. An efficient family of ratio and regression estimators were later proposed by Rao (1986, 1987) with sub- sampling the non-respondents using auxiliary information based on the population mean of the auxiliary variable. Further, transformed ratio type estimator for estimating the population mean using auxiliary information was suggested by Khare and Srivastava (1993, 1997). For situations when the population mean of the auxiliary variable is not known in advance, the double sampling scheme is often being employed for the ratio estimation of the population mean in presence of non-response. Okafor and Lee (2000) and Tabasum and Khan (2004, 2005) considered the ratio and regression type estimators, respectively.

Furthermore, an improvement of the estimation of mean in two-phase sampling was suggested by Singh and Kumar (2010) where they considered the population mean of the auxiliary variable in presence of non-response as though it improves both the situations when the auxiliary variable is independent of the problem of non-response as well as the situation when non-response exists in both the study and the auxiliary variable. Keeping in mind the estimators suggested by Singh and Kumar (2010), a more generalized class of estimators was suggested by Singh and Bhushan (2012) in two-phase sampling using auxiliary information. Shabbir and Khan (2013) proposed estimators of population mean using two auxiliary variables under four different situations under non-response.

There were lots of possibilities for developing and studying better estimation procedures under measurement error models especially when non-response is present. I have explored this area in the present research work entitled "***On the effect of measurement errors on estimation of parameters in finite population***".

This first chapter presents a brief introduction to the existing work done in the area of non-response error and measurement error. There seems to be no substantial work where the problems of non-response error and measurement error have been tackled simultaneously. The first seven chapters are devoted to this new area.

In chapter 2, we consider the problem of estimating the population mean of study variable using auxiliary information in presence of measurement error and non-response error simultaneously. The estimators in this chapter use auxiliary information to improve efficiency and we assume that response error is present in both the study and auxiliary variables. The properties of the suggested estimators are studied and compared with those of existing estimators. It is shown that the estimators t_g and t_d are most efficient among all the estimators considered under non-response and measurement error simultaneously. The efficiency of the estimators seems to be drastically curtailed with increasing measurement error.

The third chapter deals with a generalized estimator for estimating the population mean of study variable using auxiliary information in presence of measurement error and non-response error occurs simultaneously. The generalized estimators in this chapter are given as an alternative to the class of estimators proposed by Singh and Kumar (2008). The properties of suggested generalized estimators are studied and compared with those of existing estimators in presence of measurement error and non-response error. The suggested generalized estimator $\bar{y}_f$ is most efficient among all the estimators proposed by Singh and Kumar (2008) under both measurement and non-response errors occurring simultaneously. The efficiency of the proposed estimator seems to be drastically curtailed with increasing measurement error.

The chapter 4 is set with the problem of estimating the population mean under double sampling using auxiliary information in presence of measurement error and non-response error simultaneously. Some modified ratio, product and difference estimators in double sampling have been adapted from Singh and Kumar (2010) and their properties are studied presence of measurement error and non-response error simultaneously. An empirical study is carried to study the merits of the estimators over conventional unbiased estimator and other known estimators where we analysed the effect of measurement error on the adapted estimators at different levels. Both theoretical and empirical study results present the soundness and usefulness of the suggested estimators in practice under presence of measurement error and non-response error simultaneously.

The chapter 5 deals with a generalized class of double sampling estimators under measurement error and non-response error occurring simultaneously. Singh and Kumar (2010) proposed some ingenious classes of double sampling estimators which fared better in comparison to all the existing double sampling estimators under non-response. The result of the proposed generalized estimator is compared with the existing estimators theoretically in presence of measurement error and non-response error simultaneously. An empirical study is carried out to judge the merit of the proposed classes in presence of measurement error and non-response error.

In chapter 6, we have studied some cost efficient classes of estimators in the presence of measurement error and non-response for estimating population mean on the lines of the estimators proposed by Okafor and Lee (2000), Tabasum and Khan (2004) and recently by Singh and Kumar (2010). These estimators are an alternative to the double sampling estimators, when population mean of auxiliary variable is not known and fare better than the above estimators under cost efficiency criterion. The properties of these estimators have been studied in presence of measurement error and non-response error simultaneously.

In order to ascertain the soundness of these estimators under measurement error and non-response error, a comparative study is carried out both theoretically as well as empirically.

The seventh chapter deals with a couple of generalized cost efficient classes of estimators under measurement errors and non-response for estimating population mean using auxiliary information. These classes of estimators have been proposed as an alternative to the class of estimators proposed for only non-response by Singh and Kumar (2010) and Singh and Bhushan (2012). The results are derived under measurement error and non-response error simultaneously. These estimators are put to test against Singh and Kumar (2010) and Singh and Bhushan (2012) estimators under the cost efficiency criteria. A comprehensive comparative study is carried out both theoretically as well as empirically to study the effect of presence of measurement error.

Finally, in chapter 8, we have proposed three generalized classes of estimators of population mean, ratio and product of population mean using auxiliary information of two variables and one generalized class of estimator for population mean using auxiliary information on an auxiliary variable and an attribute in presence of measurement errors. Further, we also proposed the corresponding classes of unbiased estimators using the jack-knife version of Quenouille's method. The bias and mean square error of the proposed classes are obtained. We also analyzed the properties of the generalized estimators in presence of measurement errors. Also, some concluding remarks are made clearly demonstrating that some important class of estimators is special cases of the proposed study.

CHAPTER 2

ON ESTIMATION OF POPULATION MEAN UNDER MEASUREMENT ERROR AND NON-RESPONSE ERROR

SUMMARY

In this chapter we consider the problem of estimating the population mean of study variable using auxiliary information in presence of measurement error and non-response error simultaneously. The estimators in this chapter use auxiliary information to improve efficiency and we assume that response error is present in both the study and auxiliary variables. The properties of the suggested estimators are studied and compared with those of existing estimators. It is shown that the estimators t_g and t_d are most efficient among all the estimators considered under non-response and measurement error simultaneously. The efficiency of the estimators seems to be drastically curtailed with increasing measurement error.

2.1 INTRODUCTION

In survey sampling, the properties of the estimators based on data usually treated under the assumption that the observations are correct measurements on characteristics being studied. But such kind of assumption does not satisfied in many applications and data is recorded with measurement errors, such as reporting errors and computing errors. These measurement errors make the result invalid. If measurement error is very small and we can neglect it, then the statistical inferences based on observed data continue to remain valid. On the other hand if measurement error is not negligible, the inferences may not be simply invalid and

inaccurate, but may often lead to unexpected, undesirable and unfortunate consequences (see Srivastava and Shalabh (2001)).

Consider a finite population $U=(U_1,U_2,...,U_N)$ of N units. Let Y and X be the study variate and auxiliary variate, respectively. Suppose that we have a set of n paired observations obtained through simple random sampling procedure on two characteristics X and Y. Further, suppose that (x_i, y_i) for the i^{th} sampling units are observed with measurement error instead of their true values (X_i,Y_i). For a simple random sampling scheme, let (x_i, y_i) be observed values instead of the true values (X_i,Y_i) for i^{th} (i=1, 2,...n) unit such that

$$u_i = y_i - Y_i \tag{2.1.1}$$

$$v_i = x_i - X_i \tag{2.1.2}$$

where u_i and v_i are associated measurement errors which are stochastic (probabilistic) in nature with mean zero, variances $\pm_u^2$ and $\pm_v^2$ respectively. Further, we assume that u_i's and v_i's are uncorrelated although X_i's and Y_i's are correlated.

Let the population means of X and Y characteristics be $«_x$ and $«_y$.where population variances X and Y be $\pm_x^2$ and $\pm_y^2$ respectively. Let $\pm_{xy}$ be the population covariance between x and y.

The problem of non-response in sample survey is common and is more traditional in mail surveys than in personal interviews. Using the procedure of sub-sampling from non-respondents, Khare & Srivastava (1993, 1995) proposed the two-phase sampling ratio and product estimators for the population mean and studied their properties. The usual approach of non-response problem is to contact the non-respondent after the first mail attempt and then enumerating the subsample by personal interview. By using this approach we try to collect the information as much as possible from the non-respondents. A further improvement in the estimation procedure for the population mean in the presence of non-response using a auxiliary variate was considered by Khare & Srivastava (1997).

Consider a finite population of size N and a random sample of size n drawn by simple random sampling without replacement. In surveys on human populations, it is often the case that n_1 units respond and remaining $(n-n_1)$ do not any response. The initial survey may be conducted through the mail or by telephone, perhaps computer aided; see Rao (1986). In the case non-response, in the initial attempt, Hansen & Hurwitz (1946) suggested a double sampling scheme for estimating the population means comprising the following steps:

(a) A simple random sample of size n is drawn and the questionnaire is mailed to the sampled units.

(b) A sub-sample of size $r=(n/k)$, $(k>1)$ from the n_2 non-responding units in the initial step attempt is contacted through personal interviews.

Note that Hansen & Hurwitz (1946) are the pioneer of the non-response problem and they considered mail surveys in the first attempts, and personal interview in the second attempt. In the Hansen and Hurwitz method, the population of size N is supposed to be composed of two stratum, namely respondents and non-respondents, having size N_1 and $N_2\left(=N-N_1\right)$. Thus we label the data as $Y_1, Y_2, ..., Y_{N_1}$ for the response group, and as $Y_{N_1+1}, Y_{N_1+2}, ..., Y_{N_1+N_2}$ for the non-response stratum. Further, we assume that $\bar{Y}=\sum_{i=1}^{N} Y_i / N$ and $\sigma_y^2=\sum_{i=1}^{N}\left(Y_i-\bar{Y}\right)^2 / N$ denote population mean and variance, respectively. Let $\bar{Y}_1=\sum_{i=1}^{N_1} Y_i / N_1$ and $\sigma_{y_1}^2=\sum_{i=1}^{N_1}\left(Y_i-\bar{Y}_1\right)^2 / N_1$ denote the mean and variance of response stratum, respectively, and similarly, $\bar{Y}_2=\sum_{i=N_1+1}^{N_1+N_2} Y_i / N_2$ and $\sigma_{y_2}^2=\sum_{i=N_1+1}^{N_1+N_2}\left(Y_i-\bar{Y}_2\right)^2 / N_2$ denote the mean and variance of non-response stratum. The population mean can be written as $\bar{Y}=W_1\bar{Y}_1+W_2\bar{Y}_2$, where $W_1=N_1 / N$ and $W_2=N_2 / N$. The sample mean $\bar{y}_1=\sum_{i=1}^{n_1} y_i / n_1$ is unbiased for $\bar{Y}_1$, but has a bias equal to $W_2(\bar{Y}_1-\bar{Y}_2)$ in estimating the population mean $\bar{Y}$.

The sample mean $\bar{y}_{2r}=\sum_{i=1}^{r} y_i / r$ is unbiased for the mean $\bar{y}_2$ of the n_2 units.

An unbiased estimator for the population mean $\bar{Y}$ is

$$\bar{y}^*=w_1\bar{y}_1+w_2\bar{y}_{2r} \tag{2.1.3}$$

where $w_1=n_1 / n$ and $w_2=n_2 / n$.

The variance of $\bar{y}^*$ is given by

$$Var(\bar{y}^*)=\frac{1}{n}\sigma_y^2+\frac{W_2\left(k-1\right)}{n}\sigma_{y_2}^2 \tag{2.1.4}$$

Let $X_i\left(i = 1, 2,\ldots, N\right)$ represent a supplementary variate correlated with the study variate Y_i $\left(i = 1, 2,\ldots, N\right)$. The population mean of the supplementary variate x is $\bar{X} = \sum_{i=1}^{N} X_i / N$. Let $\bar{X}_1$ and $\bar{X}_2$ denote the means of response and non-response stratum, respectively. Let $\bar{x}$ denotes the mean of all n units. Further, we suppose that $\bar{x}_1$ and $\bar{x}_2$ denote the means of n_1 responding units and n_2 non-responding units, respectively. Furthermore, let, $\bar{x}_{2r} = \sum_{i=1}^{r} x_i / r$ represent the mean of the sub-sampled units and the population variance of x and y are denoted by $\pm_x^2$ and $\pm_y^2$ respectively. The population covariance of X and Y denoted by $\pm_{xy}$. Further the unbiased estimator of the population mean $\bar{X}$ of the auxiliary variate x is

$$\bar{x}^* = w_1\bar{x}_1 + w_2\bar{x}_{2r} \qquad (2.1.5)$$

and the variance of $\bar{x}^*$ is given by

$$Var(\bar{x}^*) = \frac{1}{n}\pm_x^2 + \frac{W_2\left(k-1\right)}{n}\pm_{x_2}^2 \qquad (2.1.6)$$

where $\pm_{x_2}^2 = \sum_{i=N_1+1}^{N=N_1+N_2} \left(X_i - \bar{X}_2\right)^2 / N_2$.

2.2 ADAPTED ESTIMATORS UNDER NON-RESPONSE ERROR AND MEASUREMENT ERROR SIMULTANEOUSLY

Here we describe the estimators for four different cases.

SITUATION I

Non-response error and measurement error occurs on both the study variable y and the auxiliary variable x, and the population mean $\bar{X}$ of the auxiliary variable is known. Then the conventional ratio, product and difference estimators for the estimating the mean $\bar{Y}$ of study variable y are respectively defined as

$$t_{R1} = \bar{y}^* \left(\frac{\bar{X}}{\bar{x}^*} \right) \tag{2.2.1}$$

$$t_{P1} = \bar{y}^* \left(\frac{\bar{x}^*}{\bar{X}} \right) \tag{2.2.2}$$

$$t_{lr1} = \bar{y}^* + d_1 \left(\bar{X} - \bar{x}^* \right) \tag{2.2.3}$$

where d_1 is suitably chosen constant.

SITUATION II

Non-response error occurs only on the study variable whereas measurement error occurs on both the study variable as well as on the auxiliary variable and the population mean $\bar{X}$ of the auxiliary variable is known. In this case the usual ratio, product and difference estimators for estimating the population mean $\bar{Y}$ of study variable are defined as

$$t_{R2} = \bar{y}^* \left(\frac{\bar{X}}{\bar{x}} \right) \tag{2.2.4}$$

$$t_{P2} = \bar{y}^* \left(\frac{\bar{x}}{\bar{X}} \right) \tag{2.2.5}$$

$$t_{lr2} = \bar{y}^* + d_2 \left(\bar{X} - \bar{x} \right) \tag{2.2.6}$$

where d_2 is suitably chosen constant.

SITUATION III

Non-response occurs on the study variable whereas measurement error occurs on both the study variable as well as on the auxiliary variable and the population mean $\bar{X}$ of the auxiliary variable is known. In this situation the usual ratio, product and difference estimators for estimating the population mean $\bar{Y}$ of study variate y are defined as below

$$t_1 = \bar{y}^* \left(\frac{\bar{x}}{\bar{x}^*} \right) \tag{2.2.7}$$

$$t_2 = \bar{y}^* \left(\frac{\bar{x}^*}{\bar{x}} \right) \tag{2.2.8}$$

$$t_{lr3} = \bar{y}^* + d_3 \left(\bar{x} - \bar{x}^* \right) \tag{2.2.9}$$

where d_3 is suitably chosen constant

Situation IV

We consider situation II, where the non-response errors occurs only in the study variable and measurement errors occurs both the variables study variable as well as supplementary variable. Information on the supplementary variable x is obtained from all the sample units (i.e. the initial sample units) and the population mean $\bar{X}$ of the auxiliary variable is known but some sample units are not responding about the information of study variable y. Now using this information we define the following estimators of the population mean $\bar{Y}$ as

$$t_{R4} = \bar{y}^* \left(\bar{X} / \bar{x}^* \right) \left(\bar{X} / \bar{x} \right) \tag{2.2.10}$$

$$t_{P4} = \bar{y}^* \left(\bar{x}^* / \bar{X} \right) \left(\bar{x} / \bar{X} \right) \tag{2.2.11}$$

$$t_g = \bar{y}^* \left(\bar{x}/\bar{x}^*\right)^{\sim_1} \left(\bar{X}/\bar{x}\right)^{\sim_2} \quad (2.2.12)$$

$$t_d = \bar{y}^* + d_1'\left(\bar{x} - \bar{x}^*\right) + d_2'\left(\bar{X} - \bar{x}\right) \quad (2.2.13)$$

where $\sim_i$ s *(i=1, 2)* and d_i s *(i=1, 2)* are suitably chosen constant.

Here we note that, when suggesting the estimator for the population mean $\bar{Y}$ Rao (1986) used only the information on the sample mean $\bar{x}$ and on the population mean $\bar{X}$ of the auxiliary variate *x*. However, one can also obtain the unbiased estimator $\bar{x}^* = \left(n_1/n\right)\bar{x}_1 + \left(n_2/n\right)\bar{x}_{2r}$ of $\bar{X}$ (without any extra effort) while in the process of obtaining $\bar{y}^* = \left(n_1/n\right)\bar{y}_1 + \left(n_2/n\right)\bar{y}_{2r}$, the unbiased estimator of the population mean $\bar{Y}$. Hence in situation 2, we have two unbiased estimators, $\bar{x}^*$ and $\bar{x}$, of the population mean $\bar{X}$ of the supplementary variate *x*.

2.3 BIAS AND MEAN SQUARE ERROR

For obtaining the bias and mean square error of the above estimator define in different situation, we define the following error terms

$\bar{y}^* = \bar{Y} + £_0^*$, $\quad \bar{x}^* = \bar{X} + £_1^*$ $\quad \bar{x} = \bar{X} + £_1$

such that

$E(£_0^*) = E(£_1^*) = E(£_1) = 0$

and

$$E(£_0^{*2}) = \frac{1}{n}\left(±_y^2 + ±_u^2\right) + \frac{W_2(k-1)}{n}\left(±_{y_2}^2 + ±_{u_2}^2\right)$$

$$E(£_1^{*2}) = \frac{1}{n}\left(±_x^2 + ±_v^2\right) + \frac{W_2(k-1)}{n}\left(±_{x_2}^2 + ±_{v_2}^2\right)$$

$$E(£_1^2) = \frac{1}{n}\left(±_x^2 + ±_v^2\right)$$

$$E(\pounds_0^*\pounds_1^*) = \text{cov}(\bar{y}^*,\ \bar{x}^*) = \frac{1}{n}\pm_{xy} + W_2\frac{(k-1)}{n}\pm_{xy2}$$

$$E(\pounds_0^*\pounds_1) = \text{cov}(\bar{y}^*,\ \bar{x}) = \frac{1}{n}\pm_{xy}$$

$$E(\pounds_1^*\pounds_1) = \text{cov}(\bar{x}^*,\ \bar{x}) = \frac{1}{n}\left(\pm_x^2 + \pm_v^2\right)$$

The bias of the estimators (2.2.1), (2.2.2), (2.2.4), (2.2.5), (2.2.7), (2.2.8), (2.2.10), (2.2.11) and (2.2.12) are given below while the estimators (2.2.3), (2.2.6), (2.2.9) and (2.2.13) are the unbiased estimators.

$$Bias(t_{R1}) = \frac{1}{n\bar{X}}\left[\left\{R\left(\pm_x^2 + \pm_v^2\right) - \pm_{xy}\right\} + W_2(k-1)\left\{R\left(\pm_{x_2}^2 + \pm_{v_2}^2\right) - \pm_{xy_2}\right\}\right] \quad (2.3.1)$$

$$Bias(t_{P1}) = \frac{1}{n\bar{X}}\left[\pm_{xy} + W_2(k-1)\pm_{xy_2}\right] \quad (2.3.2)$$

$$Bias(t_{R2}) = \frac{1}{n\bar{X}}\left[\left\{R\left(\pm_x^2 + \pm_v^2\right) - \pm_{xy}\right\}\right] \quad (2.3.3)$$

$$Bias(t_{P2}) = \frac{1}{n\bar{X}}\pm_{xy} \quad (2.3.4)$$

$$Bias(t_1) = \frac{W_2(k-1)}{n\bar{X}}\left[\left\{R\left(\pm_{x_2}^2 + \pm_{v_2}^2\right) - \pm_{xy_2}\right\}\right] \quad (2.3.5)$$

$$Bias(t_2) = \frac{W_2(k-1)}{n\bar{X}}\left[\pm_{xy_2}\right] \quad (2.3.6)$$

$$Bias\left(t_{R4}\right) = \frac{1}{n\bar{X}}\left[3R\left(\pm_x^2 + \pm_v^2\right) - 2\pm_{xy}\right] + \frac{W_2(k-1)}{n\bar{X}}\left[R\left(\pm_{x_2}^2 + \pm_{v_2}^2\right) - \pm_{xy_2}\right] \quad (2.3.7)$$

$$Bias\left(t_{P4}\right) = \frac{1}{n\bar{X}}\left[\left(\pm_x^2 + \pm_v^2\right) + 2\pm_{xy}\right] + \frac{W_2(k-1)}{n\bar{X}}\pm_{xy_2} \quad (2.3.8)$$

$$Bias\left(t_g\right) = \frac{1}{2n\bar{X}}\left[\sim_2\left(\sim_2 + 1\right)\left(\pm_x^2 + \pm_v^2\right) - 2\sim_2\pm_{xy}\right]$$

$$+\frac{W_2\left(k-1\right)}{2n\bar{X}}\left[\sim_1\left(\sim_1 + 1\right)\left(\pm_{x_2}^2 + \pm_{v_2}^2\right) - 2\sim_1\pm_{xy_2}\right] \quad (2.3.9)$$

The mean square error of the above estimators define in situation I to situation IV, to the first degree of approximation are given as

$$MSE\left(t_{R1}\right)=MSE\left(\bar{y}^{*}\right)+\frac{1}{n}\left[R^{2}\left(\pm_{x}^{2}+\pm_{v}^{2}\right)-2R\pm_{xy}\right]$$

$$+\frac{W_{2}\left(k-1\right)}{n}\left[R^{2}\left(\pm_{x_{2}}^{2}+\pm_{v_{2}}^{2}\right)-2R\pm_{xy(2)}\right] \tag{2.3.10}$$

$$MSE\left(t_{P1}\right)=MSE\left(\bar{y}^{*}\right)+\frac{1}{n}\left[R^{2}\left(\pm_{x}^{2}+\pm_{v}^{2}\right)+2R\pm_{xy}\right]$$

$$+\frac{W_{2}\left(k-1\right)}{n}\left[R^{2}\left(\pm_{x_{2}}^{2}+\pm_{v_{2}}^{2}\right)+2R\pm_{xy(2)}\right] \tag{2.3.11}$$

$$MSE\left(t_{lr1}\right)=MSE\left(\bar{y}^{*}\right)+\frac{1}{n}\left[d_{1}^{2}\left(\pm_{x}^{2}+\pm_{v}^{2}\right)-2d_{1}\pm_{xy}\right]$$

$$+\frac{W_{2}\left(k-1\right)}{n}\left[d_{1}^{2}\left(\pm_{x_{2}}^{2}+\pm_{v_{2}}^{2}\right)-2d_{1}\pm_{xy(2)}\right] \tag{2.3.12}$$

$$MSE\left(t_{R2}\right)=MSE\left(\bar{y}^{*}\right)+\frac{1}{n}\left[R^{2}\left(\pm_{x}^{2}+\pm_{v}^{2}\right)-2R\pm_{xy}\right]+\frac{W_{2}(k-1)}{n}\pm_{u_{2}}^{2} \tag{2.3.13}$$

$$MSE\left(t_{P2}\right)=MSE\left(\bar{y}^{*}\right)+\frac{1}{n}\left[R^{2}\left(\pm_{x}^{2}+\pm_{v}^{2}\right)+2R\pm_{xy}\right]+\frac{W_{2}(k-1)}{n}\pm_{u_{2}}^{2} \tag{2.3.14}$$

$$MSE\left(t_{lR2}\right)=MSE\left(\bar{y}^{*}\right)+\frac{1}{n}\left[d_{2}^{2}\left(\pm_{x}^{2}+\pm_{v}^{2}\right)-2d_{2}\pm_{xy}\right]+\frac{W_{2}(k-1)}{n}\pm_{u_{2}}^{2} \tag{2.3.15}$$

$$MSE\left(t_{1}\right)=MSE\left(\bar{y}^{*}\right)+\frac{1}{n}\left[R^{2}\left(\pm_{x}^{2}+\pm_{v}^{2}\right)\right]+\frac{W_{2}(k-1)}{n}\left[R^{2}\left(\pm_{x_{2}}^{2}+\pm_{v_{2}}^{2}\right)-2R\pm_{xy(2)}\right] \tag{2.3.16}$$

$$MSE\left(t_{2}\right)=MSE\left(\bar{y}^{*}\right)+\frac{1}{n}\left[R^{2}\left(\pm_{x}^{2}+\pm_{v}^{2}\right)\right]+\frac{W_{2}(k-1)}{n}\left[R^{2}\left(\pm_{x_{2}}^{2}+\pm_{v_{2}}^{2}\right)+2R\pm_{xy(2)}\right] \tag{2.3.17}$$

$$MSE\left(t_{lr3}\right)=MSE\left(\bar{y}^{*}\right)+\frac{1}{n}\left[d_{3}^{2}\left(\pm_{x}^{2}+\pm_{v}^{2}\right)\right]+\frac{W_{2}(k-1)}{n}\left[d_{3}^{2}\left(\pm_{x_{2}}^{2}+\pm_{v_{2}}^{2}\right)+2d\pm_{xy(2)}\right] \tag{2.3.18}$$

$$MSE(t_{R4})=MSE\left(\bar{y}^{*}\right)+\frac{1}{n}\left[4R^{2}\left(\pm_{x}^{2}+\pm_{v}^{2}\right)-4R\pm_{xy}\right]$$

$$+\frac{W_{2}\left(k-1\right)}{n}\left[R^{2}\left(\pm_{x}^{2}+\pm_{v}^{2}\right)-2R\pm_{xy}\right] \tag{2.3.19}$$

$$MSE(t_{P4}) = MSE\left(\bar{y}^*\right) + \frac{1}{n}\left[4R^2\left(\pm_x^2 + \pm_v^2\right) + 4R\pm_{xy}\right]$$

$$+\frac{W_2\left(k-1\right)}{n}\left[R^2\left(\pm_x^2 + \pm_v^2\right) + 2R\pm_{xy}\right] \quad (2.3.20)$$

$$MSE(t_g) = MSE\left(\bar{y}^*\right) + \frac{1}{n}\left[R^2\sim_2^2\left(\pm_x^2 + \pm_v^2\right) - 2R\sim_2\pm_{xy}\right]$$

$$+\frac{W_2\left(k-1\right)}{n}\left[R^2\sim_1^2\left(\pm_x^2 + \pm_v^2\right) - 2R\sim_1\pm_{xy_2}\right] \quad (2.3.21)$$

$$MSE(t_d) = MSE\left(\bar{y}^*\right) + \frac{d_2'}{n}\left[d_2'\left(\pm_x^2 + \pm_v^2\right) - 2\pm_{xy}\right]$$

$$+\frac{W_2(k-1)d_1'}{n}\left[d_1'\left(\pm_{x_2}^2 + \pm_{v_2}^2\right) - 2\pm_{xy_2}\right] \quad (2.3.22)$$

2.4 OPTIMUM VALUES AND MINIMUM MSE'S

Now, minimizing (2.3.12) with respect to d_1 we obtained the optimum value of d_1

$$d_1 = \frac{\pm_{xy} + W_2\left(k-1\right)\pm_{xy_2}}{\left(\pm_x^2 + \pm_v^2\right) + W_2\left(k-1\right)\left(\pm_{x_2}^2 + \pm_{v_2}^2\right)} \quad (2.4.1)$$

and the minimum MSE of (2.2.3) is

$$MSE\left(t_{lr1}\right)\min. = MSE\left(\bar{y}^*\right) - \frac{\left[\pm_{xy} + W_2\left(k-1\right)\pm_{xy_2}\right]^2}{n\left[\left(\pm_x^2 + \pm_v^2\right) + W_2\left(k-1\right)\left(\pm_{x_2}^2 + \pm_{v_2}^2\right)\right]} \quad (2.4.2)$$

On optimizing (2.3.15) the optimum value of d_2 is

$$d_2 = \frac{\pm_{xy}}{\left(\pm_x^2 + \pm_v^2\right)} \quad (2.4.3)$$

and the minimum MSE of (2.2.6) is given by

$$MSE\left(t_{lr2}\right)\min. = MSE\left(\bar{y}^*\right) - \frac{\pm_{xy}}{n\left(\pm_x^2 + \pm_v^2\right)} \quad (2.4.4)$$

On optimizing (2.3.18) the optimum value of d_3 is

$$d_3 = \frac{\pm_{xy_2}}{\left(\pm_{x_2}^2 + \pm_{v_2}^2\right)} \tag{2.4.5}$$

and the minimum MSE of t_{lr3} is given by

$$MSE\left(t_{lr3}\right)\min. = MSE\left(\bar{y}^*\right) - \frac{W_2\left(k-1\right)}{n}\frac{\pm_{xy_2}}{\left(\pm_{x_2}^2 + \pm_{v_2}^2\right)} \tag{2.4.6}$$

Differentiating (2.3.21) with respect to $\sim_1, \sim_2$ and equating the resulting derivatives to zero, we obtain the optimum value of $\sim_1$ and $\sim_2$ given as

$$\sim_1 = \frac{\pm_{xy_2}}{R\left(\pm_{x_2}^2 + \pm_{v_2}^2\right)} \text{ and } \sim_2 = \frac{\pm_{xy}}{R\left(\pm_x^2 + \pm_v^2\right)} \tag{2.4.7}$$

Substitution the values of $\sim_1$ and $\sim_2$ in the expression of MSE of t_g we get the minimum MSE of t_g given by

$$MSE(t_g)_{\min} = MSE\left(\bar{y}^*\right) - \frac{1}{n}\left[\frac{\pm_{xy}^{\ 2}}{\pm_x^2 + \pm_v^2}\right] - \frac{W_2(k-1)}{n}\left[\frac{\pm_{xy_2}^{\ 2}}{\pm_{x_2}^2 + \pm_{v_2}^2}\right] \tag{2.4.8}$$

Now, minimizing (2.3.22) with respect to d_1' and d_2', we obtained the optimum values of d_1' and d_2' as

$$d_1' = \frac{\pm_{xy_2}}{\pm_{x_2}^2 + \pm_{v_2}^2} \text{ and } d_2' = \frac{\pm_{xy}}{\pm_x^2 + \pm_v^2}\quad . \tag{2.4.9}$$

Putting (2.4.9) in (2.3.22), the minimum mean square error of t_d is

$$MSE(t_d)_{\min} = MSE\left(\bar{y}^*\right) - \frac{1}{n}\left[\frac{\pm_{xy}^{\ 2}}{\pm_x^2 + \pm_v^2}\right] - \frac{W_2(k-1)}{n}\left[\frac{\pm_{xy_2}^{\ 2}}{\pm_{x_2}^2 + \pm_{v_2}^2}\right] \tag{2.4.10}$$

which is the same as the minimum mean square error of the difference estimator t_g given by (2.4.8).

2.5 EFFICIENCY COMPARISONS

i. $MSE\left(\bar{y}^{*}\right)-MSE\left(t_{R1}\right)>0$, if

$$R>\frac{\pm_{xy}}{\pm_{x}^{2}+\pm_{v}^{2}} \text{ and } R>\frac{\pm_{xy_2}}{\pm_{x_2}^{2}+\pm_{v_2}^{2}}$$

ii. $MSE\left(\bar{y}^{*}\right)-MSE\left(t_{P1}\right)>0$, if

$$R>\frac{-\pm_{xy}}{\pm_{x}^{2}+\pm_{v}^{2}} \text{ and } R>\frac{-\pm_{xy_2}}{\pm_{x_2}^{2}+\pm_{v_2}^{2}}$$

iii. $MSE\left(\bar{y}^{*}\right)-MSE\left(t_{lr1}\right)>0$, if

$$d_1>\frac{\pm_{xy}}{\pm_{x}^{2}+\pm_{v}^{2}} \text{ and } d_1>\frac{\pm_{xy_2}}{\pm_{x_2}^{2}+\pm_{v_2}^{2}}$$

iv. $MSE\left(\bar{y}^{*}\right)-MSE\left(t_{R2}\right)>0$, if

$$R>\frac{\pm_{xy}}{\pm_{x}^{2}+\pm_{v}^{2}}$$

v. $MSE\left(\bar{y}^{*}\right)-MSE\left(t_{P1}\right)>0$, if

$$R>\frac{-\pm_{xy}}{\pm_{x}^{2}+\pm_{v}^{2}}$$

vi. $MSE\left(\bar{y}^{*}\right)-MSE\left(t_{lr2}\right)>0$, if

$$d_2>\frac{\pm_{xy}}{\pm_{x}^{2}+\pm_{v}^{2}}$$

vii. $MSE\left(\bar{y}^{*}\right)-MSE\left(t_{1}\right)>0$, if

$$R>\frac{\pm_{xy_2}}{\pm_{x_2}^{2}+\pm_{v_2}^{2}}$$

viii. $MSE\left(\bar{y}^{*}\right)-MSE\left(t_{2}\right)>0$, if

$$R > \frac{-\pm_{xy_2}}{\pm_{x_2}^2 + \pm_{v_2}^2}$$

ix. $MSE\left(\bar{y}^*\right) - MSE\left(t_{lr3}\right) > 0$, if

$$d_3 > \frac{\pm_{xy_2}}{\pm_{x_2}^2 + \pm_{v_2}^2}$$

x. $MSE\left(\bar{y}^*\right) - MSE\left(t_{R4}\right) > 0$, if

$$R > \frac{\pm_{xy}}{\pm_x^2 + \pm_v^2} \text{ and } R > \frac{\pm_{xy_2}}{\pm_{x_2}^2 + \pm_{v_2}^2}$$

xi. $MSE\left(\bar{y}^*\right) - MSE\left(t_{P4}\right) > 0$, if

$$R > \frac{-\pm_{xy}}{\pm_x^2 + \pm_v^2} \text{ and } R > \frac{-\pm_{xy_2}}{\pm_{x_2}^2 + \pm_{v_2}^2}$$

xii. $MSE\left(\bar{y}^*\right) - MSE\left(t_g\right) > 0$, if

$$R > \frac{\pm_{xy}}{\sim_2\left(\pm_x^2 + \pm_v^2\right)} \text{ and } R > \frac{\pm_{xy_2}}{\sim_1\left(\pm_{x_2}^2 + \pm_{v_2}^2\right)}$$

xiii. $MSE\left(\bar{y}^*\right) - MSE\left(t_d\right) > 0$, if

$$d_1' > \frac{\pm_{xy_2}}{\pm_{x_2}^2 + \pm_{v_2}^2} \text{ and } d_2' > \frac{\pm_{xy}}{\pm_x^2 + \pm_v^2}$$

2.6 OPTIMUM VALUE OF n AND k

The expected total cost of the survey apart from the overhead cost is given by

$$C = n\left(c + c_1 W_1 + \frac{c_2 W_2}{k}\right) \tag{2.6.1}$$

where c is the cost per unit of the first attempt with the sample, n ; c_1 is the cost per unit for processing the respondent data at the first attempt in n_1 and c_2 is the cost per unit associated with the sub sample r of n_2.

Let the MSE of the estimators is

$$MSE(t_i) = \frac{V_1 - W_2V_2}{n} - \frac{W_2V_2k}{n}$$

CASE I: FIXED VARIANCE

To determine the optimum values of n and k that minimize the cost for a fixed variance V_0 , we consider the function,

$$¤ = C^* + ª\left\{MSE\left(\bar{y}^*\right) - V_0\right\}$$

$$= n\left[c + c_1W_1 + \frac{c_2W_2}{k}\right] + ª\left[\bar{Y}^2\left\{\frac{(V_1 - W_2V_2)}{n} + \frac{W_2V_2k}{n}\right\} - V_0\right] \qquad (2.6.2)$$

where V_1 is the term of coefficient of $\frac{1}{n}$ and V_2 is the term of coefficient of $\frac{W_2(k-1)}{n}$ and ª is Lagrange's multiplier.

Now, differentiating (2.6.2) with respect to n and k, and on equating them with zero, we get

$$n = \sqrt{\frac{ª\left\{V_1 + (k-1)W_2V_2\right\}}{\left\{c + c_1W_1 + \frac{c_2W_2}{k}\right\}}} \qquad (2.6.3)$$

$$\frac{n}{k} = \sqrt{\frac{ª\bar{Y}^2V_2}{c_2}} \qquad (2.6.4)$$

On putting (2.6.3) in (2.6.4), we get

$$k_{opt} = \sqrt{\frac{c_2\left(V_1 - W_2 V_2\right)}{\left(c + c_1 W_1\right) V_2}} \tag{2.6.5}$$

which is required optimum value of k. Further, substituting the value of n and k in the expression of MSE, we get

$$\sqrt{\text{ª}} = \frac{\sqrt{\left\{V_1 + \left(k-1\right) W_2 V_2\right\}\left\{c + c_1 W_1 + \frac{c_2 W_2}{k}\right\}}}{V_0} \tag{2.6.6}$$

On using this value of ª, we get the optimum value of n given by,

$$n_{opt} = \frac{\left\{V_1 + \left(k_{opt} - 1\right) W_2 V_2\right\}}{V_0} \tag{2.6.7}$$

On substituting the optimum value of n and k in (A.2.1), we get the minimum cost for fixed variance V_0 given by

$$C^* = \frac{\left\{V_1 + \left(k_{opt} - 1\right) W_2 V_2\right\}\left\{c + c_1 W_1 + \frac{c_2 W_2}{k_{opt}}\right\}}{V_0} \tag{2.6.8}$$

CASE II: FIXED COST

In order to determine the optimum values of n and k that minimize the $MSE(\bar{y}^*)$ for a fixed cost C_0, we consider the following function using the Lagrange's principle of maxima and minima

$$¤^* = MSE(\bar{y}^*) + \text{ª}\left\{\left(c + c_1 W_1 + \frac{c_2 W_2}{k}\right) - C_0\right\}$$

$$= \left\{\frac{V_1 - W_2 V_2}{n} - \frac{W_2 V_2 k}{n}\right\} + \text{ª}\left\{\left(c + c_1 W_1 + \frac{c_2 W_2}{k}\right) - C_0\right\} \tag{2.6.9}$$

where V_1 is the term of coefficient of $\frac{1}{n}$ and V_2 is the term of coefficient of $\frac{W_2\left(k-1\right)}{n}$ and ª is Lagrange's multiplier.

Now, differentiating (2.6.9) with respect to n and k, and equating them to zero, we get

$$n = \sqrt{\frac{\left\{V_1 + (k-1)W_2V_2\right\}}{a\left\{c + c_1W_1 + \dfrac{c_2W_2}{k}\right\}}} \tag{2.6.10}$$

$$\frac{n}{k} = \sqrt{\frac{V_2}{a\,c_2}} \tag{2.6.11}$$

On using (2.6.10) in (2.6.11), we get

$$k_{opt} = \sqrt{\frac{c_2\left(V_1 - W_2V_2\right)}{\left(c + c_1W_1\right)V_2}} \tag{2.6.12}$$

Further, substituting the values of n and k in the expression of expected cost, we get

$$\sqrt{a} = \sqrt{\frac{\left\{V_1 + \left(k_{opt} - 1\right)W_2V_2\right\}\left\{c + c_1W_1 + \dfrac{c_2W_2}{k_{opt}}\right\}}{C_0}} \tag{2.6.13}$$

Similarly on substituting the value of a in (2.6.11), we get the optimum value of n as

$$n_{opt} = \frac{C_0}{\left\{c + c_1W_1 + \dfrac{c_2W_2}{k_{opt}}\right\}} \tag{2.6.14}$$

Substituting the optimum value of n and k, we get the mean square error of $\bar{y}^*$ for fixed cost C_0 given by

$$MSE\left(\bar{y}^*\right) = \left[\frac{\left\{V_1 + \left(k_{opt} - 1\right)W_2V_2\right\}\left\{c + c_1W_1 + \dfrac{c_2W_2}{k_{opt}}\right\}}{C_0}\right] \tag{2.6.15}$$

2.7 EMPIRICAL STUDY

The present data belong to the data on physical growth of upper socio-economic group of 95 school going children of Varanasi under an ICMR study, Department of Pediatrics, BHU during 1983-84 has been taken under study, Khare and Sinha (2007). The first 25% (i.e. 24 children) units have been considered as non-response units. The values of parameters related to the study characters y (weight of children in kg), the auxiliary character x (chest circumference of the children in cm) have been given as follows:

$\bar{Y}_2 = 19.4968;\ \bar{X} = 55.8611;\ \pm_y = 3.0435;\ \pm_x = 3.2735;\ \pm_{y(2)} = 02.3552;\ \pm_{x(2)} = 2.5137;$

$\pm_{yx} = 8.428611;\ \pm_{yx(2)} = 4.315874.$

The problem considered is to estimate the weight of the male children aged 6-7 years using chest circumference as the auxiliary character.

Table 2.7.1 *PRE & MSE under measurement errors & non-response*

↓ Estimators		$1/k$			
	↓ME %	1/2	1/3	1/4	1/5
$\overline{y}^*$	0%	100(0.3043)	100(0.3439)	100(0.3835)	100(0.4231)
	1%	99(0.3073)	99(0.3473)	99(0.3874)	99(0.4274)
	5%	95(0.3195)	95(0.3611)	95(0.4027)	95(0.4443)
	10%	91(0.3347)	91(0.3783)	91(0.4219)	91(0.4655)
	15%	87(0.3499)	87(0.3955)	87(0.4410)	87(0.4866)
	20%	83(0.3651)	83(0.4127)	83(0.4602)	83(0.5078)
t_{R1}	↓ME %	1/2	1/3	1/4	1/5
	0%	193(0.1574)	190(0.1810)	187(0.2046)	185(0.2282)
	1%	189(0.1609)	186(0.1850)	183(0.2090)	182(0.2331)
	5%	174(0.1748)	171(0.2007)	169(0.2265)	168(0.2524)
	10%	158(0.1922)	156(0.2203)	154(0.2484)	153(0.2765)
	15%	145(0.2095)	143(0.2399)	142(0.2702)	141(0.3006)
	20%	134(0.2269)	133(0.2595)	131(0.2921)	130(0.3247)
t_{P1}	↓ME %	1/2	1/3	1/4	1/5
	0%	56.69(0.5367)	56.99(0.6033)	57.24(0.6699)	57.44(0.7366)
	1%	56.33(0.5402)	56.63(0.6073)	56.87(0.6743)	57.07(0.7414)
	5%	54.92(0.5540)	55.21(0.6229)	55.44(0.6918)	55.62(0.7607)
	10%	53.25(0.5714)	53.52(0.6425)	53.74(0.7137)	53.91(0.7848)
	15%	51.68(0.5888)	51.93(0.6622)	52.14(0.7356)	52.31(0.8090)
	20%	50.20(0.6061)	50.44(0.6818)	50.63(0.7574)	50.79(0.8331)
t_{lr1}	↓ME %	1/2	1/3	1/4	1/5
	0%	323(0.0942)	304(0.1132)	290(0.1321)	280(0.1510)
	1%	306(0.0993)	289(0.1189)	277(0.1384)	268(0.1579)
	5%	255(0.1194)	243(0.1413)	235(0.1632)	229(0.1851)
	10%	212(0.1437)	204(0.1685)	198(0.1933)	194(0.2182)
	15%	182(0.1673)	177(0.1948)	172(0.2224)	169(0.2500)
	20%	160(0.1901)	156(0.2204)	153(0.2507)	151(0.2810)
t_{R2}	↓ME %	1/2	1/3	1/4	1/5
	0%	175(0.1735)	161(0.2131)	152(0.2527)	145(0.2923)
	1%	172(0.1769)	159(0.2169)	149(0.2569)	143(0.2969)
	5%	160(0.1905)	148(0.2322)	140(0.2738)	134(0.3154)
	10%	147(0.2076)	137(0.2512)	130(0.2948)	125(0.3384)
	15%	135(0.2247)	127(0.2703)	121(0.3158)	117(0.3614)
	20%	126(0.2418)	119(0.2893)	114(0.3369)	110(0.3844)

	↓ME %	1/2	1/3	1/4	1/5
t_{P2}	0%	59.70(0.5097)	62.61(0.5493)	65.12(0.5889)	67.32(0.6285)
	1%	59.30(0.5131)	62.18(0.5531)	64.66(0.5931)	66.83(0.6331)
	5%	57.76(0.5268)	60.51(0.5684)	62.88(0.6100)	64.94(0.6516)
	10%	55.95(0.5438)	58.54(0.5874)	60.78(0.6310)	62.73(0.6746)
	15%	54.25(0.5609)	56.70(0.6065)	58.82(0.6520)	60.66(0.6976)
	20%	52.64(0.5780)	54.98(0.6255)	56.98(0.6731)	58.72(0.7206)
t_{lr2}	↓ME %	1/2	1/3	1/4	1/5
	0%	265(0.1149)	223(0.1545)	198(0.1941)	181(0.2337)
	1%	254(0.1198)	215(0.1598)	192(0.1998)	176(0.2398)
	5%	219(0.1391)	190(0.1807)	173(0.2223)	160(0.2640)
	10%	187(0.1625)	167(0.2061)	154(0.2497)	144(0.2933)
	15%	164(0.1852)	149(0.2308)	139(0.2763)	131(0.3219)
	20%	147(0.2073)	135(0.2548)	127(0.3024)	121(0.3499)
t_1	↓ME %	1/2	1/3	1/4	1/5
	0%	93(0.3255)	98(0.3492)	103(0.3728)	108(0.3964)
	1%	92(0.3317)	97(0.3557)	101(0.3798)	105(0.4038)
	5%	85(0.3561)	90(0.3820)	94(0.4078)	98(0.4337)
	10%	79(0.3867)	83(0.4148)	87(0.4429)	90(0.4711)
	15%	73(0.4173)	77(0.4477)	80(0.4780)	83(0.5084)
	20%	68(0.4479)	72(0.4805)	75(0.5131)	78(0.5458)
t_2	↓ME %	1/2	1/3	1/4	1/5
	0%	83(0.3686)	79(0.4352)	76(0.5019)	74(0.5685)
	1%	81(0.3747)	78(0.4418)	75(0.5089)	73(0.5760)
	5%	76(0.3992)	73(0.4681)	71(0.5370)	70(0.6059)
	10%	71(0.4298)	69(0.5009)	67(0.5721)	66(0.6432)
	15%	66(0.4603)	64(0.5338)	63(0.66072)	62(0.6806)
	20%	62(0.4909)	61(0.5666)	60(0.6423)	59(0.7179)
t_{lr3}	↓ME %	1/2	1/3	1/4	1/5
	0%	-	-	-	-
	1%	-	-	-	-
	5%	-	-	-	-
	10%	-	-	-	-
	15%	-	-	-	-
	20%	-	-	-	-

	↓ME %	1/2	1/3	1/4	1/5
t_{R4}	0%	301(0.1012)	276(0.1248)	258(0.1484)	246(0.1720)
	1%	288(0.1058)	265(0.1299)	249(0.1539)	238(0.1780)
	5%	245(0.1242)	229(0.1500)	218(0.1760)	210(0.2018)
	10%	207(0.1471)	196(0.1752)	189(0.2034)	183(0.2315)
	15%	179(0.1701)	172(0.2004)	166(0.2308)	162(0.2612)
	20%	158(0.1930)	152(0.2257)	148(0.2583)	145(0.2909)
	↓ME %	1/2	1/3	1/4	1/5
t_{P4}	0%	37.26(0.8167)	38.93(0.8833)	40.37(0.9500)	41.62(1.0166)
	1%	37.05(0.8213)	38.71(0.8884)	40.14(0.9554)	41.38(1.0225)
	5%	36.24(0.8396)	37.85(0.9085)	39.24(0.9774)	40.44(1.0463)
	10%	35.28(0.8626)	36.83(0.9337)	38.17(1.0049)	39.32(1.0760)
	15%	34.36(0.8855)	35.86(0.9589)	37.15(1.0323)	38.27(1.1057)
	20%	33.49(0.9085)	34.94(0.9841)	36.19(1.0598)	37.27(1.1355)
	↓ME %	1/2	1/3	1/4	1/5
t_g	0%	324(0.0938)	306(0.1124)	293(0.1309)	283(0.1495)
	1%	308(0.0989)	291(0.1181)	279(0.1373)	271(0.1564)
	5%	256(0.1190)	245(0.1406)	237(0.1621)	230(0.1837)
	10%	212(0.1434)	205(0.1678)	199(0.1922)	195(0.2167)
	15%	182(0.1669)	177(0.1942)	173(0.2214)	170(0.2487)
	20%	160(0.1897)	157(0.2197)	154(0.2497)	151(0.2797)
	↓ME %	1/2	1/3	1/4	1/5
t_d	0%	324(0.0938)	306(0.1124)	293(0.1309)	283(0.1495)
	1%	308(0.0989)	291(0.1181)	279(0.1373)	271(0.1564)
	5%	256(0.1190)	245(0.1406)	237(0.1621)	230(0.1837)
	10%	212(0.1434)	205(0.1678)	199(0.1922)	195(0.2167)
	15%	182(0.1669)	177(0.1942)	173(0.2214)	170(0.2487)
	20%	160(0.1897)	157(0.2197)	154(0.2497)	151(0.2797)

Table 2.7.2 *PRE(MSE) with measurement error and without non-response*

Estimator	0%	1%	5%	10%	15%	20%
$\bar{y}^*$	100(0.2647)	99(0.2673)	95(0.2779)	91(0.2911)	87(0.3043)	83(0.3176)
t_{R1}	198(0.1338)	193(0.1367)	178(0.1489)	161(0.1640)	148(0.1791)	136(0.1942)
t_{P1}	56(0.4701)	56(0.4731)	55(0.4851)	53(0.5002)	51(0.5153)	50(0.5304)
t_{lr1}	352(0.0753)	332(0.0798)	271(0.0975)	222(0.1189)	189(0.1397)	166(0.1598)
t_{R2}	198(0.1338)	193(0.1369)	178(0.1489)	161(0.1640)	148(0.1791)	136(0.1942)
t_{P2}	56(0.4701)	56(0.4731)	55(0.4851)	53(0.5002)	51(0.5153)	50(0.5304)
t_{lr2}	352(0.0752)	332(0.0798)	271(0.0975)	223(0.1189)	190(0.1396)	166(0.1597)
t_1	88(0.3019)	92(0.3076)	80(0.3303)	74(0.3586)	68(0.3869)	64(0.4153)
t_2	88(0.3019)	92(0.3076)	80(0.3303)	74(0.3586)	68(0.3869)	64(0.4153)
t_{lr3}	-	-	-	-	-	-
t_{R4}	341(0.0776)	324(0.0818)	269(0.0983)	222(0.1190)	189(0.1397)	165(0.1604)
t_{P4}	35(0.7500)	35(0.7542)	34(0.7707)	33(0.7914)	33(0.8121)	32(0.8328)
t_g	352(0.0752)	332(0.0798)	271(0.0975)	223(0.1189)	190(0.1396)	166(0.1597)
t_d	352(0.0752)	332(0.0798)	271(0.0975)	223(0.1189)	190(0.1396)	166(0.1597)

Table 2.7.3 *PRE (MSE) with measurement error for optimum value of k*

Estimators ↓	PRE(MSE)				
Per cent of ME →	1%	5%	10%	15%	20%
$\bar{y}^*$	99(0.3361)	95(0.3494)	91(0.3660)	87(0.3826)	83(0.3993)
t_{R1}	189(0.1759)	174(0.1911)	158(0.2101)	145(0.2291)	134(0.2481)
t_{P1}	56(0.5903)	55(0.6055)	53(0.6245)	52(0.6435)	50(0.6625)
t_{lr1}	311(0.1069)	257(0.1293)	213(0.1563)	183(0.1822)	160(0.2174)
t_{R2}	178(0.1867)	164(0.2023)	150(0.2217)	138(0.2409)	128(0.2601)
t_{P2}	61(0.5497)	59(0.5648)	57(0.5836)	55(0.6024)	54(0.6213)
t_{lr2}	301(0.1104)	245(0.1357)	202(0.1648)	173(0.1922)	152(0.2182)
t_1	94(0.3537)	850.3561)	81(0.4125)	75(0.4451)	70(0.4778)
t_2	82(0.4064)	77(0.4339)	71(0.4681)	66(0.5022)	62(0.5361)
t_{lr3}	-	-	-	-	-
t_{R4}	298(0.1116)	251(0.1327)	210(0.1587)	180(0.1845)	158(0.2101)
t_{P4}	38(0.8824)	37(0.9023)	36(0.9273)	35(0.9522)	34(0.9772)
t_g	312(0.1066)	258(0.1290)	213(0.1559)	183(0.1819)	161(0.2071)
t_d	312(0.1066)	258(0.1290)	213(0.1559)	183(0.1819)	161(0.2071)

Table 2.7.4 *PRE(MSE) without measurement error and with non-response error for optimum value of k*

Estimators	MSE
$\bar{y}^*$	100(0.3327)
t_{R1}	193(0.1721)
t_{P1}	57(0.5866)
t_{lr1}	329(0.1011)
t_{R2}	182(0.1828)
t_{P2}	61(0.5460)
t_{lr2}	321(0.1037)
t_1	96(0.3472)
t_2	83(0.3995)
t_{lr3}	-
t_{R4}	313(0.1063)
t_{P4}	38(0.8774)
t_g	330(0.1008)
t_d	330(0.1008)

Table 2.7.5 *PRE(MSE) without measurement error and with non-response error*

Estimators	1/2	1/3	1/4	1/5
$\bar{y}^*$	100(0.3043)	100(0.3439)	100(0.3835)	100(0.4231)
t_{R1}	193(0.1574)	190(0.1810)	187(0.2046)	185(0.2282)
t_{P1}	56.69(0.5367)	57(0.6033)	57.24 (0.67)	57.44(0.7366)
t_{lr1}	323(0.0942)	304(0.1131)	291(0.1320)	280(0.1510)
t_{R2}	193(0.1574)	190(0.1810)	187(0.2046)	185(0.2282)
t_{P2}	56.69(0.5367)	56.99(0.6033)	57.24(0.6699)	57.44(0.7366)
t_{lr2}	323(0.0942)	304(0.1132)	290(0.1322)	280(0.1512)
t_1	106(0.2883)	110(0.3119)	114(0.3355)	118(0.3591)
t_2	92(0.3313)	86(0.3979)	83(0.4646)	80(0.5312)
t_{lr3}	-	-	-	-
t_{R4}	301(0.1012)	275(0.1248)	258(0.1484)	246(0.1720)
t_{P4}	37(0.8167)	34(1.0225)	33(1.1567)	33(1.2911)
t_g	172(0.1772)	123(0.2792)	101(0.3811)	88(0.4831)
t_d	172(0.1772)	123(0.2792)	101(0.3811)	88(0.4831)

2.8 CONCLUDING REMARKS

1. It is evident from the expressions (2.3.10) to (2.3.22) of MSE's of the estimators that the measurement errors seem to have inflated the MSE of these estimators and thereby decreasing the efficiency.

2. The expressions (2.3.10) to (2.3.22) of MSE can be broken into 4 major components owing to non-response and measurement error are given below:

$$MSE = A + B + C + D$$

where A = Component of MSE due to sampling error without measurement error and non-response,

B = Component of MSE due to sampling error with measurement error and without non-response,

C = Component of MSE due to sampling error without measurement error and with non-response, and

D = Component of MSE due to sampling error with measurement error and without non-response.

For Example: Consider the expression of MSE of t_d given by

$$MSE(t_d) = \underbrace{\frac{1}{n}\left(\pm_y^2 + d_2'^2 \pm_x^2 - 2d_2' \pm_{xy}\right)}_{A}$$

$$+ \underbrace{\frac{1}{n}\left(\pm_u^2 + d_2'^2 \pm_v^2\right)}_{B}$$

$$+ \underbrace{\frac{W_2(k-1)}{n}\left(\pm_{y_2}^2 + d_1'^2 \pm_{x_2}^2 - 2d_1' \pm_{xy_2}\right)}_{C}$$

$$+\underbrace{\frac{W_2(k-1)}{n}\left(\sigma_{u_2}^2+d_1'^2\sigma_{v_2}^2\right)}_{D}$$

3. If the measurement error is absent then we get the expression of the MSE of conventional estimators under non-response from the results of this study thereby the present study provides a more general and pragmatic approach for the estimation of population mean. For example: When $u_i = 0 = v_i$, for each i, then $\sigma_u^2=\sigma_v^2=\sigma_{u_2}^2=\sigma_{v_2}^2=0$ and we get

$$MSE(t_d)=\underbrace{\frac{1}{n}\left(\sigma_y^2+d_2'^2\sigma_x^2-2d_2'\sigma_{xy}\right)}_{A}$$
$$+\underbrace{\frac{W_2(k-1)}{n}\left(\sigma_{y_2}^2+d_1'^2\sigma_{x_2}^2-2d_1'\sigma_{xy_2}\right)}_{C}$$

so that only components A and C are left, which is same expression as proposed by Singh and Kumar (2008) while proposing t_d.

4. If the measurement error is absent then we get the expression of the optimum values of the characterizing scalars and minimum MSE of conventional estimators under non-response from the results of this study thereby the present study. For example: When $u_i = 0 = v_i$, for each i, then $\sigma_u^2=\sigma_v^2=\sigma_{u_2}^2=\sigma_{v_2}^2=0$ and we get optimum values of d_1' and d_2' as

$$d_1'=\sigma_{xy_2}/\sigma_{x_2}^2 \text{ and } d_2'=\sigma_{xy}/\sigma_x^2 \quad .$$

and the minimum mean square error of t_d as

$$MSE(t_d)_{\min}=\frac{1}{n}\left(1-\rho^2\right)\sigma_y^2+\frac{W_2(k-1)}{n}\left(1-\rho_2^2\right)\sigma_{y_2}^2$$

which is same expression of minimum MSE as proposed by Singh and Kumar (2008) while proposing t_d.

5. The measurement error seems to have affected all the estimators but the optimal estimators t_d and t_g perform far better than the remaining estimators where the auxiliary information was not properly utilized. For example: This is w.r.t. the empirical results in table 2.7.1 where t_d utilized the auxiliary information optimally and outperformed all the remaining estimators.

6. The measurement error seems to have affected the better estimators more where the auxiliary information was properly utilized than those estimators where the auxiliary information was not properly utilized. For example: This is w.r.t. the empirical results in table 2.7.1 where t_d utilized the auxiliary information optimally and has lost 164% efficiency with $1/2$ sub-sampling fraction which is far more in comparison to t_{P4}, estimator where the auxiliary information was not properly utilized, lost only 4% efficiency with $1/2$ sub-sampling fraction or even $\bar{y}*$ which lost 17% efficiency with $1/2$ sub-sampling fraction.

7. The gains in efficiency of various estimators were calculated w. r. t. $\bar{y}*$ with no non-response error in table 2.7.2. The best choice, which is in consonance with the theoretical results, is t_d and t_g at all levels of measurement error.

8. The percent relative efficiency of various estimators were calculated w. r. t. $\bar{y}^*$ with optimum sub-sampling fraction $1/k$ in table 2.7.3. The best choice, which is in consonance with the theoretical results, is t_d and t_g at all levels of measurement error.

9. The gains in efficiency of various estimators were calculated w. r. t. $\bar{y}^*$ with no measurement error in table 2.7.4. The best choice, which is in consonance with the theoretical results, is t_d and t_g at all levels of measurement error.

10. The percent relative efficiency of various estimators were calculated w. r. t. $\bar{y}^*$ without measurement error and with different non-respondent sub-sampling fraction $1/k$ in table 2.7.5. The best choice is t_d and t_g which is in consonance with the theoretical results of Singh and Kumar (2008).

CHAPTER 3

A CHAIN TYPE GENERALIZED ESTIMATOR OF POPULATION MEAN UNDER MEASUREMENT ERROR AND NON-RESPONSE ERROR

SUMMARY

In this chapter, we study a generalized estimator for estimating the population mean of study variable using auxiliary information in presence of measurement error and non-response error occurs simultaneously. The generalized estimators in this chapter are given as an alternative to the class of estimators proposed by Singh and Kumar (2008). The properties of suggested generalized estimators are studied and compared with those of existing estimators in presence of measurement error and non-response error. The suggested generalized estimator $\bar{y}_f$ is most efficient among all the estimators proposed by Singh and Kumar (2008) under both measurement and non-response errors occurring simultaneously. The efficiency of the proposed estimator seems to be drastically curtailed with increasing measurement error.

3.1 INTRODUCTION

Auxiliary information is often used for the purpose of improving upon the efficiency of the estimates of the population parameters. The purpose of making use of the information based on an auxiliary variable is to obtain increased precision by taking advantage of the correlation between the study variable and the auxiliary variables. When the study variable and the auxiliary variable are positively correlated it is better to use the ratio estimator however the correlation between the study and auxiliary variable is negative one may make use of the product estimator. Thus in this regard it can be said that the ratio and product method are good examples for estimating the population mean $\bar{Y}$ if the population variable is readily available and accounts to more precise estimates.

In survey sampling, the properties of the estimators based on data usually treated under the assumption that the observations are correct measurements on characteristics being studied. But such kind of assumption does not satisfied in many applications and data is recorded with measurement errors, such as reporting errors and computing errors. These measurement errors make the result invalid. If measurement error is very small and we can neglect it, then the statistical inferences based on observed data continue to remain valid. On the other hand if measurement error is not negligible, the inferences may not be simply invalid and inaccurate, but may often lead to unexpected, undesirable and unfortunate consequences (see Srivastava and Shalabh (2001)).

Consider a finite population $U = (U_1, U_2, ..., U_N)$ of N units. Let Y and X be the study variate and auxiliary variate, respectively. Suppose that we have a set of n paired observations obtained through simple random sampling procedure on two characteristics X and Y. Further, suppose that (x_i, y_i) for the i^{th} sampling units are observed with measurement error instead of their true values (X_i, Y_i). For a simple random sampling scheme, let $(x_i,\ y_i)$ be observed values instead of the true values $(X_i,\ Y_i)$ for i^{th} $(i = 1, 2, ...n)$ unit such as

$$u_i = y_i - Y_i \tag{3.1.1}$$

$v_i = x_i - X_i$ (3.1.2)

where u_i and v_i are associated measurement errors which are stochastic (probabilistic) in nature with mean zero, variances $\pm_u^2$ and $\pm_v^2$ respectively. Further, we assume that u_i and v_i are uncorrelated although X_i and Y_i are correlated. Let the population mean of X and Y characteristics be «$_x$ and «$_y$ where population variances of X and Y characteristics be $\pm_x^2$ and $\pm_y^2$ respectively. Let $\pm_{xy}$ be the population covariance between x and y.

Consider a finite population of size N and a random sample of size n drawn by simple random sampling without replacement. In surveys on human populations, it is often the case that n_1 units respond and remaining $(n-n_1)$ do not any response. The initial survey may be conducted through the mail or by telephone, perhaps computer aided; see Rao (1986). In the case non-response, in the initial attempt, Hansen & Hurwitz (1946) suggested a double sampling scheme for estimating the population means comprising the following steps.

(a) A simple random sample of size n is drawn and the questionnaire is mailed to the sampled units.

(b) A sub-sample of size $r = (n/k)$, $(k > 1)$ from the n_2 non-responding units in the initial step attempt is contacted through personal interviews.

Note that Hansen & Hurwitz (1946) are the pioneer of the non-response problem and they considered mail surveys in the first attempts, and personal interview in the second attempt. In the Hansen and Hurwitz method, the population of size N is supposed to be composed of two stratum, namely respondents and non-respondents, having size N_1 and $N_2\left(= N - N_1\right)$. Thus we label the data as $Y_1, Y_2, ..., Y_{N_1}$ for the response stratum, and as $Y_{N_1+1}, Y_{N_1+2}, \ldots, Y_{N_1+N_2}$ for the non-response stratum. Further, we assume that $\bar{Y} = \sum_{i=1}^{N} Y_i / N$ and $\pm_y^2 = \sum_{i=1}^{N}\left(Y_i - \bar{Y}\right)^2 / N$ denote population mean and variance, respectively. Let $\bar{Y}_1 = \sum_{i=1}^{N_1} y_i / N_1$ and $\pm_{y_1}^2 = \sum_{i=1}^{N_1}\left(y_i - \bar{Y}_1\right)^2 / N_1$ denote the mean and variance of response stratum, respectively, and similarly, $\bar{Y}_2 = \sum_{i=N_1+1}^{N_1+N_2} Y_i / N_2$ and $\pm_{y_2}^2 = \sum_{i=N_1+1}^{N_1+N_2}\left(Y_i - \bar{Y}_2\right)^2 / N_2$ denote the mean and variance of non-response stratum respectively. The population mean can be written as $\bar{Y} = W_1\bar{Y}_1 + W_2\bar{Y}_2$, where $W_1 = N_1 / N$ and $W_2 = N_2 / N$. The sample mean $\bar{y}_1 = \sum_{i=1}^{n_1} y_i / n_1$ is unbiased for $\bar{Y}_1$, but has a bias equal to $W_2(\bar{Y}_1 - \bar{Y}_2)$ in estimating the population mean $\bar{Y}$.

The sample mean $\bar{y}_{2r} = \sum_{i=1}^{r} y_i / r$ is unbiased for the mean $\bar{y}_2$ of the n_2 units. An unbiased estimator for the population mean $\bar{Y}$ is

$$\bar{y}^* = W_1\bar{y}_1 + W_2\bar{y}_{2r} \tag{3.1.3}$$

where $W_1 = n_1 / n$ and $W_2 = n_2 / n$.

The variance of $\bar{y}^*$ is given by

$$Var(\bar{y}^*) = \frac{1}{n}\pm_y^2 + \frac{W_2\left(k - 1\right)}{n}\pm_{y_2}^2 \tag{3.1.4}$$

Let $X_i\left(i = 1, 2, ... N\right)$ represent an supplementary variate correlated with the study variate Y_i $(i = 1,2,...N)$. The population mean of the supplementary variate x is $\bar{X} = \sum_{i=1}^{N} x_i / N$. Let $\bar{X}_1$ and $\bar{X}_2$ denote the means of response and non-response groups, respectively. Let $\bar{x}$ denotes the mean of all n units. Further, we suppose that $\bar{x}_1$ and $\bar{x}_2$ denote the means of n_1 responding units and n_2 non-responding units, respectively. Furthermore, let, $\bar{x}_{2r} = \sum_{i=1}^{r} x_i / r$ represent the mean of the sub-sampled units and the population variance of X and Y are denoted by $\pm_x^2$ and $\pm_y^2$ respectively. The population covariance between X and Y denoted by $\pm_{xy}$. The unbiased estimator of the population mean $\bar{X}$ of the auxiliary variate x is

$$\bar{x}^* = W_1\bar{x}_1 + W_2\bar{x}_{2r} \tag{3.1.5}$$

and the variance of $\bar{x}^*$ is given by

$$Var(\bar{x}^*) = \frac{1}{n}\pm_x^2 + \frac{W_2(k-1)}{n}\pm_{x_2}^2 \tag{3.1.6}$$

where $\pm_{x_2}^2 = \sum_{i=N_1+1}^{N=N_1+N_2} (X_i - X_2)^2 / N_2$.

GENERALIZED ESTIMATORS UNDER NON-REAPONSE ERROR AND MEASUREMENT ERROR OCCURING SIMULTANEOUSLY

Here we consider the following different situations.

SITUATION I

Cochran (1977) defined the ratio, product and difference estimators under non-response error. Here we consider on the same lines under non-response error and measurement error occurs on both the study variable y and the auxiliary variable x, and the population mean $\bar{X}$ of the auxiliary variable is known. Then the conventional ratio, product and difference estimators for the estimating the mean $\bar{Y}$ of study variable y are respectively defined as

$$t_{R1} = \bar{y}^{*}\left(\frac{\bar{X}}{\bar{x}^{*}}\right) \tag{3.1.7}$$

$$t_{P1} = \bar{y}^{*}\left(\frac{\bar{x}^{*}}{\bar{X}}\right) \tag{3.1.8}$$

$$t_{lr1} = \bar{y}^{*} + d_1\left(\bar{X} - \bar{x}^{*}\right) \tag{3.1.9}$$

where d_1 is suitably chosen constant.

SITUATION II

Perusing the above idea, later Rao (1986) proposed some more estimators in which the Non-response error occurs only on the study variable whereas measurement error occurs on both the study variable as well as on the auxiliary variable and the population mean $\bar{X}$ of the auxiliary variable is known. In this case the usual ratio, product and difference estimators for estimating the population mean $\bar{Y}$ of study variable are defined as

$$t_{R2} = \bar{y}^{*}\left(\frac{\bar{X}}{\bar{x}}\right) \tag{3.1.10}$$

$$t_{P2} = \bar{y}^{*}\left(\frac{\bar{x}}{\bar{X}}\right) \tag{3.1.11}$$

$$t_{lr2} = \bar{y}^{*} + d_2\left(\bar{X} - \bar{x}\right) \tag{3.1.12}$$

where d_2 is suitably chosen constant.

SITUATION III

Further non-response occurs on the study variable whereas measurement error occurs on both the study variable as well as on the auxiliary variable and the population mean $\bar{X}$ of the auxiliary variable is known. In this situation the usual ratio, product and difference estimators for estimating the population mean $\bar{Y}$ of study variate y are defined as below

$$t_1 = \bar{y}^* \left(\frac{\bar{x}}{\bar{x}^*} \right) \tag{3.1.13}$$

$$t_2 = \bar{y}^* \left(\frac{\bar{x}^*}{\bar{x}} \right) \tag{3.1.14}$$

$$t_{lr3} = \bar{y}^* + d_3 \left(\bar{x} - \bar{x}^* \right) \tag{3.1.15}$$

where d_3 is suitably chosen constant.

SITUATION IV

We consider situation II, on the lines of Singh and Kumar (2008) where the non-response errors occurs only in the study variable and measurement errors occurs both the variables study variable as well as supplementary variable. Information on the supplementary variable x is obtained from all the sample units (i.e. the initial sample units) and the population mean $\bar{X}$ of the auxiliary variable is known but some sample units are not responding about the information of study variable y. Now using this information we define the following estimators of the population mean $\bar{Y}$ as

$$t_{R4} = \bar{y}^* \left(\bar{X} / \bar{x}^* \right) \left(\bar{X} / \bar{x} \right) \tag{3.1.16}$$

$$t_{P4} = \bar{y}^* \left(\bar{x}^* / \bar{X} \right) \left(\bar{x} / \bar{X} \right) \tag{3.1.17}$$

$$t_g = \bar{y}^* \left(\bar{x} / \bar{x}^* \right)^{\sim_1} \left(\bar{X} / \bar{x} \right)^{\sim_2} \tag{3.1.18}$$

$$t_d = \bar{y}^* + d_1' \left(\bar{x} - \bar{x}^* \right) + d_2' \left(\bar{X} - \bar{x} \right) \tag{3.1.19}$$

where $\sim_i s$ *(i=1, 2)* and $d_i s$ *(i=1, 2)* are suitably chosen constant.

Here we note that, when suggesting the estimator for the population mean $\bar{Y}$ Rao (1986) used only the information on the sample mean $\bar{x}$ and on the population mean $\bar{X}$ of the auxiliary variate *x*. However, one can also obtain the unbiased estimator $\bar{x}^* = (n_1/n)\bar{x}_1 + (n_2/n)\bar{x}_{2r}$ of $\bar{X}$ (without any extra effort) while in the process of obtaining $\bar{y}^* = (n_1/n)\bar{y}_1 + (n_2/n)\bar{y}_{2r}$, the unbiased estimator of the population mean $\bar{Y}$. Hence in situation II, we have two unbiased estimators, $\bar{x}^*$ and $\bar{x}$, of the population mean $\bar{X}$ of the supplementary variate *x*.

3.2 PROPOSED GENERALISED CLASS

Here we have proposed a generalized cost efficient class of estimators for estimating the population mean when some sample units fail to supply information on the study variable but the information on the auxiliary variable is obtained on all the sample units and its population mean is also known. We proposed the following generalized class of estimators given b

$$\bar{y}_f = f\left(\bar{y}^*, \bar{x}, \bar{x}^*\right) \tag{3.2.1}$$

where $f(.)$ being a bounded function satisfy the following regularity conditions such that

(i) $f\left(\bar{Y}, \bar{X}, \bar{X}\right) = \bar{Y}$ (3.2.2)

(iii) First order partial derivative with respect to $\bar{y}^*$ at $R \equiv \left(\bar{Y}, \bar{X}, \bar{X}\right)$ is unity, that is,

$$f_0 = 1 \tag{3.2.3}$$

(iii) $f_{00} = 0$ (3.2.4)

(iv) First order partial derivative of $f\left(\bar{y}^*, \bar{x}, \bar{x}^*\right)$ with respect to $\bar{x}$, and $\bar{x}^*$ respectively at $R \equiv \left(\bar{Y}, \bar{X}, \bar{X}\right)$ satisfy

$$f_1 = -\left(f_2 + \yen\right) \tag{3.2.5}$$

where ¥ is some suitably chosen constant and the subscript 0, 1 and 2 denote the derivative of the concerned function with respect to $\bar{y}^*$, $\bar{x}$ and $\bar{x}^*$ respectively.

3.3 BIAS AND MEAN SQUARE ERROR

For obtaining the bias and mean square error of the above estimator define in different situation, we define the following error terms

$\bar{y}^* = \bar{Y} + \pounds_0^*$, $\quad \bar{x}^* = \bar{X} + \pounds_1^*$ $\quad \bar{x} = \bar{X} + \pounds_1$

such that

$E(\pounds_0^*) = E(\pounds_1^*) = E(\pounds_1) = 0$

and

$$E(\pounds_0^{*2}) = \frac{1}{n}\left(\pm_y^2 + \pm_u^2\right) + \frac{W_2(k-1)}{n}\left(\pm_{y_2}^2 + \pm_{u_2}^2\right)$$

$$E(\pounds_1^{*2}) = \frac{1}{n}\left(\pm_x^2 + \pm_v^2\right) + \frac{W_2(k-1)}{n}\left(\pm_{x_2}^2 + \pm_{v_2}^2\right)$$

$$E(\pounds_1^2) = \frac{1}{n}\left(\pm_x^2 + \pm_v^2\right)$$

$$E(\pounds_0^*\pounds_1^*) = \mathrm{cov}(\bar{y}^*, \bar{x}^*) = \frac{1}{n}\pm_{xy} + W_2\frac{(k-1)}{n}\pm_{xy2}$$

$$E(\pounds_0^*\pounds_1) = \mathrm{cov}(\bar{y}^*, \bar{x}) = \frac{1}{n}\pm_{xy}$$

$$E(\pounds_1^*\pounds_1) = \mathrm{cov}(\bar{x}^*, \bar{x}) = \frac{1}{n}\left(\pm_x^2 + \pm_v^2\right)$$

$$Bias(t_{R1})=\frac{1}{n\bar{X}}\left[\left\{R\left(\pm_x^2+\pm_v^2\right)-\pm_{xy}\right\}+W_2(k-1)\left\{R\left(\pm_{x_2}^2+\pm_{v_2}^2\right)-\pm_{xy_2}\right\}\right] \quad (3.3.1)$$

$$Bias(t_{P1})=\frac{1}{n\bar{X}}\left[\pm_{xy}+W_2(k-1)\pm_{xy_2}\right] \quad (3.3.2)$$

$$Bias(t_{R2})=\frac{1}{n\bar{X}}\left[\left\{R\left(\pm_x^2+\pm_v^2\right)-\pm_{xy}\right\}\right] \quad (3.3.3)$$

$$Bias(t_{P2})=\frac{1}{n\bar{X}}\pm_{xy} \quad (3.3.4)$$

$$Bias(t_{R3})=\frac{1}{n\bar{X}}\left[W_2(k-1)\left\{R\left(\pm_{x_2}^2+\pm_{v_2}^2\right)-\pm_{xy_2}\right\}\right] \quad (3.3.5)$$

$$Bias(t_{P3})=\frac{1}{n\bar{X}}\left[W_2(k-1)\pm_{xy_2}\right] \quad (3.3.6)$$

$$Bias\left(t_{R4}\right)=\frac{1}{n\bar{X}}\left[3R\left(\pm_x^2+\pm_v^2\right)-2\pm_{xy}\right]+\frac{W_2(k-1)}{n\bar{X}}\left[R\left(\pm_{x_2}^2+\pm_{v_2}^2\right)-\pm_{xy_2}\right] \quad (3.3.7)$$

$$Bias\left(t_{P4}\right)=\frac{1}{n\bar{X}}\left[\left(\pm_x^2+\pm_v^2\right)+2\pm_{xy}\right]+\frac{W_2(k-1)}{n\bar{X}}\pm_{xy_2} \quad (3.3.8)$$

$$Bias\left(t_g\right)=\frac{1}{2n\bar{X}}\left[\sim_2\left(\sim_2+1\right)\left(\pm_x^2+\pm_v^2\right)-2\sim_2\pm_{xy}\right]$$

$$+\frac{W_2\left(k-1\right)}{2n\bar{X}}\left[\sim_1\left(\sim_1+1\right)\left(\pm_{x_2}^2+\pm_{v_2}^2\right)-2\sim_1\pm_{xy_2}\right] \quad (3.3.9)$$

$$Bias\left(\bar{y}_f\right)=\frac{1}{n}\left[\left(\pm_x^2+\pm_v^2\right)\left(f_{11}+f_{22}\right)+\pm_{xy}\left(f_{01}+f_{02}\right)\right]$$

$$+\frac{W_2\left(k-1\right)}{n}\left[\left(\pm_{x(2)}^2+\pm_{v(2)}^2\right)f_{22}+\pm_{xy(2)}f_{02}\right] \quad (3.2.10)$$

To the first order of approximation the MSE of the proposed estimators is given by

$$MSE\left(t_{R1}\right)=MSE\left(\bar{y}^*\right)+\frac{1}{n}\left[R^2\left(\pm_x^2+\pm_v^2\right)-2R\pm_{xy}\right]$$

$$+\frac{W_2\left(k-1\right)}{n}\left[R^2\left(\pm_{x_2}^2+\pm_{v_2}^2\right)-2R\pm_{xy(2)}\right] \quad (3.3.11)$$

$$MSE\left(t_{P1}\right)=MSE\left(\bar{y}^{*}\right)+\frac{1}{n}\left[R^{2}\left(\pm_{x}^{2}+\pm_{v}^{2}\right)+2R\pm_{xy}\right]$$

$$+\frac{W_{2}\left(k-1\right)}{n}\left[R^{2}\left(\pm_{x_{2}}^{2}+\pm_{v_{2}}^{2}\right)+2R\pm_{xy_{(2)}}\right] \qquad (3.3.12)$$

$$MSE\left(t_{lr1}\right)=MSE\left(\bar{y}^{*}\right)+\frac{1}{n}\left[d_{1}^{2}\left(\pm_{x}^{2}+\pm_{v}^{2}\right)-2d_{1}\pm_{xy}\right]$$

$$+\frac{W_{2}\left(k-1\right)}{n}\left[d_{1}^{2}\left(\pm_{x_{2}}^{2}+\pm_{v_{2}}^{2}\right)-2d_{1}\pm_{xy_{(2)}}\right] \qquad (3.3.13)$$

$$MSE\left(t_{R2}\right)=MSE\left(\bar{y}^{*}\right)+\frac{1}{n}\left[R^{2}\left(\pm_{x}^{2}+\pm_{v}^{2}\right)-2R\pm_{xy}\right]+\frac{W_{2}(k-1)}{n}\pm_{u_{2}}^{2} \qquad (3.3.14)$$

$$MSE\left(t_{P2}\right)=MSE\left(\bar{y}^{*}\right)+\frac{1}{n}\left[R^{2}\left(\pm_{x}^{2}+\pm_{v}^{2}\right)+2R\pm_{xy}\right]+\frac{W_{2}(k-1)}{n}\pm_{u_{2}}^{2} \qquad (3.3.15)$$

$$MSE\left(t_{lR2}\right)=MSE\left(\bar{y}^{*}\right)+\frac{1}{n}\left[d_{2}^{2}\left(\pm_{x}^{2}+\pm_{v}^{2}\right)-2d_{2}\pm_{xy}\right]+\frac{W_{2}(k-1)}{n}\pm_{u_{2}}^{2} \qquad (3.3.16)$$

$$MSE\left(t_{1}\right)=MSE\left(\bar{y}^{*}\right)+\frac{1}{n}\left[R^{2}\left(\pm_{x}^{2}+\pm_{v}^{2}\right)\right]+\frac{W_{2}(k-1)}{n}\left[R^{2}\left(\pm_{x_{2}}^{2}+\pm_{v_{2}}^{2}\right)-2R\pm_{xy_{(2)}}\right] \qquad (3.3.17)$$

$$MSE\left(t_{2}\right)=MSE\left(\bar{y}^{*}\right)+\frac{1}{n}\left[R^{2}\left(\pm_{x}^{2}+\pm_{v}^{2}\right)\right]+\frac{W_{2}(k-1)}{n}\left[R^{2}\left(\pm_{x_{2}}^{2}+\pm_{v_{2}}^{2}\right)+2R\pm_{xy_{(2)}}\right] \qquad (3.3.18)$$

$$MSE\left(t_{lr3}\right)=MSE\left(\bar{y}^{*}\right)+\frac{1}{n}\left[d_{3}^{2}\left(\pm_{x}^{2}+\pm_{v}^{2}\right)\right]+\frac{W_{2}(k-1)}{n}\left[d_{3}^{2}\left(\pm_{x_{2}}^{2}+\pm_{v_{2}}^{2}\right)+2d\pm_{xy_{(2)}}\right] \qquad (3.3.18)$$

$$MSE(t_{R4})=MSE\left(\bar{y}^{*}\right)+\frac{1}{n}\left[4R^{2}\left(\pm_{x}^{2}+\pm_{v}^{2}\right)-4R\pm_{xy}\right]$$

$$+\frac{W_{2}\left(k-1\right)}{n}\left[R^{2}\left(\pm_{x}^{2}+\pm_{v}^{2}\right)-2R\pm_{xy}\right] \qquad (3.3.20)$$

$$MSE(t_{P4})=MSE\left(\bar{y}^{*}\right)+\frac{1}{n}\left[4R^{2}\left(\pm_{x}^{2}+\pm_{v}^{2}\right)+4R\pm_{xy}\right]$$

$$+\frac{W_{2}\left(k-1\right)}{n}\left[R^{2}\left(\pm_{x}^{2}+\pm_{v}^{2}\right)+2R\pm_{xy}\right] \qquad (3.3.21)$$

$$MSE(t_{g})=MSE\left(\bar{y}^{*}\right)+\frac{1}{n}\left[R^{2}\sim_{2}^{2}\left(\pm_{x}^{2}+\pm_{v}^{2}\right)-2R\sim_{2}\pm_{xy}\right]$$

$$+\frac{W_2(k-1)}{n}\left[R^2 \sim_1^2 \left(\pm_x^2 + \pm_v^2\right) - 2R \sim_1 \pm_{xy_2}\right] \qquad (3.3.22)$$

$$MSE(t_d) = MSE\left(\bar{y}^*\right) + \frac{d_2'}{n}\left[d_2'\left(\pm_x^2 + \pm_v^2\right) - 2\pm_{xy}\right]$$

$$+\frac{W_2(k-1)d_1'}{n}\left[d_1'\left(\pm_{x_2}^2 + \pm_{v_2}^2\right) - 2\pm_{xy_2}\right] \qquad (3.3.23)$$

$$MSE\left(\bar{y}_f\right) = \frac{1}{n}\left\{\left(\pm_y^2 + \pm_u^2\right) + \text{¥}^2\left(\pm_x^2 + \pm_v^2\right) - 2\text{¥}\pm_{xy}\right\}$$

$$+\frac{(k-1)W_2}{n}\left\{\left(\pm_{y(2)}^2 + \pm_{u(2)}^2\right) + f_2^2\left(\pm_{x(2)}^2 + \pm_{v(2)}^2\right) + 2f_2\pm_{xy(2)}\right\} \qquad (3.3.24)$$

3.4 OPTIMUM VALUES AND MINIMUM MSE

Now minimizing (3.3.13) with respect to d_1 we obtained the optimum value of d_1 is given by

$$d_1 = \frac{\pm_{xy} + W_2(k-1)\pm_{xy_2}}{\left(\pm_x^2 + \pm_v^2\right) + W_2(k-1)\left(\pm_{x_2}^2 + \pm_{v_2}^2\right)} \qquad (3.4.1)$$

and the minimum MSE of (3.1.9) is given by

$$MSE\left(t_{lr1}\right)\min = MSE\left(\bar{y}^*\right) - \frac{\left[\pm_{xy} + W_2(k-1)\pm_{xy_2}\right]^2}{n\left[\left(\pm_x^2 + \pm_v^2\right) + W_2(k-1)\left(\pm_{x_2}^2 + \pm_{v_2}^2\right)\right]} \qquad (3.4.2)$$

On optimizing (3.3.16) optimum value of d_2 is

$$d_2 = \frac{\pm_{xy}}{\left(\pm_x^2 + \pm_v^2\right)} \qquad (3.4.3)$$

Minimum MSE of (3.1.12) is given by

$$MSE\left(t_{lr2}\right)\min = MSE\left(\bar{y}^*\right) - \frac{\pm_{xy}}{n\left(\pm_x^2 + \pm_v^2\right)} \qquad (3.4.4)$$

On optimizing (3.3.19) optimum value of d_3 is given by

$$d_3 = \frac{\pm_{xy_2}}{\left(\pm_{x_2}^2 + \pm_{v_2}^2\right)} \tag{3.4.5}$$

and the minimum MSE of t_{lr3} is given by

$$MSE\left(t_{lr3}\right)\min = MSE\left(\bar{y}^*\right) - \frac{W_2\left(k-1\right)}{n}\frac{\pm_{xy_2}}{\left(\pm_{x_2}^2 + \pm_{v_2}^2\right)} \tag{3.4.6}$$

Differentiating (3.2.22) with respect to $\sim_1$, $\sim_2$ and equating the resulting derivatives to zero, we obtain the optimum value of $\sim_1$ and $\sim_2$ given as

$$\sim_1 = \frac{\pm_{xy_2}}{R\left(\pm_{x_2}^2 + \pm_{v_2}^2\right)} \text{ and } \sim_2 = \frac{\pm_{xy}}{R\left(\pm_{x}^2 + \pm_{v}^2\right)} \tag{3.4.7}$$

Putting these values of $\sim_1$ and $\sim_2$ in the expression of MSE of t_g we get the minimum MSE of t_g given as

$$MSE(t_g)_{\min} = MSE\left(\bar{y}^*\right) - \frac{1}{n}\left[\frac{\left(\pm_{xy}\right)^2}{\pm_x^2 + \pm_v^2}\right] - \frac{W_2(k-1)}{n}\left[\frac{\left(\pm_{xy_2}\right)^2}{\pm_{x_2}^2 + \pm_{v_2}^2}\right] \tag{3.4.8}$$

Now minimizing (3.3.23) with respect to d_1' and d_2', we obtained the optimum values of d_1' and d_2' given by

$$d_1' = \frac{\pm_{xy_2}}{\pm_{x_2}^2 + \pm_{v_2}^2} \text{ and } d_2' = \frac{\pm_{xy}}{\pm_x^2 + \pm_v^2}\ . \tag{3.4.9}$$

Putting these values of d_1' and d_2' in (3.3.23) the minimum mean square error of t_d is given by

$$MSE(t_d)_{\min} = MSE\left(\bar{y}^*\right) - \frac{1}{n}\left[\frac{\left(\pm_{xy}\right)^2}{\pm_x^2 + \pm_v^2}\right] - \frac{W_2(k-1)}{n}\left[\frac{\left(\pm_{xy_2}\right)^2}{\pm_{x_2}^2 + \pm_{v_2}^2}\right] \tag{3.4.10}$$

Now minimizing (3.3.24) with respect to f_2 and¥ , we obtained the optimum values of f_2 and¥ is given by

$$f_{2(opt)} = -\frac{\pm_{xy(2)}}{\pm^2_{x(2)} + \pm^2_{v(2)}} \tag{3.4.12}$$

$$¥_{(opt)} = \frac{\pm_{xy}}{\pm^2_x + \pm^2_u} \tag{3.4.13}$$

Putting these values of f_2 and¥ in (3.3.24) the minimum mean square error of $\bar{y}_f$ is given by

$$MSE_{\min}(\bar{y}_f) = MSE(\bar{y}^*) - \left[\frac{(\pm_{xy})^2}{(\pm^2_x + \pm^2_v)} + \frac{W_2(k-1)}{n} \frac{(\pm_{xy(2)})^2}{(\pm^2_{x(2)} + \pm^2_{v(2)})} \right] \tag{3.4.14}$$

which is the same as the minimum mean square error of the difference estimator t_d and t_g given by (3.4.8) and (3.4.10) respectively.

OPTIMUM VALUE OF *n* AND *k*

The expected total cost of the survey apart from the overhead cost is given by

$$C = n\left(c + c_1 W_1 + \frac{c_2 W_2}{k} \right) \tag{3.4.15}$$

where c is the cost per unit of the first attempt with the sample, n ; c_1 is the cost per unit for processing the respondent data at the first attempt in n_1 and c_2 is the cost per unit associated with the sub sample r of n_2.

Let the MSE of the estimators is

$$MSE(t_i) = \frac{V_1 - W_2 V_2}{n} - \frac{W_2 V_2 k}{n}$$

CASE I: FIXED VARIANCE

To determine the optimum values of n and k that minimize the cost for a fixed variance V_0 , we consider the function,

$$¤ = C^* + ª\left\{MSE\left(\bar{y}^*\right) - V_0\right\}$$

$$= n\left[c + c_1W_1 + \frac{c_2W_2}{k}\right] + ª\left[\left\{\frac{\left(V_1 - W_2V_2\right)}{n} + \frac{W_2V_2k}{n}\right\} - V_0\right] \tag{3.4.16}$$

where, V_1 is the term of coefficient of $\frac{1}{n}$ and V_2 is the term of coefficient of $\frac{W_2\left(k-1\right)}{n}$ and ª is Lagrange's multiplier.

Now differentiating (3.4.16) with respect to n and k, and on equating them with zero, we get

$$n = \sqrt{\frac{ª\left\{V_1 + \left(k-1\right)W_2V_2\right\}}{\left\{c + c_1W_1 + \frac{c_2W_2}{k}\right\}}} \tag{3.4.17}$$

$$\frac{n}{k} = \sqrt{\frac{ªV_2}{c_2}} \tag{3.4.18}$$

On putting (3.4.17) in (3.4.18) we get,

$$k_{opt} = \sqrt{\frac{c_2\left(V_1 - W_2V_2\right)}{\left(c + c_1W_1\right)V_2}} \tag{3.4.19}$$

which is required optimum value of k. Further substituting the value of n and k in the expression of MSE, we get

$$\sqrt{ª} = \frac{\sqrt{\left\{V_1 + \left(k-1\right)W_2V_2\right\}\left\{c + c_1W_1 + \frac{c_2W_2}{k}\right\}}}{V_0} \tag{3.4.20}$$

On using this value of ª , we get the optimum value of n given by,

$$n_{opt} = \frac{\left\{V_1 + \left(k_{opt} - 1\right)W_2V_2\right\}}{V_0} \tag{3.4.21}$$

On substituting the optimum value of n and k in (3.4.15), we get the minimum cost for fixed variance V_0 given by

$$C^* = \frac{\left\{V_1 + \left(k_{opt} - 1\right)W_2V_2\right\}\left\{c + c_1W_1 + \dfrac{c_2W_2}{k_{opt}}\right\}}{V_0} \qquad (3.4.22)$$

CASE II: FIXED COST

In order to determine the optimum values of n and k that minimize the $MSE(\bar{y}^*)$ for a fixed cost C_0, we consider the following function using the Lagrange's principle of maxima and minima

$$\text{¤}^* = MSE(\bar{y}^*) + \text{ª}\left\{\left(c + c_1W_1 + \frac{c_2W_2}{k}\right) - C_0\right\}$$

$$= \left\{\frac{V_1 - W_2V_2}{n} - \frac{W_2V_2k}{n}\right\} + \text{ª}\left\{\left(c + c_1W_1 + \frac{c_2W_2}{k}\right) - C_0\right\} \qquad (3.4.23)$$

where V_1 is the term of coefficient of $\frac{1}{n}$ and V_2 is the term of coefficient of $\frac{W_2\left(k-1\right)}{n}$ and ª is Lagrange's multiplier.

Now, differentiating (3.4.23) with respect to n and k, and equating them to zero, we get

$$n = \sqrt{\frac{\left\{V_1 + \left(k-1\right)W_2V_2\right\}}{\text{ª}\left\{c + c_1W_1 + \dfrac{c_2W_2}{k}\right\}}} \qquad (3.4.24)$$

$$\frac{n}{k} = \sqrt{\frac{V_2}{\text{ª}c_2}} \qquad (3.4.25)$$

On using (3.4.24) in (3.4.25), we get

$$k_{opt} = \sqrt{\frac{c_2\left(V_1 - W_2V_2\right)}{\left(c + c_1W_1\right)V_2}} \qquad (3.4.26)$$

Further, substituting the values of n and k in the expression of expected cost, we get

$$\sqrt{a} = \sqrt{\frac{\left\{V_1 + \left(k_{opt} - 1\right)W_2 V_2\right\}\left\{c + c_1 W_1 + \frac{c_2 W_2}{k_{opt}}\right\}}{C_0}} \tag{3.4.27}$$

Similarly on substituting the value of a in (3.4.25), we get the optimum value of n as

$$n_{opt} = \frac{C_0}{\left\{c + c_1 W_1 + \frac{c_2 W_2}{k_{opt}}\right\}} \tag{3.4.28}$$

Substituting the optimum value of n and k, we get the mean square error of $\bar{y}^*$ for fixed cost C_0 given by

$$MSE\left(\bar{y}^*\right) = \left[\frac{\left\{V_1 + \left(k_{opt} - 1\right)W_2 V_2\right\}\left\{c + c_1 W_1 + \frac{c_2 W_2}{k_{opt}}\right\}}{C_0}\right] \tag{3.4.29}$$

3.5 EMPIRICAL STUDY

The present data belong to the data on physical growth of upper socio-economic group of 95 school going children of Varanasi under an ICMR study, Department of Pediatrics, BHU during 1983-84 has been taken under study, Khare and Sinha (2007). The first 25% (i.e. 24 children) units have been considered as non-response units. The values of parameters related to the study characters y (weight of children in kg), the auxiliary character x (chest circumference of the children in cm) have been given as follows:

$\bar{Y}_2 = 19.4968;\ \bar{X} = 55.8611;\ \pm_y = 3.0435;\ \pm_x = 3.2735;\ \pm_{y_{(2)}} = 02.3552;\ \pm_{x_{(2)}} = 2.5137;$

$\pm_{yx} = 8.428611;\ \pm_{yx_{(2)}} = 4.315874.$

The problem considered is to estimate the weight of the male children aged 6-7 years using chest circumference as the auxiliary character.

Table 3.5.1 *PRE & MSE under measurement error & non-response*

↓ Estimators		$1/k$			
	↓ME %	1/2	1/3	1/4	1/5
$\overline{y}^*$	0%	100(0.2958)	100(0.3343)	100(0.3729)	100(0.4114)
	1%	99(0.2988)	99(0.3377)	99(0.6766)	99(0.4155)
	5%	95(0.3106)	95(0.3511)	95(0.3915)	95(0.4320)
	10%	91(0.3254)	91(0.3678)	91(0.4102)	91(0.4525)
	15%	87(0.3402)	87(0.3845)	87(0.4288)	87(0.4731)
	20%	83(0.3550)	83(0.4012)	83(0.4474)	83(0.4937)
t_{R1}	↓ME %	1/2	1/3	1/4	1/5
	0%	193(0.1531)	190(0.1760)	187(0.1990)	185(0.2219)
	1%	189(0.1564)	186(0.1798)	183(0.2032)	182(0.2266)
	5%	174(0.1699)	171(0.1951)	169(0.2202)	168(0.2454)
	10%	158(0.1868)	156(0.2141)	154(0.2415)	153(0.2688)
	15%	145(0.2037)	143(0.2332)	142(0.2627)	141(0.2923)
	20%	134(0.2206)	133(0.2523)	131(0.2840)	130(0.3157)
t_{P1}	↓ME %	1/2	1/3	1/4	1/5
	0%	56.69(0.5218)	56.99(0.5866)	57.24(0.6514)	57.44(0.7161)
	1%	56.33(0.5252)	56.63(0.5904)	56.87(0.6556)	57.07(0.7208)
	5%	54.92(0.5387)	55.21(0.6054)	55.44(0.6726)	55.62(0.7396)
	10%	53.25(0.5555)	53.52(0.6247)	53.74(0.6939)	53.91(0.7630)
	15%	51.68(0.5724)	51.94(0.6438)	52.14(0.7151)	52.31(0.7865)
	20%	50.20(0.5893)	50.44(0.6628)	50.63(0.7363)	50.79(0.8099)
t_{lr1}	↓ME %	1/2	1/3	1/4	1/5
	0%	223(0.0916)	304(0.1100)	290(0.1284)	280(0.1468)
	1%	306(0.0966)	289(0.1156)	277(0.1346)	268(0.1536)
	5%	255(0.1161)	243(0.1374)	235(0.1584)	229(0.1800)
	10%	212(0.1398)	204(0.1638)	198(0.1879)	194(0.2120)
	15%	182(0.1626)	177(0.1894)	172(0.2162)	169(0.2430)
	20%	160(0.1848)	156(0.2143)	153(0.2437)	151(0.2732)
t_{R2}	↓ME %	1/2	1/3	1/4	1/5
	0%	175(0.1687)	161(0.2072)	152(0.2457)	145(0.2842)
	1%	172(0.1720)	159(0.2109)	149(0.2498)	143(0.2887)
	5%	160(0.1853)	148(0.2257)	140(0.2661)	134(0.3066)
	10%	147(0.2019)	137(0.2442)	130(0.2866)	125(0.3290)
	15%	135(0.2185)	127(0.2628)	121(0.3071)	117(0.3514)
	20%	126(0.2351)	119(0.2813)	114(0.3275)	110(0.3737)

	↓ME %	1/2	1/3	1/4	1/5
t_{P2}	0%	59.70(0.4955)	62.61(0.5340)	65.12(0.5726)	67.32(0.6111)
	1%	59.30(0.4988)	62.18(0.5377)	64.66(0.5766)	66.83(0.6156)
	5%	57.76(0.5121)	60.51(0.5526)	62.88(0.5930)	64.94(0.6335)
	10%	55.95(0.5287)	58.54(0.5711)	60.78(0.6135)	62.73(0.6558)
	15%	54.25(0.5453)	56.70(0.5896)	58.82(0.6339)	60.66(0.6782)
	20%	52.64(0.5619)	54.98(0.6082)	56.98(0.6544)	58.72(0.7006)
t_{lr2}	↓ME %	1/2	1/3	1/4	1/5
	0%	265(0.1117)	223(0.1502)	198(0.1887)	181(0.2272)
	1%	254(0.1164)	215(0.1554)	192(0.1943)	176(0.2332)
	5%	219(0.1352)	190(0.1757)	173(0.2161)	160(0.2566)
	10%	187(0.1580	167(0.2004)	154(0.2427)	144(0.2851)
	15%	164(0.1801)	149(0.2244)	139(0.2687)	131(0.3130)
	20%	147(0.2015)	135(0.2478)	127(0.2940)	121(0.3402)
t_1	↓ME %	1/2	1/3	1/4	1/5
	0%	93(0.3165)	98(0.3394)	103(0.3624)	107(0.3853)
	1%	92(0.3199)	97(0.3433)	102(0.3666)	105(0.3900)
	5%	89(0.3334)	93(0.3585)	97(0.3836)	101(0.4088)
	10%	84(0.3502)	89(0.3776)	92(0.4049)	95(0.4322)
	15%	81 (0.3671)	84(0.3966)	87(0.4262)	90(0.4557)
	20%	77(0.3839)	80(0.4157)	83(0.4474)	86(0.4791)
t_2	↓ME %	1/2	1/3	1/4	1/5
	0%	83(0.3583)	79(0.4231)	76(0.4879)	74(0.5527)
	1%	81 (0.3617)	78(0.4269)	76(0.4922)	74(0.5574)
	5%	79(0.3752)	76(0.4422)	73(0.5092)	71(0.5762)
	10%	75(0.3921)	72(0.4613)	70(0.5304)	69(0.5996)
	15%	72(0.4089)	70(0.4803)	68(0.5517)	66(0.6231)
	20%	69(0.4258)	67(0.4993)	65(0.5729)	64(0.6465)
t_{lr3}	↓ME %	1/2	1/3	1/4	1/5
	-	-	-	-	-
	-	-	-	-	-
	-	-	-	-	-
	-	-	-	-	-
	-	-	-	-	-
	-	-	-	-	-

	↓ME %	1/2	1/3	1/4	1/5
t_{R4}	0%	301(0.0984)	275(0.1214)	258(0.1443)	246(0.1673)
	1%	288(0.1029)	265(0.1263)	249(0.1497)	238(0.1730)
	5%	245(0.1207)	229(0.1459)	218(0.1710)	210(0.1961)
	10%	207(0.1430)	196(0.1704)	189(0.1977)	183(0.2250)
	15%	179(0.1654)	172(0.1949)	166(0.2244)	162(0.2539)
	20%	158(0.1877)	152(0.2194)	148(0.2511)	145(0.2828)
t_{P4}	↓ME %	1/2	1/3	1/4	1/5
	0%	37.26(0.7940)	38.93(0.8588)	40.37(0.9236)	41.62(0.9883)
	1%	38.11(0.7985)	39.82(0.8637)	41.29(0.9289)	42.56(0.9941)
	5%	37.27(0.8163)	37.93(0.8833)	40.36(0.9503)	41.60(1.0172)
	10%	36.28(0.8386)	37.88(0.9078)	39.26(0.9770)	40.45(1.0461)
	15%	35.34(0.8609)	36.89(0.9923)	38.21(1.0037)	39.36(1.0750)
	20%	34.45(0.8832)	35.94(0.9568)	37.22(1.0304)	38.33(1.1039)
t_g	↓ME %	1/2	1/3	1/4	1/5
	0%	324(0.0912)	306(0.1092)	293(0.1273)	283(0.1453)
	1%	308(0.0962)	291(0.1148)	279(0.1335)	270(0.1521)
	5%	256(0.1157)	245(0.1367)	237(0.1576)	230(0.1786)
	10%	212(0.1394)	205(0.1631)	199(0.1869)	195(0.2769)
	15%	182(0.1623)	177(0.1888)	173(0.2153)	170(0.2418)
	20%	160(0.1845)	157(0.2136)	154(0.2428)	151(0.2720)
t_d	↓ME %	1/2	1/3	1/4	1/5
	0%	324(0.0912)	306(0.1092)	293(0.1273)	283(0.1453)
	1%	308(0.0962)	291(0.1148)	279(0.1335)	270(0.1521)
	5%	256(0.1157)	245(0.1367)	237(0.1576)	230(0.1786)
	10%	212(0.1394)	205(0.1631)	199(0.1869)	195(0.2769)
	15%	182(0.1623)	177(0.1888)	173(0.2153)	170(0.2418)
	20%	160(0.1845)	157(0.2136)	154(0.2428)	151(0.2720)
$\overline{y}_f$	↓ME %	1/2	1/3	1/4	1/5
	0%	324(0.0912)	306(0.1092)	293(0.1273)	283(0.1453)
	1%	308(0.0962)	291(0.1148)	279(0.1335)	270(0.1521)
	5%	256(0.1157)	245(0.1367)	237(0.1576)	230(0.1786)
	10%	212(0.1394)	205(0.1631)	199(0.1869)	195(0.2769)
	15%	182(0.1623)	177(0.1888)	173(0.2153)	170(0.2418)
	20%	160(0.1845)	157(0.2136)	154(0.2428)	151(0.2720)

Table 3.5.2 *PRE(MSE) with measurement error and without non-response*

Estimator	0%	1%	5%	10%	15%	20%
$\bar{y}^*$	100(0.2573)	99(0.2599)	95(0.2702)	91(0.2830)	87(0.2959)	83(0.3088)
t_{R1}	198(0.1301)	193(0.1331)	178(0.1448)	161(0.1595)	148(0.1742)	136(0.1888)
t_{P1}	56(0.4570)	56(0.4599)	55(0.4717)	53(0.4864)	51(0.5010)	50(0.5157)
t_{lr1}	352(0.0732)	332(0.0776)	271(0.0948)	222(0.1157)	189(0.1358)	166(0.1553)
t_{R2}	198(0.1301)	193(0.1331)	178(0.1448)	161(0.1595)	148(0.1742)	136(0.1888)
t_{P2}	56(0.4570)	56(0.4599)	55(0.4717)	53(0.4864)	51(0.5010)	50(0.5157)
t_{lr2}	352(0.0731)	332(0.0775)	271(0.0948)	223(0.1156)	190(0.1358)	166(0.1553)
t_1	100(0.2573)	99(0.2599)	95(0.2702)	91(0.2830)	87(0.2959)	83(0.3087)
t_2	100(0.2573)	99(0.2599)	95(0.2702)	91(0.2830)	87(0.2959)	83(0.3087)
t_{R4}	341(0.0755)	324(0.0795)	269(0.0956)	222(0.1157)	189(0.1358)	165(0.1559)
t_{P4}	35(0.7292)	36(0.7332)	35(0.7493)	34(0.7694)	34(0.7896)	33(0.8097)
t_g	352(0.0731)	332(0.0775)	271(0.0948)	223(0.1156)	190(0.1358)	166(0.1553)
t_d	352(0.0731)	332(0.0775)	271(0.0948)	223(0.1156)	190(0.1358)	166(0.1553)
$\bar{y}_f$	352(0.0731)	332(0.0775)	271(0.0948)	223(0.1156)	190(0.1358)	166(0.1553)

Table 3.5.3 *PRE (MSE) with measurement error for optimum value of k*

Estimators ↓	PRE(MSE)				
Per cent of ME →	1%	5%	10%	15%	20%
$\bar{y}^*$	99(0.4302)	95(0.4472)	91(0.4685)	87(0.4898)	83(0.5111)
t_{R1}	190(0.2246)	175(0.2441)	159(0.2684)	146(0.2927)	134(0.3170)
t_{P1}	56(0.7562)	55(0.7757)	53(0.7999)	52(0.8243)	50(0.8485)
t_{lr1}	314(0.1358)	259(0.1645)	214(0.1991)	183(0.2324)	161(0.2646)
t_{R2}	180(0.2365)	166(0.2564)	151(0.2812)	139(0.3059)	129(0.3305)
t_{P2}	60(0.7084)	59(0.7278)	57(0.7519)	55(0.7760)	53(0.8001)
t_{lr2}	308(0.1381)	250(0.1705)	205(0.2079)	175(0.2429)	154(0.2763)
t_1	94(0.4527)	90(0.4726)	85(0.4974)	82(0.5222	78(0.5467)
t_2	83(0.5133)	80(0.5330)	76(0.5575)	73(0.5820)	70(0.6065)
t_{R4}	301(0.1413)	253(0.1684)	211(0.2018)	181(0.2349)	159(0.2677)
t_{P4}	37(1.1365)	37(1.1622)	36(1.1943)	35(1.2263)	34(1.2584)
t_g	314(0.1355)	259(0.1642)	214(0.1987)	184(0.2320)	161(0.2642)
t_d	314(0.1355)	259(0.1642)	214(0.1987)	184(0.2320)	161(0.2642)
$\bar{y}_f$	314(0.1355)	259(0.1642)	214(0.1987)	184(0.2320)	161(0.2642)

Table 3.5.4 *PRE(MSE) without measurement error and with non-response error for optimum value of k*

Estimators	MSE
$\bar{y}^*$	100(0.4259)
t_{R1}	194(0.2197)
t_{P1}	57(0.7514)
t_{lr1}	332(0.1284)
t_{R2}	184(0.2314)
t_{P2}	61(0.7036)
t_{lr2}	329(0.1295)
t_1	95(0.4477)
t_2	84(0.5084)
t_{R4}	317(0.1345)
t_{P4}	38(1.1301)
t_g	332(0.1281)
t_d	332(0.1281)
$\bar{y}_f$	332(0.1281)

Table 3.5.5 *PRE(MSE) without measurement error and with non-response error*

Estimators	1/2	1/3	1/4	1/5
$\bar{y}^*$	100(0.2958)	100(0.3343)	100(0.3729)	100(0.4114)
t_{R1}	193(0.1531)	190(0.1760)	187(0.1990)	185(0.2219)
t_{P1}	56.69(0.5218)	56.99(0.5866)	57.24 (0.6514)	57.44(0.7161)
t_{lr1}	323(0.0916)	304(0.1100)	290(0.1284)	280(0.1468)
t_{R2}	193(0.1531)	190(0.1860)	187(0.1990)	185(0.2219)
t_{P2}	56.69(0.5218)	56.99(0.5866)	57.24 (0.6514)	57.44(0.7161)
t_{lr2}	323(0.0917)	303(0.1102)	290(0.1287)	279(0.1472)
t_1	106(0.2802)	110(0.3032)	114(0.3261)	118(0.3491)
t_2	92(0.3221)	86(0.3869)	83(0.4517)	80(0.5165)
t_{R4}	301(0.0984)	275(0.1214)	258(0.1443)	246(0.1673)
t_{P4}	37(0.7940)	34(0.9884)	33(1.1179)	33(1.2475)
t_g	171(0.1731)	122(0.2730)	100(0.3730)	87(0.4729)
t_d	171(0.1731)	122(0.2730)	100(0.3730)	87(0.4729)
$\bar{y}_f$	171(0.1731)	122(0.2730)	100(0.3730)	87(0.4729)

3.6 CONCLUDING REMARKS

1. It is evident from the expressions (3.3.11) to (3.3.22) of MSE's of the estimators that the measurement errors seem to have inflated the MSE of these estimators and thereby decreasing the efficiency.

2. The expressions (3.3.11) to (3.3.22) of MSE can be broken into 4 major components owing to non-response and measurement error are given below:

$$MSE = A + B + C + D$$

where A = Component of MSE due to sampling error without measurement error and non-response,

B = Component of MSE due to sampling error with measurement error and without non-response,

C = Component of MSE due to sampling error without measurement error and with non-response, and

D = Component of MSE due to sampling error with measurement error and without non-response.

For Example: Consider the expression of MSE of $\bar{y}_f$ given by

$$MSE(\bar{y}_f) = \underbrace{\frac{1}{n}\left(\pm_y^2 + \yen^2 \pm_x^2 - 2\yen \pm_{xy}\right)}_{A}$$
$$+ \underbrace{\frac{1}{n}\left(\pm_u^2 + \yen^2 \pm_v^2\right)}_{B}$$
$$+ \underbrace{\frac{W_2(k-1)}{n}\left(\pm_{y_2}^2 + f_2^2 \pm_{x_2}^2 - 2 f_2 \pm_{xy_2}\right)}_{C}$$

$$+\underbrace{\frac{W_2(k-1)}{n}\left(\pm_{u_2}^2+f_2^2\pm_{v_2}^2\right)}_{D}$$

3. If the measurement error is absent then we get the expression of the MSE of conventional estimators under non-response from the results of this study thereby the present study provides a more general and pragmatic approach for the estimation of population mean. For example: When $u_i = 0 = v_i$, for each i, then $\pm_u^2=\pm_v^2=\pm_{u_2}^2=\pm_{v_2}^2=0$ and we get

$$MSE(\bar{y}_f)=\underbrace{\frac{1}{n}\left(\pm_y^2+¥^2\pm_x^2-2¥\pm_{xy}\right)}_{A}$$

$$+\underbrace{\frac{W_2(k-1)}{n}\left(\pm_{y_2}^2+f_2^2\pm_{x_2}^2-2f_2\pm_{xy_2}\right)}_{C}$$

so that only components A and C are left, which is same expression as proposed by Bhushan and Naqvi (2015) while proposing t_d of Singh and Kumar (2008) as a special case of $\bar{y}_f$.

4. If the measurement error is absent then we get the expression of the optimum values of the characterizing scalars/derivatives and minimum MSE of conventional estimators under non-response from the results of this study thereby the present study. For example: When $u_i = 0 = v_i$, for each i, then $\pm_u^2=\pm_v^2=\pm_{u_2}^2=\pm_{v_2}^2=0$ and we get optimum values of f_2 and ¥ as

$f_2=-\pm_{xy_2}/\pm_{x_2}^2$ and $¥=\pm_{xy}/\pm_x^2$.

and the minimum mean square error of $\bar{y}_f$ as

$$MSE(\bar{y}_f)_{\min}=\frac{1}{n}\left(1-°^2\right)\pm_y^2+\frac{W_2(k-1)}{n}\left(1-°_2^2\right)\pm_{y_2}^2$$

which is same expression of minimum MSE as proposed by Bhushan and Naqvi (2015) while proposing t_d of Singh and Kumar (2008) as a special case of $\bar{y}_f$.

5. The measurement error seems to have affected all the estimators but the optimal generalized estimator $\bar{y}_f$ and their optimal special cases t_d and t_g perform far better than the remaining estimators where the auxiliary information was not properly utilized. For example: This is w.r.t. the empirical results in table 3.5.1 where $\bar{y}_f$ utilized the auxiliary information optimally and outperformed all the remaining estimators.

6. The measurement error seems to have affected the better estimators more where the auxiliary information was properly utilized than those estimators where the auxiliary information was not properly utilized. For example: This is w.r.t. the empirical results in table 3.5.1 where $\bar{y}_f$ utilized the auxiliary information optimally and has lost 132% efficiency with 1/5 sub-sampling fraction which is far more in comparison to t_{P4}, estimator where the auxiliary information was not properly utilized, lost only 3% efficiency with 1/5 sub-sampling fraction or even $\bar{y}*$ which lost 17% efficiency with 1/5 sub-sampling fraction.

7. The gains in efficiency of various estimators were calculated w. r. t. $\bar{y}*$ with no non-response error in table 3.5.2. The best choice, which is in consonance with the theoretical results, is $\bar{y}_f$ at all levels of measurement error.

8. The percent relative efficiency of various estimators were calculated w. r. t. $\bar{y}^*$ with optimum sub-sampling fraction $1/k$ in table 3.5.3. The best choice, which is in consonance with the theoretical results, is $\bar{y}_f$ at all levels of measurement error.

9. The gains in efficiency of various estimators were calculated w. r. t. $\bar{y}^*$ with no measurement error in table 3.5.4. The best choice, which is in consonance with the theoretical results, is $\bar{y}_f$ at all levels of measurement error.

10. The percent relative efficiency of various estimators were calculated w. r. t. $\bar{y}^*$ without measurement error and with different non-respondent sub-sampling fraction $1/k$ in table 3.5.5. The best choice is $\bar{y}_f$ which is in consonance with the theoretical results of Bhushan and Naqvi (2015).

CHAPTER 4

ON SOME ESTIMATORS OF POPULATION MEAN UNDER DOUBLE SAMPLING WITH MEASUREMENT ERROR AND NON-RESPONSE

SUMMARY

In this chapter we consider the problem of estimating the population mean under double sampling using auxiliary information in presence of measurement error and non-response error simultaneously. Some modified ratio, product and difference estimators in double sampling have been adapted from Singh and Kumar (2010) and their properties are studied presence of measurement error and non-response error simultaneously. An empirical study is carried to study the merits of the estimators over conventional unbiased estimator and other known estimators where we analyzed the effect of measurement error on the adapted estimators at different levels. Both theoretical and empirical study results present the soundness and usefulness of the suggested estimators in practice under presence of measurement error and non-response error simultaneously.

4.1 INTRODUCTION

In survey sampling, it is presumed that the observations are recorded correct measurements on characteristics being studied. But such kind of assumption does not satisfied in many applications and data is recorded with measurement errors, such as reporting errors and computing errors. These measurement errors make the result invalid. If measurement error is very small and we can neglect it, then the statistical inferences based on observed data continue to remain valid. On the contrary if measurement error is not negligible, the inferences may not be simply invalid and

inaccurate, but may often lead to unexpected, undesirable and unfortunate consequences (see Srivastava and Shalabh (2001)).

Consider a finite population $U = (U_1, U_2, ..., U_N)$ of N units. Let Y and X be the study variate and auxiliary variate, respectively. Suppose that we have a set of n paired observations obtained through simple random sampling procedure on two characteristics X and Y. Further, suppose that (x_i, y_i) for the ith sampling units are observed with measurement error instead of their true values (X_i, Y_i). For a simple random sampling scheme, let (x_i, y_i) be observed values instead of the true values (X_i, Y_i). for ith (i=1, 2,...n) unit such that

$$u_i = y_i - Y_i \tag{4.1.1}$$

$$v_i = x_i - X_i \tag{4.1.2}$$

where u_i and v_i are associated measurement errors which are stochastic (probabilistic) in nature with mean zero, variances $\pm_u^2$ and $\pm_v^2$ respectively. Further, we assume that u_i and v_i are uncorrelated although X_i are Y_i are correlated. Let the population means of X and Y characteristics be «$_x$ and «$_y$ where population variances of X and Y be $\pm_x^2$ and $\pm_y^2$. Let $\pm_{xy}$ be the population covariance between x and y.

In surveys covering human populations, information is in most cases not obtained from all the units in the survey even after some call-backs. An estimate obtained from such incomplete data may be misleading especially when the respondents differ from the non-respondents because the estimate can be biased. Hansen and Hurwits (1946) envisaged a simple technique of sub-sampling the non-respondents in order to adjust for the non-response in a mail survey. In estimating population parameters like the mean, total or ratio, sample survey experts sometimes use auxiliary information to improve the precision. When the population mean $\bar{X}$ of the auxiliary variable x is known and in presence of non-response, the problem of estimation of population mean $\bar{Y}$ of the study variate y has been dealt by Cochran (1977), Rao (1986, 1987) and Khare and Srivastava (1993, 1997). However , many situations of practical importance the problem of estimation of population mean $\bar{Y}$ of the study variate y assumes importance when the population mean $\bar{X}$ of auxiliary variate x is not known in presence of non-response. But, if such information is missing then such situation we generally resort to double sampling as suggested by Okafor and Lee (2000), Tabasum and Khan (2004) and recently by Singh and Kumar (2010) among others. If the population mean $\bar{X}$ of the auxiliary variable is not known then Okafor and Lee (2000) proposed to use the sample mean $\bar{x}'$ obtained from a large first phase preliminary sample of size n' drawn from N units by simple random sampling without replacement (SRSWOR). It is noteworthy to mention here that it is assumed that all the first phase sample units supplied the auxiliary information; see Singh and Kumar (2010). Then a second phase sample of size n $(n < n')$ is drawn from the n' by simple random sampling without replacement (SRSWOR) and study variable y is measured on it. At the second phase from the sample of size n, let n_1 units respond and n_2 units refuse to respond. Now, we use Hansen Hurwitz (1946) sampling strategy to sub-sample r units from n_2 non-responding units and enumerated by direct interview such that $r = n_2 / k$, $k > 1$. It is again implicitly assumed that these r units respond to the direct interview. Okafor and Lee (2000) and Tabasum and Khan (2004) have mentioned that the procedure of two phase sampling can be applied to a household survey where household size is an auxiliary variable for the estimation of family expenditure. The information such as family size might be completely known while there may be a non-response on the household expenditure. A similar example would be regarding disposable income survey where personal taxes paid are known due to tax laws while there may be non-response on the study variable i.e. disposable income.

In this chapter throughout we assume the deterministic set up of non-response exactly on the similar lines as that of Singh and Kumar (2010) and assume that the whole population (denoted by Ω) is stratified into two strata: one is the stratum (denoted by Ω_1) of N_1 units, which would respond on the first call at the second phase and the other stratum (denoted by Ω_2) of N_2 units, which would not respond on the first call but would respond on the second call. Let the first and second phase samples be denoted by s and s' respectively, and let $s_1 = s \cap \Omega_1$ and $s_2 = s \cap \Omega_2$. The sub-sample of s_2 will be denoted by s_{2m}. Summation over the units in the set s will be denoted by $\sum_s$. As tradition goes, throughout in this chapter, the population parameters are denoted by capital letters and the sample statistics are denoted by small letters.

4.2 ADAPTED ESTIMATORS

With this background, the usual ratio, product and difference estimators using two phase sampling in presence of non-response are respectively defined by

$$T_1 = \frac{\bar{y}^*}{\bar{x}^*}\bar{x}' \qquad (4.2.1)$$

$$T_2 = \frac{\bar{y}^*}{\bar{x}'}\bar{x}^* \qquad (4.2.2)$$

$$T_3 = \frac{\bar{y}^*}{\bar{x}}\bar{x}' \qquad (4.2.3)$$

$$T_4 = \frac{\bar{y}^*}{\bar{x}'}\bar{x} \qquad (4.2.4)$$

$$T_5 = \bar{y}^*\left(\frac{\bar{x}'}{\bar{x}^*}\right)\left(\frac{\bar{x}'}{\bar{x}}\right) \qquad (4.2.5)$$

$$T_6 = \bar{y}^*\left(\frac{\bar{x}^*}{\bar{x}'}\right)\left(\frac{\bar{x}}{\bar{x}'}\right) \qquad (4.2.6)$$

$$T_7 = \bar{y}^*\left(\frac{\bar{x}}{\bar{x}^*}\right)^{\sim_1}\left(\frac{\bar{x}'}{\bar{x}}\right)^{\sim_2} \qquad (4.2.7)$$

$$T_8 = \bar{y}^* + d_1\left(\bar{x} - \bar{x}^*\right) + d_2\left(\bar{x}' - \bar{x}\right) \qquad (4.2.8)$$

$$T_9 = \bar{y}^* + d_3\left(\bar{x}^* - \bar{x}'\right) \qquad (4.2.9)$$

$$T_{10} = \bar{y}^* + d_4\left(\bar{x} - \bar{x}'\right) \qquad (4.2.10)$$

where $\sim_1, \sim_2, d_i\ (i = 1,2,3,4)$ are characterizing scalars to be suitably chosen.

4.3 BIAS AND MEAN SQUARE ERROR

The bias of these estimators is given below while estimators (4.1.8), (4.1.9) and (4.1.10) are unbiased.

$$Bias(T_1)=\left(\frac{1}{n}-\frac{1}{n'}\right)\frac{1}{\bar{X}}\left[R\left(\pm_x^2+\pm_v^2\right)-\pm_{xy}\right]+\frac{W_2(k-1)R}{n\bar{X}}\left(\pm_{x_{(2)}}^2+\pm_{v_{(2)}}^2\right) \qquad (4.3.1)$$

$$Bias(T_2)=\left(\frac{1}{n}-\frac{1}{n'}\right)\frac{1}{\bar{X}}\pm_{xy}+\frac{W_2(k-1)}{n\bar{X}}\pm_{xy_{(2)}} \qquad (4.3.2)$$

$$Bias(T_4)=\left(\frac{1}{n}-\frac{1}{n'}\right)\frac{1}{\bar{X}}\left[R\left(\pm_x^2+\pm_v^2\right)+\pm_{xy}\right] \qquad (4.3.3)$$

$$Bias(T_5)=\left(\frac{1}{n}-\frac{1}{n'}\right)\frac{1}{\bar{X}}\left[R\left(\pm_x^2+\pm_v^2\right)-\pm_{xy}\right] \qquad (4.3.3)$$

$$Bias(T_7)=\left(\frac{1}{n}-\frac{1}{n'}\right)\frac{1}{\bar{X}}\left[3R\left(\pm_x^2+\pm_v^2\right)-2\pm_{xy}\right]+\frac{W_2(k-1)}{n\bar{X}}\left[R\left(\pm_{x_{(2)}}^2+\pm_{v_{(2)}}^2\right)-2\pm_{xy_{(2)}}\right] \qquad (4.3.4)$$

$$Bias(T_8)=\left(\frac{1}{n}-\frac{1}{n'}\right)\frac{1}{\bar{X}}\left[R\left(\pm_x^2+\pm_v^2\right)+\pm_{xy}\right]+\frac{W_2(k-1)}{n\bar{X}}\pm_{xy_{(2)}} \qquad (4.3.5)$$

$$Bias(T_9)=\left(\frac{1}{n}-\frac{1}{n'}\right)\frac{1}{\bar{X}}\left[R\sim_2(\sim_2+1)\left(\pm_x^2+\pm_v^2\right)-\sim_2\pm_{xy}\right]$$

$$-\frac{W_2(k-1)\sim_1}{\bar{X}}\pm_{xy_{(2)}} \qquad (4.3.6)$$

The MSE's of these estimators is given by

$$MSE(T_1)=MSE\left(\bar{y}^*\right)+\left[\left(\frac{1}{n}-\frac{1}{n'}\right)\left\{\left(\pm_x^2+\pm_v^2\right)-2\pm_{yx}\right\}\right.$$

$$\left.+\frac{W_2(k-1)}{n}\left\{\left(\pm_{x_{(2)}}^2+\pm_{v_{(2)}}^2\right)-2\pm_{yx_{(2)}}\right\}\right] \qquad (4.3.7)$$

$$MSE(T_2)=MSE\left(\bar{y}^*\right)+\left[\left(\frac{1}{n}-\frac{1}{n'}\right)\left\{\left(\pm_x^2+\pm_v^2\right)+2\pm_{yx}\right\}\right.$$

$$\left.+\frac{W_2(k-1)}{n}\left\{\left(\pm_{x_{(2)}}^2+\pm_{v_{(2)}}^2\right)+2\pm_{yx_{(2)}}\right\}\right] \qquad (4.3.8)$$

$$MSE(T_3)=MSE\left(\bar{y}^*\right)+\left[\left(\frac{1}{n}-\frac{1}{n'}\right)\left\{\left(\pm_x^2+\pm_v^2\right)-2\pm_{yx}\right\}\right] \qquad (4.3.9)$$

$$MSE(T_4)=MSE\left(\bar{y}^*\right)+\left[\left(\frac{1}{n}-\frac{1}{n'}\right)\left\{\left(\pm_x^2+\pm_v^2\right)+2\pm_{yx}\right\}\right] \tag{4.3.10}$$

$$MSE(T_5)=MSE\left(\bar{y}^*\right)+\left[\left(\frac{1}{n}-\frac{1}{n'}\right)\left\{4\left(\pm_x^2+\pm_v^2\right)-4\pm_{yx}\right\}\right.$$
$$\left.+\frac{W_2\left(k-1\right)}{n}\left\{\left(\pm_{x_{(2)}}^2+\pm_{v_{(2)}}^2\right)-2\pm_{yx_{(2)}}\right\}\right] \tag{4.3.11}$$

$$MSE(T_6)=MSE\left(\bar{y}^*\right)+\left[\left(\frac{1}{n}-\frac{1}{n'}\right)\left\{4\left(\pm_x^2+\pm_v^2\right)+4\pm_{yx}\right\}\right.$$
$$\left.+\frac{W_2\left(k-1\right)}{n}\left\{\left(\pm_{x_{(2)}}^2+\pm_{v_{(2)}}^2\right)+2\pm_{yx_{(2)}}\right\}\right] \tag{4.3.12}$$

$$MSE(T_7)=MSE\left(\bar{y}^*\right)+\left[\left(\frac{1}{n}-\frac{1}{n'}\right)\left\{\sim_2^2\left(\pm_x^2+\pm_v^2\right)-2\sim_2\pm_{yx}\right\}\right.$$
$$\left.+\frac{W_2\left(k-1\right)}{n}\left\{\sim_1^2\left(\pm_{x_{(2)}}^2+\pm_{v_{(2)}}^2\right)-2\sim_1\pm_{yx_{(2)}}\right\}\right] \tag{4.3.13}$$

$$MSE(T_8)=MSE\left(\bar{y}^*\right)+\left[\left(\frac{1}{n}-\frac{1}{n'}\right)\left\{d_2^2\left(\pm_x^2+\pm_v^2\right)-2d_2^2\pm_{yx}\right\}\right.$$
$$\left.+\frac{W_2\left(k-1\right)}{n}\left\{d_1^2\left(\pm_{x_{(2)}}^2+\pm_{v_{(2)}}^2\right)-2d_1\pm_{yx_{(2)}}\right\}\right] \tag{4.3.14}$$

$$MSE(T_9)=MSE\left(\bar{y}^*\right)+\left[\left(\frac{1}{n}-\frac{1}{n'}\right)\left\{d_3^2\left(\pm_x^2+\pm_v^2\right)+2d_3\pm_{yx}\right\}\right.$$
$$\left.+\frac{W_2\left(k-1\right)}{n}\left\{d_3^2\left(\pm_{x_{(2)}}^2+\pm_{v_{(2)}}^2\right)+2d_3\pm_{yx_{(2)}}\right\}\right] \tag{4.3.15}$$

$$MSE(T_{10})=MSE\left(\bar{y}^*\right)+\left[\left(\frac{1}{n}-\frac{1}{n'}\right)\left\{d_4^2\left(\pm_x^2+\pm_v^2\right)+2d_4\pm_{yx}\right\}\right] \tag{4.3.16}$$

The optimum values which minimizes the MSE is

$$\sim_1=\frac{\pm_{yx_{(2)}}}{\left(\pm_{x_{(2)}}^2+\pm_{x_{(2)}}^2\right)} \tag{4.3.17}$$

$$\sim_2 = \frac{\pm_{yx}}{\left(\pm_x^2 + \pm_v^2\right)} \tag{4.3.18}$$

$$d_1 = \frac{\pm_{yx_{(2)}}}{\left(\pm_{x_{(2)}}^2 + \pm_{x_{(2)}}^2\right)} \tag{4.3.19}$$

$$d_2 = \frac{\pm_{yx}}{\left(\pm_x^2 + \pm_v^2\right)} \tag{4.3.20}$$

$$d_3 = -\frac{\left[\left(\frac{1}{n} - \frac{1}{n'}\right)\pm_{yx} + \frac{W_2(k-1)}{n}\pm_{yx_{(2)}}\right]}{\left[\left(\frac{1}{n} - \frac{1}{n'}\right)\left(\pm_x^2 + \pm_v^2\right) + \frac{W_2(k-1)}{n}\left(\pm_{x_{(2)}}^2 + \pm_{v_{(2)}}^2\right)\right]} \tag{4.3.21}$$

$$d_4 = -\frac{\pm_{yx}}{\left(\pm_x^2 + \pm_v^2\right)} \tag{4.3.22}$$

and the minimum MSE's are found to be

$$MSE(T_7)_{min} = MSE\left(\overline{y}^*\right) - \left[\left(\frac{1}{n} - \frac{1}{n'}\right)\frac{\left(\pm_{yx}\right)^2}{\left(\pm_x^2 + \pm_v^2\right)} + \frac{W_2\left(k-1\right)}{n}\frac{\left(\pm_{yx_{(2)}}\right)^2}{\left(\pm_{x_{(2)}}^2 + \pm_{v_{(2)}}^2\right)}\right] \tag{4.3.23}$$

$$MSE(T_8)_{min} = MSE\left(\overline{y}^*\right) - \left[\left(\frac{1}{n} - \frac{1}{n'}\right)\frac{\left(\pm_{yx}\right)^2}{\left(\pm_x^2 + \pm_v^2\right)} + \frac{W_2\left(k-1\right)}{n}\frac{\left(\pm_{yx_{(2)}}\right)^2}{\left(\pm_{x_{(2)}}^2 + \pm_{v_{(2)}}^2\right)}\right] \tag{4.3.24}$$

$$MSE(T_9)_{min} = MSE\left(\overline{y}^*\right) - \left[\frac{\left\{\left(\frac{1}{n} - \frac{1}{n'}\right)\pm_{yx} + \frac{k'}{n}\pm_{yx_{(2)}}\right\}^2}{\left\{\left(\frac{1}{n} - \frac{1}{n'}\right)\left(\pm_x^2 + \pm_v^2\right) + \frac{k'}{n}\left(\pm_{x_{(2)}}^2 + \pm_{v_{(2)}}^2\right)\right\}}\right] \tag{4.3.25}$$

$$MSE(T_{10})_{min} = MSE\left(\overline{y}^*\right) - \left[\left(\frac{1}{n} - \frac{1}{n'}\right)\frac{\left(\pm_{yx}\right)^2}{\left(\pm_x^2 + \pm_v^2\right)}\right] \tag{4.3.26}$$

It can be easily seen that T_7 and T_8 have the same minimum mean square error.

$$MSE_{min}\left(T_7\right)=MSE_{min}\left(T_8\right)$$

$$=MSE\left(\bar{y}^*\right)-\left[\left(\frac{1}{n}-\frac{1}{n'}\right)\frac{\left(\pm_{yx}\right)^2}{\left(\pm_x^2+\pm_v^2\right)}+\frac{W_2\left(k-1\right)}{n}\frac{\left(\pm_{yx_{(2)}}\right)^2}{\left(\pm_{x_{(2)}}^2+\pm_{v_{(2)}}^2\right)}\right] \qquad (4.3.27)$$

4.4 DETERMINATION OF OPTIMUM VALUES OF *n, n′* AND *k*

The expected total cost of the survey apart from the overhead cost is given by

$$C=c'n'+n\left(c+c_1W_1+\frac{c_2W_2}{k}\right) \qquad (4.4.1)$$

where c' is the cost per unit of the first phase sample of size, n' ; c is the cost per unit of the first attempt with the sample, n ; c_1 is the cost per unit for processing the respondent data at the first attempt in n_1 and c_2 is the cost per unit associated with the sub sample r of n_2.

CASE I: FIXED VARIANCE

To determine the optimum values of n , n' and k that minimize the cost for a fixed variance V_0, we consider the function

$$\text{¤}=C^*+\text{ª}\left\{MSE\left(T_g\right)-V_0\right\} \qquad (4.4.2)$$

$$=c'n'+n\left[c+c_1W_1+\frac{c_2W_2}{k}\right]+\text{ª}\left\{\left[\frac{\left(N-n\right)}{Nn}\left(\left(\pm_y^2+\pm_u^2\right)+\left(\pm_x^2+\pm_v^2\right)g_3^2-2\pm_{yx}g_3\right)\right.\right.$$

$$+\frac{(k-1)N_2}{Nn}\left(\pm_{y_{(2)}}^2+\pm_{y_{(2)}}^2+\left(\pm_{x_{(2)}}^2+\pm_{v_{(2)}}^2\right)g_1^2+2\pm_{yx_{(2)}}g_1\right)-\frac{\left(N-n'\right)}{Nn'}\left(\pm_x^2+\pm_v^2\right)g_3^2$$

$$\left.\left.-2\pm_{yx}g_3\right)\right]-V_0\right\}$$

$$= c'n' + n\left[c + c_1W_1 + \frac{c_2W_2}{k}\right] + \text{a}\left\{\left[\frac{\{U_1 + (k-1)W_2U_2 - U_3\}}{n} + \frac{U_3}{n'} - \frac{U_1}{N}\right] - V_0\right\}$$

(4.4.3)

where $U_1 = \left[\pm_y^2 + \pm_u^2 + \left(\pm_x^2 + \pm_v^2\right)g_3^2 - 2\pm_{yx}g_3\right]$,

$U_2 = \left[\left(\pm_{y(2)}^2 + \pm_{u(2)}^2\right) + \left(\pm_{x(2)}^2 + \pm_{x(2)}^2\right)g_1^2 + 2\pm_{yx(2)}g_1\right]$ and $U_3 = \left(-\pm_x^2 g_3^2 + 2\pm_{yx}g_3\right)$ and a is Lagrange's multiplier.

Now differentiating (4.4.3) with respect to n, n' and k, and on equating with zero, we get

$$n = \sqrt{\frac{\text{a}\bar{Y}^2\{U_1 + (k-1)W_2U_2 - U_3\}}{\left\{c + c_1W_1 + \frac{c_2W_2}{k}\right\}}}$$

(4.4.4)

$$\frac{n}{k} = \sqrt{\frac{\text{a}\bar{Y}^2U_2}{c_2}}$$

(4.4.5)

$$n' = \sqrt{\frac{\text{a}\bar{Y}^2U_3}{c'}}$$

(4.4.6)

On putting (4.4.4) in (4.4.5), we get

$$k_{opt} = \sqrt{\frac{c_2(U_1 - W_2U_2 - U_3)}{(c + c_1W_1)U_2}}$$

(4.4.7)

which is required optimum value of k. Further on substituting the value of n and k in the expression of MSE, we get

$$\sqrt{\text{a}} = \frac{\sqrt{\{U_1 + (k-1)W_2U_2 - U_3\}\left\{c + c_1W_1 + \frac{c_2W_2}{k}\right\}} + \sqrt{c'U_3}}{\left\{\frac{V_0}{\bar{Y}} + \frac{\bar{Y}U_1}{N}\right\}}$$

(4.4.8)

On using this value of ª we get the optimum value of n, as

$$n_{opt} = \frac{\left\{U_1 + \left(k_{opt} - 1\right)W_2U_2 - U_3\right\}\sqrt{\left\{c + c_1W_1 + \frac{c_2W_2}{k_{opt}}\right\}} + \sqrt{c'\left\{U_1 + \left(k_{opt} - 1\right)W_2U_2 - U_3\right\}U_3}}{\sqrt{\left\{c + c_1W_1 + \frac{c_2W_2}{k_{opt}}\right\}}\left\{\frac{V_0}{\bar{Y}^2} + \frac{U_1}{N}\right\}}$$

(4.4.9)

Similarly, on using this value of ª we get the optimum value of n', as

$$n'_{opt} = \frac{\sqrt{\left\{U_1 + \left(k_{opt} - 1\right)W_2U_2 - U_3\right\}U_3\left\{c + c_1W_1 + \frac{c_2W_2}{k_{opt}}\right\}} + \sqrt{c'}U_3}{\sqrt{c'}\left\{\frac{V_0}{\bar{Y}^2} + \frac{U_1}{N}\right\}} \qquad (4.4.10)$$

On substituting the optimum value of n, n' and k also ignoring the terms of $O\left(\frac{1}{N}\right)$ we get the minimum cost for fixed variance V_0 given as

$$C^* = \frac{\bar{Y}^2}{V_0}\left[2\left(\sqrt{c'\left\{U_1 + \left(k_{opt} - 1\right)W_2U_2 - U_3\right\}\left\{c + c_1W_1 + \frac{c_2W_2}{k_{opt}}\right\}U_3}\right) + c'U_3 + \left\{U_1 + \left(k_{opt} - 1\right)W_2U_2 - U_3\right\}\left\{c + c_1W_1 + \frac{c_2W_2}{k_{opt}}\right\}\right] \qquad (4.4.11)$$

CASE II: FOR FIXED COST

To determine the optimum values of n, n' and k that minimize the $MSE(T_g)$ for a fixed cost ($C^* < C_0$), we consider the function

$$¤^* = MSE\left(T_g\right) + ª\left\{c'n' + \left(c + c_1W_1 + \frac{c_2W_2}{k}\right)\right\} \qquad (4.4.12)$$

where ª is Lagrange's multiplier.

Now differentiating (4.4.12) with respect to n, n' and k on equating with zero, we get

$$n = \sqrt{\frac{\bar{Y}^2\left\{U_1+(k-1)W_2U_2-U_3\right\}}{\text{a}\left\{c+c_1W_1+\frac{c_2W_2}{k}\right\}}} \qquad (4.4.13)$$

$$\frac{n}{k} = \sqrt{\frac{\bar{Y}^2U_2}{\text{a}\,c_2}} \qquad (4.4.14)$$

$$n' = \sqrt{\frac{\bar{Y}^2U_3}{\text{a}\,c'}} \qquad (4.4.15)$$

On using (4.4.13) in (4.4.14) we get,

$$k_{opt} = \sqrt{\frac{c_2\left(U_1-W_2U_2-U_3\right)}{\left(c+c_1W_1\right)U_2}} \qquad (4.4.16)$$

Further, on substituting the values of n, n' and k in the expression of expected cost, we get

$$\sqrt{\text{a}} = \frac{\sqrt{\bar{Y}^2\left\{U_1+\left(k_{opt.}-1\right)W_2U_2-U_3\right\}\left\{c+c_1W_1+\frac{c_2W_2}{k_{opt.}}\right\}}+\sqrt{c'U_3}}{C_0} \qquad (4.4.17)$$

Again, on substituting the value of a in (4.4.13), we get the optimum value of n as

$$n_{opt} = \frac{C_0\sqrt{\left\{U_1+\left(k_{opt}-1\right)W_2U_2-U_3\right\}}}{\left[\left\{c+c_1W_1+\frac{c_2W_2}{k_{opt}}\right\}\sqrt{\left\{U_1+\left(k_{opt}-1\right)W_2U_2-U_3\right\}}+\sqrt{c'\left\{c+c_1W_1+\frac{c_2W_2}{k_{opt}}\right\}U_3}\right]}$$

(4.4.18)

Similarly, on substituting the value of a in (4.4.15), we get the optimum value of n' as

$$n'_{opt} = \frac{C_0\sqrt{U_3}}{\left[\sqrt{c'\left\{U_1+\left(k_{opt}-1\right)W_2U_2-U_3\right\}\left\{c+c_1W_1+\frac{c_2W_2}{k_{opt}}\right\}}+c'\sqrt{U_3}\right]} \tag{4.4.19}$$

On substituting the optimum value of n, n' and k, we get the mean square error of T_g for fixed cost ($C^* < C_0$) as

$$MSE\left(T_g\right) = \frac{1}{C_0}\left[\left\{U_1+\left(k_{opt}-1\right)W_2U_2-U_3\right\}\left\{c+c_1W_1+\frac{c_2W_2}{k_{opt}}\right\}+c'U_3 + 2\left\{\sqrt{c'\left\{U_1+\left(k_{opt}-1\right)W_2U_2+U_3\right\}\left\{c+c_1W_1+\frac{c_2W_2}{k_{opt}}\right\}U_3}\right\} - \frac{U_1C_0}{N}\right] \tag{4.4.20}$$

Ignoring the terms of order $\frac{1}{N}$ we get the mean square error of T_g for fixed cost ($C^* < C_0$) as

$$MSE\left(T_g\right) = \frac{1}{C_0}\left[\left\{U_1+\left(k_{opt}-1\right)W_2U_2-U_3\right\}\left\{c+c_1W_1+\frac{c_2W_2}{k_{opt}}\right\}+c'U_3 + 2\left\{\sqrt{c'\left\{U_1+\left(k_{opt}-1\right)W_2U_2-U_3\right\}\left\{c+c_1W_1+\frac{c_2W_2}{k_{opt}}\right\}U_3}\right\}\right] \tag{4.4.21}$$

From (4.4.1) to (4.4.21) it is clear that on replacing any other estimator with T_g we will get different value of U_1 being the coefficient of $\left(\frac{1}{n}-\frac{1}{N}\right)$, U_2 being the coefficient of $\frac{W_2\left(k-1\right)}{n}$ and U_3 being the coefficient of $\left(\frac{1}{n'}-\frac{1}{N}\right)$, respectively in the expression for mean square error of T_g and hence the optimum values of n, n' and k can be obtained for both the cases of fixed cost as well as for fixed precision. For the sake of brevity we are omitting the derivations.

4.5 EMPIRICAL STUDY

The present data belong to the data on physical growth of upper socio-economic group of 95 school going children of Varanasi under an ICMR study, Department of Pediatrics, BHU during 1983-84 has been taken under study, Khare and Sinha (2007). The first 25% (i.e. 24 children) units have been considered as non-response units. The values of parameters related to the study characters y (weight of children in kg), the auxiliary character x (chest circumference of the children in cm) have been given as follows:

$\bar{Y}_2 = 19.4968;\ \bar{X} = 55.8611;\ \pm_y = 3.0435;\ \pm_x = 3.2735;\ \pm_{y_{(2)}} = 02.3552;\ \pm_{x_{(2)}} = 2.5137;$

$\pm_{yx} = 8.428611;\ \pm_{yx_{(2)}} = 4.315874.$

The problem considered is to estimate the weight of the male children aged 6-7 years using chest circumference as the auxiliary character.

Table 4.5.1 *PRE & MSE under measurement error & non-response*

↓ Estimators		1/k			
	↓ME %	1/2	1/3	1/4	1/5
$\bar{y}^*$	0%	0.3207(100)	0.3608(100)	0.4008(100)	0.4408(100)
	1%	0.3239(99)	0.3644(99)	0.4048(99)	0.4453(99)
	5%	0.3368(95)	0.3788 (95)	0.4208(95)	0.4629(95)
	10%	0.3528(91)	0.3968(91)	0.4409(91)	0.4849(91)
	15%	0.3688(87)	0.4149(87)	0.4603(87)	0.5070(87)
	20%	0.3849(83)	0.4329(83)	0.4809(83)	0.5290(83)
T_1	↓ME %	1/2	1/3	1/4	1/5
	0%	0.2347(132)	0.2551(135)	0.2789(138)	0.3028(140)
	1%	0.2346(137)	0.2590(139)	0.2833(141)	0.3076(143)
	5%	0.2486(129)	0.2747(131)	0.3008(133)	0.3270(135)
	10%	0.2659(121)	0.2943(123)	0.3227(124)	0.3512(126)
	15%	0.2833(113)	0.3140(115)	0.3447(116)	0.3754(117)
	20%	0.3007(107)	0.3336(108)	0.3666109)	0.3995(110)
T_3	↓ME %	1/2	1/3	1/4	1/5
	0%	0.2474(123)	0.2874(120)	0.3275(118)	0.3675(116)
	1%	0.2508(128)	0.2913(124)	0.3317(121)	0.3721(118)
	5%	0.2645(121)	0.3065(118)	0.3486(115)	0.3906(113)
	10%	0.2816(114)	0.3256(111)	0.3697(108)	0.4137(107)
	15%	0.2986(107)	0.3447(105)	0.3907(103)	0.4368(101)
	20%	0.3157(102)	0.3638(99)	0.4118(97)	0.4599(96)
T_5	↓ME %	1/2	1/3	1/4	1/5
	0%	0.1997(153)	0.2235(154)	0.2474(156)	0.2712(157)
	1%	0.1977(162)	0.2171(166)	0.2365(169)	0.2558(172)
	5%	0.2202(146)	0.2463(146)	0.2725(147)	0.2986(148)
	10%	0.2407(133)	0.2691(134)	0.2975(135)	0.3259(135)
	15%	0.2612(123)	0.2919(124)	0.3226(124)	0.3533(125)
	20%	0.2817(114)	0.3147(115)	0.3476(115)	0.3806(116)
T_7	↓ME %	1/2	1/3	1/4	1/5
	0%	0.1933(158)	0.2120(163)	0.2308(167)	0.2495(170)
	1%	0.1977(162)	0.2171(166)	0.2365(169)	0.2558(172)
	5%	0.2154(149)	0.2371(152)	0.2589(155)	0.2807(157)
	10%	0.2369(135)	0.2616(138)	0.2863(140)	0.3110(142)
	15%	0.2580(124)	0.2855(126)	0.3131(128)	0.3406(129)
	20%	0.2787(115)	0.3090(117)	0.3393(118)	0.3696(119)

	↓ME %	1/2	1/3	1/4	1/5
T_9	0%	0.1938(157)	0.2127(162)	0.2316(166)	0.2505(170)
	1%	0.1982(162)	0.2178(166)	0.2373(169)	0.2568(172)
	5%	0.2159(149)	0.2378(152)	0.2597(154)	0.2816(157)
	10%	0.2374(135)	0.2622(138)	0.2871(140)	0.3120(141)
	15%	0.2585(124)	0.2861(126)	0.3138(128)	0.3415(129)
	20%	0.2791(115)	0.3095(117)	0.3400(118)	0.3704(119)
T_{10}	↓ME %	1/2	1/3	1/4	1/5
	0%	0.2145(142)	0.2546(135)	0.2946(131)	0.3347(127)
	1%	0.2188(147)	0.2592(139)	0.2997(134)	0.3401(130)
	5%	0.2356(136)	0.2777(130)	0.3197(125)	0.3618(122)
	10%	0.2563(125)	0.3003(120)	0.3444(116)	0.3884(113)
	15%	0.2765(116)	0.3225(112)	0.3686(109)	0.4146(106)
	20%	0.2964(108)	0.3444(105)	0.3925(102)	0.4405(100)
T_8	↓ME %	1/2	1/3	1/4	1/5
	0%	0.1933(158)	0.2120(163)	0.2308(167)	0.2495(170)
	1%	0.1977(162)	0.2171(166)	0.2365(169)	0.2558(172)
	5%	0.2153(149)	0.2371(152)	0.2589(155)	0.2807(157)
	10%	0.2369(135)	0.2616(138)	0.2863(140)	0.3110(142)
	15%	0.2580(124)	0.2855(126)	0.3131(128)	0.3406(129)
	20%	0.2787(115)	0.3090(117)	0.3393(118)	0.3696(119)

Table 4.5.2 *PRE(MSE) with measurement error and without non-response*

Estimator	0%	1%	5%	10%	15%	20%
$\bar{y}^*$	0.2807(100)	0.2835(101)	0.2947(105)	0.3088(110)	0.3228(115)	0.3368(120)
T_1	0.1823(154)	0.1868(67)	0.2049(73)	0.2276(81)	0.2502(89)	0.2728 (97)
T_3	0.7223(39)	0.7269(259)	0.7450(265)	0.7676(273)	0.7902(282)	0.8128(290)
T_5	0.1759(160)	0.1795(64)	0.1941(69)	0.2123(76)	0.2305(82)	0.2487(89)
T_7	0.5439(52)	0.5441(194)	0.5454(194)	0.5480(195)	0.5517(197)	0.5562(198)
T_9	0.1749(160)	0.1788(64)	0.1940(69)	0.2126(76)	0.2308(82)	0.2487(89)
T_{10}	0.1749(160)	0.1788(64)	0.1940(69)	0.2126(76)	0.2308(82)	0.2489(89)
T_8	0.1745(161)	0.1784(64)	0.1936(69)	0.2122(76)	0.2305(82)	0.2483(88)

Table 4.5.3 *PRE (MSE) with measurement error for optimum value of k*

Estimators ↓	MSE(PRE)				
Percent of ME →	1%	5%	10%	15%	20%
$\bar{y}^*$	0.4175(99)	0.4340(95)	0.4546(91)	0.4753(87)	0.4960(83)
T_1	0.4894(84)	0.5156(80)	0.5477(75)	0.5792(71)	0.6101(68)
T_3	0.4988(83)	0.5257(79)	0.5584(74)	0.5903(70)	0.6215(67)
T_5	0.4149(100)	0.4545(91)	0.5004(83)	0.5430(76)	0.5831(71)
T_7	0.4094(101)	0.4507(92)	0.4976(83)	0.5404(76)	0.5801(71)
T_9	0.4107(101)	0.4519 (91)	0.4985(83)	0.5412(76)	0.5808(71)
T_{10}	0.4034(102)	0.4510(92)	0.5024(82)	0.5479(75)	0.5894(70)
T_8	0.4094(101)	0.4507(92)	0.4976(83)	0.5404(76)	0.5801(71)

Table 4.5.4 *PRE(MSE) without measurement error and with non-response error for optimum value of k*

Estimators	MSE
$\bar{y}^*$	0.4133(100)
T_1	0.4827(86)
T_3	0.4920(84)
T_5	0.4045(102)
T_7	0.3983(104)
T_9	0.3997(103)
T_{10}	0.3901(106)
T_8	0.3983(104)

Table 4.5.5 *PRE(MSE) without measurement error and with non-response error*

Estimators	1/2	1/3	1/4	1/5
$\bar{y}^*$	0.3207(100)	0.3608(100)	0.4008(100)	0.4408(100)
T_1	0.2312(132)	0.2551(135)	0.2789(138)	0.3028(140)
T_3	0.2474(123)	0.2874(120)	0.3275(118)	0.3675(116)
T_5	0.1997(153)	0.2235(154)	0.2474(156)	0.2712(157)
T_7	0.1933(158)	0.2120(163)	0.2308(1167)	0.2495(170)
T_9	0.1938(157)	0.2127(162)	0.2316(166)	0.2505(170)
T_{10}	0.2145(142)	0.2546(135)	0.2946(131)	0.3347(127)
T_8	0.1933(158)	0.2120(163)	0.2308(167)	0.2495(170)

4.6 CONCLUDING REMARKS

1. It is evident from the expressions (4.3.7) to (4.3.16) of MSE's of the estimators that the measurement errors seem to have inflated the MSE of these estimators and thereby decreasing the efficiency.

2. The expressions (4.3.7) to (4.3.16) of MSE can be broken into 4 major components owing to non-response and measurement error are given below:

$$MSE = A + B + C + D$$

where A = Component of MSE due to sampling error without measurement error and non-response,

B = Component of MSE due to sampling error with measurement error and without non-response,

C = Component of MSE due to sampling error without measurement error and with non-response, and

D = Component of MSE due to sampling error with measurement error and without non-response.

For Example: Consider the expression of MSE of T_8 given by

$$MSE(T_8) = \underbrace{\left(\frac{1}{n} - \frac{1}{n'}\right)\left(\pm_y^2 + d_2^2 \pm_x^2 - 2d_2 \pm_{xy}\right) + \frac{1}{n'}\pm_y^2}_{A}$$

$$+ \underbrace{\frac{1}{n}\left(\pm_u^2 + d_2^2 \pm_v^2\right)}_{B}$$

$$+ \underbrace{\frac{W_2(k-1)}{n}\left(\pm_{y(2)}^2 + d_1^2 \pm_{x(2)}^2 - 2d_1 \pm_{xy(2)}\right)}_{C}$$

$$+\underbrace{\frac{W_2(k-1)}{n}\left(\pm^2_{u_{(2)}} + d_1{}^2\pm^2_{v_{(2)}}\right)}_{D}$$

3. If the measurement error is absent then we get the expression of the MSE of conventional estimators under non-response from the results of this study thereby the present study provides a more general and pragmatic approach for the estimation of population mean. For example: When $u_i = 0 = v_i$, for each i, then $\pm^2_u = \pm^2_v = \pm^2_{u_2} = \pm^2_{v_2} = 0$ and we get

$$MSE(T_8) = \underbrace{\left(\frac{1}{n} - \frac{1}{n'}\right)\left(\pm^2_y + d_2{}^2\pm^2_x - 2d_2\pm_{xy}\right) + \frac{1}{n'}\pm^2_y}_{A}$$
$$+\underbrace{\frac{W_2(k-1)}{n}\left(\pm^2_{y_{(2)}} + d_1{}^2\pm^2_{x_{(2)}} - 2d_1\pm_{xy_{(2)}}\right)}_{C}$$

so that only components A and C are left, which is same expression as proposed by Singh and Kumar (2010) while proposing T_8.

4. If the measurement error is absent then we get the expression of the optimum values of the characterizing scalars and minimum MSE of conventional estimators under non-response from the results of this study thereby the present study. For example: When $u_i = 0 = v_i$, for each i, then $\pm^2_u = \pm^2_v = \pm^2_{u_2} = \pm^2_{v_2} = 0$ and we get optimum values of d_1 and d_2 as

$$d_1 = \pm_{xy_{(2)}} / \pm^2_{x_{(2)}} \text{ and } d_2 = \pm_{xy} / \pm^2_x \quad .$$

and the minimum mean square error of T_8 as

$$MSE(T_8)_{\min} = \left(\frac{1}{n} - \frac{1}{n'}\right)\left(1 - \circ^2\right)\pm^2_y + \frac{W_2(k-1)}{n}\left(1 - \circ^2_{(2)}\right)\pm^2_{y_{(2)}} + \frac{1}{n'}\pm^2_y$$

which is same expression of minimum MSE as proposed by Singh and Kumar (2010) while proposing T_8.

5. The measurement error seems to have affected all the estimators but the optimal estimators T_7 and T_8 perform far better than the remaining estimators where the auxiliary information was not properly utilized. For example: This is w.r.t. the empirical results in table 4.5.1 where T_8 utilized the auxiliary information optimally and outperformed all the remaining estimators.

6. The measurement error seems to have affected the better estimators more where the auxiliary information was properly utilized than those estimators where the auxiliary information was not properly utilized. For example: This is w.r.t. the empirical results in table 4.5.1 where T_8 utilized the auxiliary information optimally and has lost 51% efficiency with 1/5 sub-sampling fraction which is far more in comparison to T_3, estimator where the auxiliary information was not properly utilized, lost only 22% efficiency with 1/5 sub-sampling fraction or even $\bar{y}*$ which lost 17% efficiency with 1/5 sub-sampling fraction.

7. The gains in efficiency of various estimators were calculated w. r. t. $\bar{y}*$ with no non-response error in table 4.5.2. The best choice, which is in consonance with the theoretical results, is T_7 and T_8 at all levels of measurement error.

8. The percent relative efficiency of various estimators were calculated w. r. t. $\bar{y}*$ with optimum sub-sampling fraction $1/k$ in table 4.5.3. The best choice, which is in consonance with the theoretical results, is T_7 and T_8 at all levels of measurement error.

9. The gains in efficiency of various estimators were calculated w. r. t. $\bar{y}*$ with no measurement error in table 4.5.4. The best choice, which is in consonance with the theoretical results, is T_7 and T_8 at all levels of measurement error.

10. The percent relative efficiency of various estimators were calculated w. r. t. $\bar{y}*$ without measurement error and with different non-respondent sub-sampling fraction $1/k$ in table 4.5.5. The best choice is T_7 and T_8 which is in consonance with the theoretical results of Singh and Kumar (2010).

CHAPTER 5

ON GENERALIZED CHAIN TYPE ESTIMATORS OF POPULATION MEAN UNDER DOUBLE SAMPLING WITH MEASUREMENT ERROR AND NONRESPONSE

SUMMARY

In this chapter we study a generalized class of double sampling estimators under measurement error and non-response error occurring simultaneously. Singh and Kumar (2010) proposed some ingenious classes of double sampling estimators which fared better in comparison to all the existing double sampling estimators under non-response. The result of the proposed generalized estimator is compared with the existing estimators theoretically in presence of measurement error and non-response error simultaneously. An empirical study is carried out to judge the merit of the proposed classes in presence of measurement error and non-response error.

5.1. INTRODUCTION

When any sample survey has been conducted, then the properties of the estimators based on data usually treated under the assumption that the observations are correct measurements on characteristics being studied. But such kind of supposition does not met in many applications and data is recorded with measurement errors, such as reporting errors and computing errors. These measurement errors make the result invalid. If measurement error is very small and we can neglect it, then the statistical inferences based on recorded data continue to remain valid. Contrary if measurement error is not negligible, the inferences may not be simply invalid and

inaccurate, but may often lead to unexpected, undesirable and unfortunate consequences (see Srivastava and Shalabh (2001)).

We consider a finite population $U = (U_1, U_2, ..., U_N)$ of N units. Let Y and X be the study variate and auxiliary variate, respectively. Suppose that we have a set of n paired observations obtained through simple random sampling procedure on two characteristics X and Y. Further, suppose that (x_i, y_i) for the i[th] sampling units are observed with measurement error instead of their true values (X_i, Y_i). For a simple random sampling scheme, let (x_i, y_i) be observed values instead of the true values (X_i, Y_i) for i[th] (i=1, 2,...n) unit such that

$$u_i = y_i - Y_i \tag{5.1.1}$$

$$v_i = x_i - X_i \tag{5.1.2}$$

where u_i and v_i are associated measurement errors which are stochastic (probabilistic) in nature with mean zero, variances $\pm_u^2$ and $\pm_v^2$ respectively. Further, we assume that u_i and v_i are uncorrelated although X_i are Y_i are correlated.

Let the population means of X and Y characteristics be $«_x$, $«_y$. Population variances of (x, y) be $\pm_x^2$ and $\pm_y^2$. Let $\pm_{xy}$ be the population covariance between x and y.

Most of the methods using auxiliary information like ratio type methods, regression type methods and difference type methods assume that the sample data contains no missing observations and the information regarding the auxiliary variable is either known or can be easily obtained. However, in most of the sample surveys involving human respondents, it has been observed that the information cannot be obtained from all the units which are selected in the sample. An estimator based on such incomplete information is generally biased and the inferences based on such incomplete information may be grossly misleading when the respondents set differs considerably from the non-respondents set. In their seminal paper Hansen and Hurwitz (1946) considered a technique of sub-sampling the non-respondents in order to adjust for the non-response bias in a mail survey.

Let Y_i, i=(1,2,...,N) and X_i, i=(1,2,...,N) be the non-negative value of i^th unit of the population on the study character y and the auxiliary character x with their population means $\bar{Y}$ and $\bar{X}$ respectively. The whole population is supposed to be divided into two parts- the response stratum and the non-response stratum having N_1 and N_2 units respectively. Here the population mean $\bar{X}$ of the auxiliary character x is not known.

If the information about the population mean $\bar{X}$ is not known then Khare and Srivastava (1995), Okafor and Lee (2000), Tabassum and Khan (2004), Singh and Kumar (2010) among others suggested the use of double sampling. In double sampling, an estimate of the population mean $\bar{X}$ is obtained on the basis of a large first phase sample of size n' drawn from the finite population of size N by simple random sampling without replacement. Then a second phase sample of size $n(n<n')$ is drawn from n' by simple random sampling without replacement and the information on study

variable is measured on it. Since all the sample units do not respond to the call of the interviewer, by using Hansen and Hurwitz (1946) approach, a sub-sample, from n_2 non-respondents, r units are selected at random and enumerated by direct interview, such that $r=(n_2/k,\ k>1)$, where $n=n_1+n_2$ and n_1 is the number of respondents. It is noteworthy to mention that all the above authors have assumed that the auxiliary information is available without any non-response. The estimator proposed by Hansen and Hurwitz (1946) is given by $\bar{y}^*=\frac{n_1}{n}\bar{y}_{(1)}+\frac{n_2}{n}\bar{y}^*_{(2)}$ $\left(\text{let}\, W_1=\frac{n_1}{n} \text{ and } W_2=\frac{n_2}{n}\right)$ where $\bar{y}_{(1)}$ and $\bar{y}^*_{(2)}$ denote the sample mean of y based on n_1 and r units respectively.

Singh and Kumar (2010), while proposing their estimators, considered the estimators proposed by Khare and Srivastava (1993, 1995), Okafor and Lee (2000), Tabassum and Khan (2004) given by

$$T_1=\frac{\bar{y}^*}{\bar{x}^*}\bar{x}' \tag{5.1.3}$$

$$T_2=\frac{\bar{y}^*}{\bar{x}'}\bar{x}^* \tag{5.1.4}$$

$$T_3=\frac{\bar{y}^*}{\bar{x}}\bar{x}' \tag{5.1.5}$$

$$T_4=\frac{\bar{y}^*}{\bar{x}'}\bar{x} \tag{5.1.6}$$

$$T_5=\bar{y}^*\left(\frac{\bar{x}'}{\bar{x}^*}\right)\left(\frac{\bar{x}'}{\bar{x}}\right) \tag{5.1.7}$$

$$T_6=\bar{y}^*\left(\frac{\bar{x}^*}{\bar{x}'}\right)\left(\frac{\bar{x}}{\bar{x}'}\right) \tag{5.1.8}$$

$$T_7 = \bar{y}^* \left(\frac{\bar{x}}{\bar{x}^*} \right)^{\sim_1} \left(\frac{\bar{x}'}{\bar{x}} \right)^{\sim_2} \tag{5.1.9}$$

$$T_8 = \bar{y}^* + d_1 \left(\bar{x} - \bar{x}^* \right) + d_2 \left(\bar{x}' - \bar{x} \right) \tag{5.1.10}$$

$$T_9 = \bar{y}^* + d_3 \left(\bar{x}^* - \bar{x}' \right)$$

(5.1.11)

$$T_{10} = \bar{y}^* + d_4 \left(\bar{x} - \bar{x}' \right) \tag{5.1.12}$$

where $\sim_1, \sim_2, d_i \; (i = 1,2,3,4)$ are characterizing scalars to be suitably chosen.

The MSE's of the Singh and Kumar (2010) estimators are also given as

$$MSE(T_1) = MSE\left(\bar{y}^*\right) + \left[\left(\frac{1}{n} - \frac{1}{n'} \right) \left\{ \left(\pm_x^2 + \pm_v^2 \right) - 2\pm_{yx} \right\} \right.$$

$$\left. + \frac{W_2 (k-1)}{n} \left\{ \left(\pm_{x_{(2)}}^2 + \pm_{v_{(2)}}^2 \right) - 2\pm_{yx_{(2)}} \right\} \right] \tag{5.1.13}$$

$$MSE(T_2) = MSE\left(\bar{y}^*\right) + \left[\left(\frac{1}{n} - \frac{1}{n'} \right) \left\{ \left(\pm_x^2 + \pm_v^2 \right) + 2\pm_{yx} \right\} \right.$$

$$\left. + \frac{W_2 (k-1)}{n} \left\{ \left(\pm_{x_{(2)}}^2 + \pm_{v_{(2)}}^2 \right) + 2\pm_{yx_{(2)}} \right\} \right] \tag{5.1.14}$$

$$MSE(T_3) = MSE\left(\bar{y}^*\right) + \left[\left(\frac{1}{n} - \frac{1}{n'} \right) \left\{ \left(\pm_x^2 + \pm_v^2 \right) - 2\pm_{yx} \right\} \right] \tag{5.1.15}$$

$$MSE(T_4) = MSE\left(\bar{y}^*\right) + \left[\left(\frac{1}{n} - \frac{1}{n'} \right) \left\{ \left(\pm_x^2 + \pm_v^2 \right) + 2\pm_{yx} \right\} \right] \tag{5.1.16}$$

$$MSE(T_5) = MSE\left(\bar{y}^*\right) + \left[\left(\frac{1}{n} - \frac{1}{n'} \right) \left\{ 4\left(\pm_x^2 + \pm_v^2 \right) - 4\pm_{yx} \right\} \right.$$

$$\left. + \frac{W_2 (k-1)}{n} \left\{ \left(\pm_{x_{(2)}}^2 + \pm_{v_{(2)}}^2 \right) - 2\pm_{yx_{(2)}} \right\} \right] \tag{5.1.17}$$

$$MSE(T_6)=MSE\left(\bar{y}^*\right)+\left[\left(\frac{1}{n}-\frac{1}{n'}\right)\left\{4\left(\pm_x^2+\pm_v^2\right)+4\pm_{yx}\right\}\right.$$

$$\left.+\frac{W_2\left(k-1\right)}{n}\left\{\left(\pm_{x_{(2)}}^2+\pm_{v_{(2)}}^2\right)+2\pm_{yx_{(2)}}\right\}\right] \qquad (5.1.18)$$

$$MSE(T_7)=MSE\left(\bar{y}^*\right)+\left[\left(\frac{1}{n}-\frac{1}{n'}\right)\left\{\sim_2^2\left(\pm_x^2+\pm_v^2\right)-2\sim_2\pm_{yx}\right\}\right.$$

$$\left.+\frac{W_2\left(k-1\right)}{n}\left\{\sim_1^2\left(\pm_{x_{(2)}}^2+\pm_{v_{(2)}}^2\right)-2\sim_1\pm_{yx_{(2)}}\right\}\right] \qquad (5.1.19)$$

$$MSE(T_8)=MSE\left(\bar{y}^*\right)+\left[\left(\frac{1}{n}-\frac{1}{n'}\right)\left\{d_2^2\left(\pm_x^2+\pm_v^2\right)-2d_2^2\pm_{yx}\right\}\right.$$

$$\left.+\frac{W_2\left(k-1\right)}{n}\left\{d_1^2\left(\pm_{x_{(2)}}^2+\pm_{v_{(2)}}^2\right)-2d_1\pm_{yx_{(2)}}\right\}\right] \qquad (5.1.20)$$

$$MSE(T_9)=MSE\left(\bar{y}^*\right)+\left[\left(\frac{1}{n}-\frac{1}{n'}\right)\left\{d_3^2\left(\pm_x^2+\pm_v^2\right)+2d_3\pm_{yx}\right\}\right.$$

$$\left.+\frac{W_2\left(k-1\right)}{n}\left\{d_3^2\left(\pm_{x_{(2)}}^2+\pm_{v_{(2)}}^2\right)+2d_3\pm_{yx_{(2)}}\right\}\right] \qquad (5.1.21)$$

$$MSE(T_{10})=MSE\left(\bar{y}^*\right)+\left[\left(\frac{1}{n}-\frac{1}{n'}\right)\left\{d_4^2\left(\pm_x^2+\pm_v^2\right)+2d_4\pm_{yx}\right\}\right] \qquad (5.1.22)$$

5.2 SUGGESTED CLASS OF ESTIMATORS- I

The main advantage of Singh and Kumar (2010) was that they were able to avail all possible information at their disposal to the best possible use as in most of the literature it is considered that non-response is present only on the study variable and not on the auxiliary variable and only the structure of HH estimator is adopted for the auxiliary character. It is also important to mention that the main idea was proposed by Singh and Kumar (2008) while Singh and Kumar (2010) incorporated the same under the double sampling scheme. We propose the following classes of estimators based on $\bar{y}^*, \bar{x}^*, \bar{x}$ and $\bar{x}'$ defined as

$$T_g = \bar{y}^* g\left(\bar{x}^*, \bar{x}, \bar{x}'\right) \tag{5.2.1}$$

where $g(.)$ is a bounded function satisfying the following regularity conditions

(i) $g(\mathbf{R}) = 1$ (5.2.2)

ii) The first order partial derivatives g_1 , g_2 and g_3 of T with respect to $\bar{x}^*$, $\bar{x}$ and $\bar{x}'$ at point $\mathbf{R} \equiv \left(\bar{X}, \bar{X}, \bar{X}\right)$, respectively, satisfy

$$g_2 = -\left(g_1 + g_3\right) \tag{5.2.3}$$

where $\bar{x}$ is the sample mean of x based on n units; the sample mean of the study variable y and the auxiliary variable x are $\bar{y}^* = \frac{n_1}{n}\bar{y}_{(1)} + \frac{n_2}{n}\bar{y}^*_{(2)}$ and $\bar{x}^* = \frac{n_1}{n}\bar{x}_{(1)} + \frac{n_2}{n}\bar{x}^*_{(2)}$ where $\left(\bar{y}_{(1)}, \bar{x}_{(1)}\right)$ and $\left(\bar{y}_{(2)}, \bar{x}_{(2)}\right)$ are the sample means based on n_1 units and the sub-sample means based on r units of the variables (y, x) respectively.

The bias and MSE's to the first order of approximation, are given by

$$Bias\left(T_g\right) = \bar{Y}\left[\frac{(N-n)}{Nn}\left\{\frac{1}{2}\left(g_{11} + g_{22} + 2g_{12} + \frac{2g_1}{\bar{X}}\right)\left(\pm_x^2 + \pm_x^2\right) - \pm_{yx} g_3\right\}\right.$$

$$+\frac{W_2(k-1)}{n}\left\{\frac{1}{2}\left(\pm_{x(2)}^2 + \pm_{x(2)}^2\right)g_{11} + \pm_{yx(2)} g_1\right\} + \frac{(N-n')}{Nn'}\left\{\frac{1}{2}\left(g_{33} + 2g_{13}\right.\right.$$

$$\left.\left.+2g_{23}\right)\left(\pm_x^2 + \pm_x^2\right) + \pm_{yx} g_3\right\}\Bigg] \tag{5.2.4}$$

$$MSE(T_g) = MSE(\bar{y}^*) + \bar{Y}^2\left[\left(\frac{1}{n} - \frac{1}{n'}\right)\left\{\left(\pm_x^2 + \pm_v^2\right)g_3^2\right\} + \frac{W_2(k-1)}{n}\left(\pm_{x(2)}^2 + \pm_{v(2)}^2\right)g_1^2\right]$$

$$-2\bar{Y}\left[\left(\frac{1}{n}-\frac{1}{n'}\right)\pm_{yx} g_3+\frac{W_2(k-1)}{n}\pm_{yx_{(2)}} g_1\right] \tag{5.2.5}$$

Since the optimum value of g_1 and g_3 minimizing the mean square error of the estimator T_g is

$$g_{3_{(opt)}}=\frac{1}{\bar{Y}}\left(\frac{\pm_{yx}}{\left(\pm_x^2+\pm_v^2\right)}\right)=D_1 \tag{5.2.6}$$

and

$$g_{1_{(opt)}}=-\frac{1}{\bar{Y}}\left(\frac{\pm_{yx_{(2)}}}{\left(\pm_{x_{(2)}}^2+\pm_{v_{(2)}}^2\right)}\right)=D_2 \tag{5.2.7}$$

and the minimum mean square error is given by

$$MSE(T_g)_{\min}=MSE(\bar{y}^*)-\left[\left(\frac{1}{n}-\frac{1}{n'}\right)\frac{\left(\pm_{yx}\right)^2}{\left(\pm_x^2+\pm_v^2\right)}+\frac{(k-1)W_2}{n}\frac{\left(\pm_{yx_{(2)}}\right)^2}{\left(\pm_{x_2}^2+\pm_{v_2}^2\right)}\right] \tag{5.2.8}$$

5.3 SUGGESTED CLASS OF ESTIMATORS II

We propose the following classes of estimators based on $\bar{y}^*, \bar{x}^*, \bar{x}$ and $\bar{x}'$ defined as

$$T_G=G\left(\bar{y}^*,\bar{x}^*,\bar{x},\bar{x}'\right) \tag{5.3.1}$$

where $G(.)$ is a bounded function satisfying the following regularity conditions

(i) $G(\mathbf{S})=\bar{Y}$ (5.3.2)

(ii) the first order partial derivatives G_0, G_1 , G_2 and G_3 of T_G with respect to $\bar{y}^*$, $\bar{x}^*$, $\bar{x}$ and $\bar{x}'$ at point $\mathbf{S}\equiv\left(\bar{Y},\bar{X},\bar{X},\bar{X}\right)$, respectively, satisfy

$$G_0=1 \tag{5.3.3}$$

$$G_2 = -(G_1 + G_3) \tag{5.3.4}$$

Where $\bar{x}$ is the sample mean of x based on n units; the sample mean of the study variable y and the auxiliary variable x are $\bar{y}^* = \frac{n_1}{n}\bar{y}_{(1)} + \frac{n_2}{n}\bar{y}^*_{(2)}$ and $\bar{x}^* = \frac{n_1}{n}\bar{x}_{(1)} + \frac{n_2}{n}\bar{x}^*_{(2)}$ where $(\bar{x}_{(1)}, \bar{y}_{(1)})$ and $(\bar{x}^*_{(2)}, \bar{y}^*_{(2)})$ are the sample means based on n_1 units and the sub-sample means based on r units of the variables (x, y) respectively. Here G_0, G_1, G_2 and G_3 are the first order partial derivatives of T_G with respect to $\bar{y}^*, \bar{x}^*$, $\bar{x}'$ and $\bar{x}$ respectively. It is important to note this class of estimator includes the classes of estimators proposed by Singh and Bhushan (2012), Singh and Kumar (2010), Tabassum and Khan (2004, 2006), Okafor and Lee (2000) among various other estimators available in literature.

The Bias and MSE, to the first order of approximation, are given by

$$Bias(T_G) = \left[\frac{(N-n)}{Nn}\left\{\frac{1}{2}(G_{11} + G_{22} + 2G_{12})\left(\pm_x^2 + \pm_v^2\right) + (G_{01} + G_{02})\pm_{yx}\right\}\right.$$

$$+\frac{(k-1)W_2}{n}\left\{\frac{1}{2}\left(\pm_{x_{(2)}}^2 + \pm_{v_{(2)}}^2\right)G_{11} + \pm_{yx_{(2)}}G_{01}\right\} + \frac{(N-n')}{Nn'}\left\{\frac{1}{2}(G_{33} + 2G_{13}\right.$$

$$\left.\left. +2G_{23})\left(\pm_x^2 + \pm_v^2\right) + \pm_{yx}G_{03}\right\}\right] \tag{5.3.5}$$

$$MSE(T_G) = MSE(\bar{y}^*) + \left[\left(\frac{1}{n} - \frac{1}{n'}\right)\left\{\left(\pm_x^2 + \pm_v^2\right)G_3^2 - 2\pm_{yx}G_3\right\}\right.$$

$$\left. + \frac{(k-1)W_2}{n}\left\{\left(\pm_{x_{(2)}}^2 + \pm_{v_{(2)}}^2\right)G_1^2 + 2\pm_{yx_{(2)}}G_1\right\}\right] \tag{5.3.6}$$

Since the optimum value of G_1 and G_3 minimizing the mean square error of the estimator T_G is

$$G_{3_{(opt)}} = \left(\frac{\pm_{yx}}{\pm_x^2 + \pm_v^2} \right) \tag{5.3.7}$$

and

$$G_{1_{(opt)}} = -\left(\frac{\pm_{yx_{(2)}}}{\pm_{x_{(2)}}^2 + \pm_{v_{(2)}}^2} \right) \tag{5.3.8}$$

and the minimum mean square error is given by

$$MSE(T_G)_{\min} = MSE(\bar{y}^*) - \left[\left(\frac{1}{n} - \frac{1}{n'} \right) \left\{ \frac{\left(\pm_{yx}\right)^2}{\pm_x^2 + \pm_v^2} \right\} + \frac{(k-1)W_2}{Nn} \left\{ \frac{\left(\pm_{yx_{(2)}}\right)^2}{\pm_{x_{(2)}}^2 + \pm_{v_{(2)}}^2} \right\} \right] \tag{5.3.9}$$

5.4 DETERMINATION OF OPTIMUM VALUES OF n, n' AND k

Let us consider a cost function for T_G as

$$C = c'n' + cn + c_1 n_1 + c_2 r$$

where c' is the cost per unit associated with the first phase sample n', c is the cost per unit of the first attempt with the sample, n; c_1 is the cost per unit for processing the respondent data at the first attempt in n_1 and c_2 is the cost per unit associated with the subsample, r of n_2.

Since the values of n_1 and r is not known until the first attempt is made, so the expected cost will be used in planning the survey. The expected values of n_1 and r are $W_1 n$ and $\frac{W_2 n}{k}$. Thus the expected cost is given by

$$E\left(C\right) = C^* = c'n' + n\left[c + c_1 W_1 + \frac{c_2 W_2}{k} \right] \tag{5.4.1}$$

Theorem 5.4.1: To the first order of approximation, the optimum values of n, n' and k that minimize the cost for a fixed variance V_0 are given by

(i) $k_{opt} = \sqrt{\frac{c_2\left(U_1 - W_2U_2 - U_3\right)}{\left(c + c_1W_1\right)U_2}}$

(ii) $$n_{opt} = \frac{\left\{U_1 + \left(k_{opt} - 1\right)W_2U_2 - U_3\right\}\sqrt{\left\{c + c_1W_1 + \frac{c_2W_2}{k_{opt}}\right\}} + \sqrt{c'\left\{U_1 + \left(k_{opt} - 1\right)W_2U_2 - U_3\right\}U_3}}{\sqrt{\left\{c + c_1W_1 + \frac{c_2W_2}{k_{opt}}\right\}}\left\{\frac{V_0}{\bar{Y}^2} + \frac{U_1}{N}\right\}}$$

(iii) $$n'_{opt} = \frac{\sqrt{\left\{U_1 + \left(k_{opt} - 1\right)W_2U_2 - U_3\right\}U_3\left\{c + c_1W_1 + \frac{c_2W_2}{k_{opt}}\right\}} + \sqrt{c'}U_3}{\sqrt{c'}\left\{\frac{V_0}{\bar{Y}^2} + \frac{U_1}{N}\right\}}$$, and

(iv) $$C^* = \frac{\bar{Y}^2}{V_0}\left[2\left(\sqrt{c'\left\{U_1 + \left(k_{opt} - 1\right)W_2U_2 - U_3\right\}\left\{c + c_1W_1 + \frac{c_2W_2}{k_{opt}}\right\}U_3}\right) + c'U_3 + \left\{U_1 + \left(k_{opt} - 1\right)W_2U_2 - U_3\right\}\left\{c + c_1W_1 + \frac{c_2W_2}{k_{opt}}\right\}\right]$$

Where $C*$ is the minimum cost.

Theorem 5.4.2: Let C_0 be the total cost (fixed) of the survey apart from overhead cost. The optimum values of n, n' and k that minimize the $MSE(T_g)$ for a fixed cost ($C^* < C_0$) along with the minimum MSE are given by

(i) $k_{opt} = \sqrt{\frac{c_2\left(U_1 - W_2U_2 - U_3\right)}{\left(c + c_1W_1\right)U_2}}$

(ii) $$n_{opt} = \frac{C_0\sqrt{\left\{U_1 + \left(k_{opt} - 1\right)W_2U_2 - U_3\right\}}}{\left[\left\{c + c_1W_1 + \frac{c_2W_2}{k_{opt}}\right\}\sqrt{\left\{U_1 + \left(k_{opt} - 1\right)W_2U_2 - U_3\right\}} + \sqrt{c'\left\{c + c_1W_1 + \frac{c_2W_2}{k_{opt}}\right\}U_3}\right]}$$

(iii) $n'_{opt} = \dfrac{C_0\sqrt{U_3}}{\left[\sqrt{c'\left\{U_1+\left(k_{opt}-1\right)W_2U_2-U_3\right\}\left\{c+c_1W_1+\dfrac{c_2W_2}{k_{opt}}\right\}}+c'\sqrt{U_3}\right]}$

(iv) $MSE\left(T_g\right) = \dfrac{\bar{Y}^2}{C_0}\left[\left\{U_1+\left(k_{opt}-1\right)W_2U_2-U_3\right\}\left\{c+c_1W_1+\dfrac{c_2W_2}{k_{opt}}\right\}+c'U_3\right.$

$$\left.+2\left\{\sqrt{c'\left\{U_1+\left(k_{opt}-1\right)W_2U_2-U_3\right\}\left\{c+c_1W_1+\dfrac{c_2W_2}{k_{opt}}\right\}U_3}\right\}\right]$$

where $U_1 = \left[\pm_y^2 + \pm_u^2 + \left(\pm_x^2 + \pm_v^2\right)g_3^2 - 2\pm_{yx}g_3\right]$,

$U_2 = \left[\left(\pm_{y(2)}^2 + \pm_{u(2)}^2\right) + \left(\pm_{x(2)}^2 + \pm_{x(2)}^2\right)g_1^2 + 2\pm_{yx(2)}g_1\right]$ $U_3 = \left(-\pm_x^2 g_3^2 + 2\pm_{yx}g_3\right)$ and ª is

5.5 AN EMPIRICAL STUDY

The present data belong to the data on physical growth of upper socio-economic group of 95 school going children of Varanasi under an ICMR study, Department of Pediatrics, BHU during 1983-84 has been taken under study, Khare and Sinha (2007). The first 25% (i.e. 24 children) units have been considered as non-response units. The values of parameters related to the study characters y (weight of children in kg), the auxiliary character x (chest circumference of the children in cm) have been given as follows:

$\bar{Y}_2 = 19.4968;\ \bar{X} = 55.8611;\ \pm_y = 3.0435;\ \pm_x = 3.2735;\ \pm_{y(2)} = 02.3552;\ \pm_{x(2)} = 2.5137;$

$\pm_{yx} = 8.428611;\ \pm_{yx_{(2)}} = 4.315874.$

The problem considered is to estimate the weight of the male children aged 6-7 years using chest circumference as the auxiliary character.

Table 5.5.1 *PRE & MSE under measurement error & non-response*

↓ Estimators		$1/k$			
	↓ME %	1/2	1/3	1/4	1/5
$\bar{y}^*$	0%	0.3046(100)	0.3447(100)	0.3847(100)	0.4248(100)
	1%	0.3156(99)	0.3560(99)	0.3965(99)	0.4369(99)
	5%	0.3281(95)	0.3701 (95)	0.4122(95)	0.4542(95)
	10%	0.3437(91)	0.3878 (91)	0.4318(91)	0.4759(91)
	15%	0.3593(87)	0.4054 (87)	0.4514(87)	0.4975(87)
	20%	0.3750(83)	0.4230 (83)	0.4711(83)	0.5191(83)
t_{1p}	↓ME %	1/2	1/3	1/4	1/5
	0%	0.2885(106)	0.3123(110)	0.3362(114)	0.3600(118)
	1%	0.2916(105)	0.3159(110)	0.3402(114)	0.3645(118)
	5%	0.3040(105)	0.3301(110)	0.3562(113)	0.3824(117)
	10%	0.3195(105)	0.3479(109)	0.3763(112)	0.4047(115)
	15%	0.3350(104)	0.3657(108)	0.3964(112)	0.4271(114)
	20%	0.3505(104)	03835(108)	0.4164(111)	0.4495(113)
t_{2p}	↓ME %	1/2	1/3	1/4	1/5
	0%	0.4389(69)	0.6133(56)	0.7876(49)	0.9619(54)
	1%	0.4420(70)	0.6168(56)	0.7916(49)	0.9665(44)
	5%	0.4545(70)	0.6311(57)	0.8077(50)	0.9843(45)
	10%	0.4700(71)	0.6489(58)	0.8278(51)	1.0067(46)
	15%	0.4855(72)	0.6666(59)	0.8478(52)	1.0290(47)
	20%	0.5010(73)	0.6844(60)	0.8679(53)	1.0514(48)
t_{3p}	↓ME %	1/2	1/3	1/4	1/5
	0%	0.2834(107)	0.3021(114)	0.3209(120)	0.3397(125)
	1%	0.2866(107)	0.3060(114)	0.3254(119)	0.3448(124)
	5%	0.2996(107)	0.3214(113)	0.3432(118)	0.3649(122)
	10%	0.3158(106)	0.3405(111)	0.3652(116)	0.3899(120)
	15%	0.3318(105)	0.3594(110)	0.3869(114)	0.4145(118)
	20%	0.3478(105)	0.3782(109)	0.4085(113)	0.4384(116)
t_{5p}	↓ME %	1/2	1/3	1/4	1/5
	0%	0.2834(106)	0.3022(114)	0.3209(120)	0.3397(125)
	1%	0.2866(107)	0.3060(114)	0.3254(119)	0.3448(124)
	5%	0.2996(107)	0.3214(113)	0.3432(118)	0.3649(122)
	10%	0.3158(106)	0.3405(111)	0.3652(116)	0.3899(120)
	15%	0.3318(105)	0.3594(110)	0.3869(114)	0.4145(118)
	20%	0.3478(105)	0.3782(109)	0.4085(113)	0.4384(116)

	↓ME %	1/2	1/3	1/4	1/5
T_1	0%	0.2231(137)	0.2469(140)	0.2707(142)	0.2946(144)
	1%	0.2304(136)	0.2447(138)	0.2790(141)	0.3033(143)
	5%	0.2439(128)	0.2700(131)	0.2962(133)	0.3223(134)
	10%	0.2608(120)	0.2892(122)	0.3176(124)	0.3460(125)
	15%	0.2777(112)	0.3084(114)	0.3391(116)	0.3698(117)
	20%	0.2946(106)	0.3275(108)	0.3605(109)	0.3935(110)
	↓ME %	1/2	1/3	1/4	1/5
T_4	0%	0.2393(127)	0.2793(123)	0.3194(120)	0.3594(118)
	1%	0.2465(127)	0.2870(123)	0.3274(120)	0.3679(118)
	5%	0.2598(120)	0.3019(117)	0.3439(114)	0.3860(112)
	10%	0.2764(113)	0.3205(110)	0.3645(108)	0.4086(106)
	15%	0.2931(107)	0.3391(104)	0.3851(102)	0.4312(100)
	20%	0.3097(101)	0.3577(99)	0.4058(97)	0.4538(95)
	↓ME %	1/2	1/3	1/4	1/5
T_7	0%	0.1950(156)	0.2188(158)	0.2426(159)	0.2665(159)
	1%	0.2012(155)	0.2256(156)	0.2499(157)	0.2742(158)
	5%	0.2171(144)	0.2433(145)	0.2694(146)	0.2955(146)
	10%	0.2370(132)	0.2654(133)	0.2938(134)	0.3222(134)
	15%	0.2568(122)	0.2875(123)	0.3182(123)	0.3489(124)
	20%	0.2767(113)	0.3096(114)	0.3526(115)	0.3756(115)
	↓ME %	1/2	1/3	1/4	1/5
T_9	0%	0.1887(161)	0.2075(166)	0.2262(170)	0.2450(173)
	1%	0.1952(160)	0.2146(164)	0.2339(168)	0.2534(171)
	5%	0.2123(147)	0.2341(151)	0.2559(153)	0.2777(156)
	10%	0.2332(134)	0.2579(137)	0.2826(139)	0.3073(141)
	15%	0.2536(123)	0.2812(125)	0.3087(127)	0.3363(129)
	20%	0.2737(114)	0.3040(116)	0.3343(117)	0.3646(119)
	↓ME %	1/2	1/3	1/4	1/5
T_{11}	0%	0.1892(161)	0.2081(166)	0.2270(170)	0.2458(173)
	1%	0.1958(160)	0.2153(164)	0.2348(167)	0.2543(170)
	5%	0.2128(147)	0.2347(150)	0.2566(153)	0.2785(155)
	10%	0.2337(134)	0.2585(136)	0.2833(139)	0.3082(140)
	15%	0.2541(123)	0.2818(125)	0.3094(127)	0.3371(128)
	20%	0.2741(114)	0.3045(116)	0.3350(117)	0.3654(118)

	↓ME %	1/2	1/3	1/4	1/5
T_{12}	0%	0.2099(145)	0.2500(138)	0.2901(133)	0.3301(129)
	1%	0.2163(144)	0.2568(137)	0.2972(132)	0.3376(128)
	5%	0.2326(134)	0.2746(128)	0.3167(124)	0.3587(121)
	10%	0.2526(124)	0.2966(119)	0.3406(115)	0.3847(112)
	15%	0.2721(115)	0.3182(111)	0.3682(108)	0.4102(105)
	20%	0.2914(107)	0.3395(104)	0.3875(101)	0.4355(99)
T_{10}	↓ME %	1/2	1/3	1/4	1/5
	0%	0.1887(161)	0.2075(166)	0.2262(170)	0.2450(173)
	1%	0.1952(160)	0.2146(164)	0.2340(168)	0.2537(171)
	5%	0.2123(147)	0.2341(151)	0.2559(153)	0.2777(156)
	10%	0.2332(134)	0.2579(137)	0.2826(139)	0.3073(141)
	15%	0.2536(123)	0.2812(125)	0.3087(127)	0.3363(129)
	20%	0.2737(114)	0.3040(116)	0.3343(117)	0.3599(119)
T_g	↓ME %	1/2	1/3	1/4	1/5
	0%	0.1909(164)	0.2097(168)	0.2284(172)	0.2472(175)
	1%	0.1952(160)	0.2146(164)	0.2339(168)	0.2534(171)
	5%	0.2123(147)	0.2341(151)	0.2559(153)	0.2777(156)
	10%	0.2332(134)	0.2579(137)	0.2826(139)	0.3073(141)
	15%	0.2536(123)	0.2812(125)	0.3087(127)	0.3363(129)
	20%	0.2737(114)	0.3040(116)	0.3343(117)	0.3646(119)
T_G	↓ME %	1/2	1/3	1/4	1/5
	0%	0.1909(164)	0.2097(168)	0.2284(172)	0.2472(175)
	1%	0.1952(160)	0.2146(164)	0.2339(168)	0.2534(171)
	5%	0.2123(147)	0.2341(151)	0.2559(153)	0.2777(156)
	10%	0.2332(134)	0.2579(137)	0.2826(139)	0.3073(141)
	15%	0.2536(123)	0.2812(125)	0.3087(127)	0.3363(129)
	20%	0.2737(114)	0.3040(116)	0.3343(117)	0.3646(119)

Table 5.5.2 *PRE(MSE) with measurement error and without non-response*

Estimator	0%	1%	5%	10%	15%	20%
$\bar{y}^*$	0.4133(100)	0.4171(100)	0.4321(100)	0.4508(100)	0.4693(100)	0.4877(100)
t_{1p}	0.3934(105)	0.3970(105)	0.4112(105)	0.4288(105)	0.4463(105)	0.4638(105)
t_{2p}	0.3836(108)	0.3893(107)	0.4118(105)	0.4391(103)	0.4656(101)	0.4916(99)
t_{3p}	0.3843(108)	0.3878(108)	0.4017(108)	0.4189(108)	0.4361(108)	0.4532(108)
t_{5p}	0.3843(108)	0.3878(108)	0.4017(108)	0.4189(108)	0.4361(108)	0.4532(108)
T_1	0.4827(86)	0.4889(85)	0.5129(84)	0.5421(83)	0.5703(82)	0.5977(82)
T_4	0.4920(84)	0.4988(84)	0.5251(82)	0.5568(81)	0.5872(80)	0.6167(79)
T_7	0.4045(102)	0.4149(101)	0.4534(95)	0.4969(91)	0.5366(87)	0.5731(85)
T_9	0.3983(104)	0.4088(102)	0.4481(96)	0.4909(92)	0.5293(89)	0.5642(86)
T_{11}	0.3997(103)	0.4101(102)	0.4485(96)	0.4911(92)	0.5293(89)	0.5642(86)
T_{12}	0.3901(106)	0.4038(103)	0.4522(96)	0.5031(90)	0.5473(86)	0.5870(83)
T_{10}	0.3983(104)	0.4089(102)	0.4481(96)	0.4909(92)	0.5293(89)	0.5642(86)
T_g	0.3983(104)	0.4089(102)	0.4481(96)	0.4909(92)	0.5293(89)	0.5642(86)
T_G	0.3983(104)	0.4089(102)	0.4481(96)	0.4909(92)	0.5293(89)	0.5642(86)

Table 5.5.3 *PRE (MSE) with measurement error for optimum value of k*

Estimators ↓	MSE(PRE)				
Per cent of ME →	1%	5%	10%	15%	20%
$\bar{y}^*$	0.4175(100)	0.4340(100)	0.4546(100)	0.4753(100)	0.4960(100)
t_{1p}	0.3977(105)	0.4148(105)	0.4361(104)	0.4574(104)	0.4787(103)
t_{2p}	0.3888(107)	0.4095(106)	0.4349(105)	0.4593(103)	0.4846(102)
t_{3p}	0.3890(107)	0.4075(107)	0.4302(106)	0.4526(105)	0.4748(104)
t_{5p}	0.3890(107)	0.4075(107)	0.4302(106)	0.4526(105)	0.4748(104)
T_1	0.4893(85)	0.5156(84)	0.5477(83)	0.5792(82)	0.6101(81)
T_4	0.4988(84)	0.5257(82)	0.5584(81)	0.5903(80)	0.6215(80)
T_7	0.4149(101)	0.4545(95)	0.5004(91)	0.5430(87)	0.5831(85)
T_9	0.4093(102)	0.4507(96)	0.4976(91)	0.5404(88)	0.5801(85)
T_{11}	0.4107(102)	0.4519 (96)	0.4985(91)	0.5412(88)	0.5808(85)
T_{12}	0.4034(103)	0.4510(96)	0.5024(90)	0.5479(87)	0.5894(84)
T_{10}	0.4093(102)	0.4507(96)	0.4976(91)	0.5404(88)	0.5801(85)
T_g	0.4093(102)	0.4507(96)	0.4976(91)	0.5404(88)	0.5801(85)
T_G	0.4093(102)	0.4507(96)	0.4976(91)	0.5404(88)	0.5801(85)

Table 5.5.4 *PRE(MSE) without measurement error and with non-response error for optimum value of k*

Estimators	**MSE**
$\bar{y}^*$	0.4133(100)
t_{1p}	0.3934(105)
t_{2p}	0.3836(108)
t_{3p}	0.3843(108)
t_{5p}	0.3843(108)
T_1	0.4827(86)
T_4	0.4920(84)
T_7	0.4045(102)
T_9	0.3983(104)
T_{11}	0.3997(103)
T_{12}	0.3901(106)
T_{10}	0.3983(104)
T_g	0.3983(104)
T_G	0.3983(104)

Table 5.5.5 *PRE(MSE) without measurement error and with non-response error*

Estimators	1/2	1/3	1/4	1/5
$\bar{y}^*$	0.3046(100)	0.3447(100)	0.3847(100)	0.4249(100)
t_{1p}	0.2885(106)	0.3123(110)	0.3362(114)	0.3600(118)
t_{2p}	0.4389(69)	0.6133(56)	0.7876(49)	0.9619(44)
t_{3p}	0.2834(108)	0.3021(114)	0.3209(120)	0.3397(125)
t_{5p}	0.2834(104)	0.3021(114)	0.3209(120)	0.3397(125)
T_1	0.2231(136)	0.2469(139)	0.2707(142)	0.2947(144)
T_4	0.2392(127)	0.2793(123)	0.3193(120)	0.3594(118)
T_7	0.1949(156)	0.2188(157)	0.2426(158)	0.2665(159)
T_9	0.1887(161)	0.2074(166)	0.2262(170)	0.2449(173)
T_{11}	0.1892(161)	0.2081(166)	0.2270(169)	0.2459(173)
T_{12}	0.2099(145)	0.2500(138)	0.2900(133)	0.3301(129)
T_{10}	0.1887(161)	0.2074(166)	0.2262(170)	0.4499(173)
T_g	0.1887(161)	0.2074(166)	0.2262(170)	0.4499(173)
T_G	0.1887(161)	0.2074(166)	0.2262(170)	0.4499(173)

5.6 CONCLUDING REMARKS

1. It is evident from the expressions (5.1.13) to (5.1.22), (5.2.5) and (5.3.6) of MSE's of the estimators that the measurement errors seem to have inflated the MSE of these estimators and thereby decreasing the efficiency.

2. The expressions (5.1.13) to (5.1.22), (5.2.5) and (5.3.6) of MSE can be broken into 4 major components owing to non-response and measurement error are given below:

$$MSE = A + B + C + D$$

where A = Component of MSE due to sampling error without measurement error and non-response,

B = Component of MSE due to sampling error with measurement error and without non-response,

C = Component of MSE due to sampling error without measurement error and with non-response, and

D = Component of MSE due to sampling error with measurement error and without non-response. For Example: Consider the expression of MSE of $\bar{y}_f$ given by

$$MSE(T_G) = \underbrace{\left(\frac{1}{n} - \frac{1}{n'}\right)\left(\pm_y^2 + G_3^2 \pm_x^2 - 2G_3 \pm_{xy}\right) + \frac{1}{n'} \pm_y^2}_{A}$$

$$+ \underbrace{\frac{1}{n}\left(\pm_u^2 + G_3^2 \pm_v^2\right)}_{B}$$

$$+ \underbrace{\frac{W_2(k-1)}{n}\left(\pm_{y_{(2)}}^2 + G_1^2 \pm_{x_{(2)}}^2 - 2G_1 \pm_{xy_{(2)}}\right)}_{C}$$

$$+\underbrace{\frac{W_2(k-1)}{n}\left(\pm^2_{u_{(2)}} + G_1^2 \pm^2_{v_{(2)}}\right)}_{D}$$

3. If the measurement error is absent then we get the expression of the MSE of conventional estimators under non-response from the results of this study thereby the present study provides a more general and pragmatic approach for the estimation of population mean. For example: When $u_i = 0 = v_i$, for each i, then $\pm^2_u = \pm^2_v = \pm^2_{u_2} = \pm^2_{v_2} = 0$ and we get

$$MSE(T_G) = \underbrace{\left(\frac{1}{n} - \frac{1}{n'}\right)\left(\pm^2_y + G_3^2 \pm^2_x - 2G_3 \pm_{xy}\right) + \frac{1}{n'} \pm^2_y}_{A}$$
$$+\underbrace{\frac{W_2(k-1)}{n}\left(\pm^2_{y_{(2)}} + G_1^2 \pm^2_{x_{(2)}} - 2G_1 \pm_{xy_{(2)}}\right)}_{C}$$

so that only components A and C are left, which is same expression as proposed by Bhushan and Naqvi (2015) while proposing T_8 of Singh and Kumar (2010) as a special case of T_G.

4. If the measurement error is absent then we get the expression of the optimum values of the characterizing scalars/derivatives and minimum MSE of conventional estimators under non-response from the results of this study thereby the present study. For example: When $u_i = 0 = v_i$, for each i, then $\pm^2_u = \pm^2_v = \pm^2_{u_2} = \pm^2_{v_2} = 0$ and we get optimum values of G_1 and G_3 as

$G_1 = -\pm_{xy_{(2)}} / \pm^2_{x_{(2)}}$ and $G_3 = \pm_{xy} / \pm^2_x$.

and the minimum mean square error of $\overline{y}_f$ as

$$MSE(T_G)_{\min} = \left(\frac{1}{n} - \frac{1}{n'}\right)\left(1-\rho^2\right)\sigma_y^2 + \frac{W_2(k-1)}{n}\left(1-\rho_{(2)}^2\right)\sigma_{y_{(2)}}^2 + \frac{1}{n'}\sigma_y^2$$

which is same expression of minimum MSE as proposed by Bhushan and Naqvi (2015) while proposing T_8 of Singh and Kumar (2008) as a special case of T_G.

5. The measurement error seems to have affected all the estimators but the optimal generalized estimator T_G and their optimal special cases T_7 and T_8 perform far better than the remaining estimators where the auxiliary information was not properly utilized. For example: This is w.r.t. the empirical results in table 5.5.1 where T_G utilized the auxiliary information optimally and outperformed all the remaining estimators.

6. The measurement error seems to have affected the better estimators more where the auxiliary information was properly utilized than those estimators where the auxiliary information was not properly utilized. For example: This is w.r.t. the empirical results in table 5.5.1 where T_G utilized the auxiliary information optimally and has lost 52% efficiency with 1/3 sub-sampling fraction which is far more in comparison to T_4, estimator where the auxiliary information was not properly utilized, lost only 24% efficiency with 1/3 sub-sampling fraction or even $\bar{y}*$ which lost 17% efficiency with 1/3 sub-sampling fraction.

7. The gains in efficiency of various estimators were calculated w. r. t. $\bar{y}*$ with no non-response error in table 5.5.2. The best choice, which is in

consonance with the theoretical results, is T_G at all levels of measurement error.

8. The percent relative efficiency of various estimators were calculated w. r. t. $\bar{y}*$ with optimum sub-sampling fraction $1/k$ in table 5.5.3. The best choice, which is in consonance with the theoretical results, is T_G at all levels of measurement error.

9. The gains in efficiency of various estimators were calculated w. r. t. $\bar{y}*$ with no measurement error in table 5.5.4. The best choice, which is in consonance with the theoretical results, is T_G at all levels of measurement error.

10. The percent relative efficiency of various estimators were calculated w. r. t. $\bar{y}*$ without measurement error and with different non-respondent sub-sampling fraction $1/k$ in table 5.5.5. The best choice is T_G which is in consonance with the theoretical results of Bhushan and Naqvi (2015).

CHAPTER 6

ON COST EFFICIENT CLASSES OF ESTIMATORS FOR POPULATION MEAN IN PRESENCE OF MEASUREMENT ERRORS AND NON-RESPONSE SIMULTANEOUSLY

SUMMARY

In this chapter we have studied some cost efficient classes of estimators in the presence of measurement error and non-response for estimating population mean on the lines of the estimators proposed by Okafor and Lee (2000), Tabasum and Khan (2004) and recently by Singh and Kumar (2010). These estimators are an alternative to the double sampling estimators, when population mean of auxiliary variable is not known and fare better than the above estimators under cost efficiency criterion. The properties of these estimators have been studied in presence of measurement error and non-response error simultaneously. In order to ascertain the soundness of these estimators under measurement error and non-response error, a comparative study is carried out both theoretically as well as empirically.

6.1 INTRODUCTION

The study of present chapter deals the impact of measurement errors and non-response error occurring simultaneously on estimation of mean μ_y of the study variable y. In theory of survey sampling the properties of estimators based on data usually presumed that the observations are the correct measurement on the characteristic being studied. When the measurement errors are negligible small, the statistical inference based on data continue to remain valid. Some authors including Singh and Karpe (2008, 2009), Shalabh(1997), Allen et al. (2003), Manisha and Singh (2001, 2002), Srivastava and Shalabh (2001), Kumar et al. (2011 a,b), Malik and Singh (2013), Malik et al. (2013) have paid their attention towards the estimation of population mean of study variable using auxiliary information in the presence of measurement errors.

Now we consider a finite population $U=\left(U_1,U_2,...U_N\right)$ of N units. Let Y and X be the study variate and auxiliary variate, respectively. Further we assume that we have a set of n paired observations obtained through simple random sampling procedure on two characteristics X and Y and suppose that x_i, and y_i for the i^{th} sampling units are observed with measurement error instead of their true values (X_i, Y_i). For a simple random sampling scheme, let (x_i, y_i) be observed values instead of the true values (X_i, Y_i) for i^{th} (i=1, 2,...n) unit such as

$$u_i = y_i - Y_i \tag{6.1.1}$$

$$v_i = x_i - X_i \tag{6.1.2}$$

where u_i and v_i are combined as measurement errors in y_i and x_i respectively which are stochastic (probabilistic) in nature with mean zero, variances $\pm_u^2$ and $\pm_v^2$ respectively. Further, we assume that u_i's and v_i's are uncorrelated although X_i's Y_i's are correlated.

The problem of non-response is very undesirable yet inevitable in sample surveys as the required information may not always be obtained from all the units selected in the survey even after some call-backs. A sampling strategy based on such incomplete information is generally biased and the results may be enormously misleading when the respondents differ from the non-respondents. Hansen and Hurwitz (1946) considered the problem of non-response for estimating the population mean by taking a sub-sample from the non-respondent stratum with the help of some extra efforts and an estimator was proposed by combining the information available from response and non-response strata.

The use of auxiliary information for making accurate estimation is a well-known practice of survey statisticians. Auxiliary information is often used for the purpose of improving upon the efficiency of the estimates of the population parameters. The purpose of making use of the information based on an auxiliary variable is to obtain increased precision by taking advantage of the correlation between the study variable and the auxiliary variables. A considerable amount of literature is available in this area, some of the important recent references are Kadilar and Cingi (2004, 2005, 2006), Bhushan (2013), Yadav and Kadilar (2013, 2014) etc. If the population mean $\bar{X}$ of the auxiliary variable is known then the problem of estimation of population mean in presence of deterministic non-response has been dealt initially by Cochran (1977), Rao (1986, 1987), Khare and Srivastava (1993, 1997) among others. But, if such information is missing then such situation we generally resort to double sampling as suggested by Okafor and Lee (2000), Tabasum and Khan (2004) and recently by Singh and Kumar (2010) among others. If the population mean $\bar{X}$ of the auxiliary variable is not known then Okafor and Lee (2000) proposed to use the sample mean $\bar{x}'$ obtained from a large first phase preliminary sample of size n' drawn from N units by simple random sampling without replacement (SRSWOR). It is noteworthy to mention here that it is assumed that all the first phase sample units supplied the auxiliary information; see Singh and Kumar (2010). Then a second phase sample of size n $(n < n')$ is drawn from the n' by simple random sampling without replacement (SRSWOR) and study variable y is measured on it. At the second phase from the sample of size n, let n_1 units respond and n_2 units refuse to respond. Now, we use Hansen Hurwitz (1946) sampling strategy to sub-sample r units from n_2 non-responding units and enumerated by direct interview such that $r = n_2 / k$, $k > 1$. It is again implicitly assumed that these r units respond to the direct interview. Okafor and Lee (2000) and Tabasum and Khan (2004) have mentioned that the procedure of two phase sampling can be applied to a household survey where household size is an auxiliary variable for the estimation of family expenditure. The information such as family size might be completely known while there may be a non-response on the household expenditure. A similar example would be regarding disposable income survey where personal taxes paid are known due to tax laws while there may be non-response on the study variable i.e. disposable income.

In this chapter throughout we assume the deterministic set up of non-response exactly on the similar lines as that of Okafor and Lee (2000) and assume that the whole population (denoted by Ω) is stratified into two strata: one is the stratum (denoted by Ω_1) of N_1 units, which would respond on the first call at the second phase and the other stratum (denoted by Ω_2) of N_2 units, which would not respond on the first call but would respond on the second call. Let the first and second phase samples be denoted by s and s' respectively, and let $s_1 = s \cap \Omega_1$ and $s_2 = s \cap \Omega_2$. The sub-sample of s_2 will be denoted by s_{2m}. Summation over the units in the set s will be denoted by $\sum_s$. As consuetude goes, throughout in this chapter, the population parameters are denoted by capital letters and the sample statistics are denoted by small letters.

The Khare and Srivastava (1993) estimator, revisited by Okafor and Lee (2000), Tabasum and Khan (2004), is given by

$$T_1 = \frac{\bar{y}^*}{\bar{x}^*}\bar{x}' \tag{6.1.3}$$

where $\bar{y}^* = W_1\bar{y}_{(1)} + W_2\bar{y}^*_{(2)}$, $\bar{x}^* = W_1\bar{x}_{(1)} + W_2\bar{x}^*_{(2)}$ $\left(\text{where } W_1 = \frac{n_1}{n} \text{ and } W_2 = \frac{n_2}{n}\right)$ and $\bar{x}' = \frac{1}{n'}\sum(x_i)$ with $\left(\bar{x}_{(1)}, \bar{y}_{(1)}\right)$ and $\left(\bar{x}^*_{(2)}, \bar{y}^*_{(2)}\right)$ being the sample means based on n_1 units and sub-sample means based on r units of the variates (x, y) respectively.

In this study, we use almost all the important double sampling estimators under non-response in our study for the purpose of comparison with the proposed estimators given in the next section 2 so that no scope of doubt remains in the mind of practitioners regarding the choice of the estimator if mean of auxiliary variable is not known. Also, it is important to mention that Singh and Kumar (2010) proposed estimators T_7 to T_{10} in comparison to the estimators T_1 to T_6 . For the sake of comprehensiveness, we have also included the difference estimators T_9 and T_{10} in this study.

$$T_2 = \frac{\bar{y}^*}{\bar{x}'}\bar{x}^* \tag{6.1.4}$$

$$T_3 = \frac{\overline{y}^*}{\overline{x}} \overline{x}' \qquad (6.1.5)$$

$$T_4 = \frac{\overline{y}^*}{\overline{x}'} \overline{x} \qquad (6.1.6)$$

$$T_5 = \overline{y}^* \left(\frac{\overline{x}'}{\overline{x}^*}\right)\left(\frac{\overline{x}'}{\overline{x}}\right) \qquad (6.1.7)$$

$$T_6 = \overline{y}^* \left(\frac{\overline{x}^*}{\overline{x}'}\right)\left(\frac{\overline{x}}{\overline{x}'}\right) \qquad (6.1.8)$$

$$T_7 = \overline{y}^* \left(\frac{\overline{x}}{\overline{x}^*}\right)^{\sim_1} \left(\frac{\overline{x}'}{\overline{x}}\right)^{\sim_2} \qquad (6.1.9)$$

$$T_8 = \overline{y}^* + d_1\left(\overline{x} - \overline{x}^*\right) + d_2\left(\overline{x}' - \overline{x}\right) \qquad (6.1.10)$$

$$T_9 = \overline{y}^* + d_3\left(\overline{x}^* - \overline{x}'\right)$$

(6.1.11)

$$T_{10} = \overline{y}^* + d_4\left(\overline{x} - \overline{x}'\right) \qquad (6.1.12)$$

where $\sim_1, \sim_2, d_i \ (i=1,2,3,4)$ are characterizing scalars to be suitably chosen;.

BIAS AND MSE

The bias and mean square of the above estimators are given as

$$MSE(T_1) = MSE\left(\overline{y}^*\right) + \left[\left(\frac{1}{n} - \frac{1}{n'}\right)\left\{\left(\pm_x^2 + \pm_v^2\right) - 2\pm_{yx}\right\}\right.$$

$$\left. + \frac{W_2(k-1)}{n}\left\{\left(\pm_{x(2)}^2 + \pm_{v(2)}^2\right) - 2\pm_{yx(2)}\right\}\right] \qquad (6.1.13)$$

$$MSE(T_2) = MSE\left(\overline{y}^*\right) + \left[\left(\frac{1}{n} - \frac{1}{n'}\right)\left\{\left(\pm_x^2 + \pm_v^2\right) + 2\pm_{yx}\right\}\right.$$

$$\left. + \frac{W_2(k-1)}{n}\left\{\left(\pm_{x(2)}^2 + \pm_{v(2)}^2\right) + 2\pm_{yx(2)}\right\}\right] \qquad (6.1.14)$$

$$MSE(T_3) = MSE\left(\bar{y}^*\right) + \left[\left(\frac{1}{n} - \frac{1}{n'}\right)\left\{\left(\pm_x^2 + \pm_v^2\right) - 2\pm_{yx}\right\}\right] \quad (6.1.15)$$

$$MSE(T_4) = MSE\left(\bar{y}^*\right) + \left[\left(\frac{1}{n} - \frac{1}{n'}\right)\left\{\left(\pm_x^2 + \pm_v^2\right) + 2\pm_{yx}\right\}\right] \quad (6.1.16)$$

$$MSE(T_5) = MSE\left(\bar{y}^*\right) + \left[\left(\frac{1}{n} - \frac{1}{n'}\right)\left\{4\left(\pm_x^2 + \pm_v^2\right) - 4\pm_{yx}\right\}\right.$$

$$\left. + \frac{W_2(k-1)}{n}\left\{\left(\pm_{x_{(2)}}^2 + \pm_{v_{(2)}}^2\right) - 2\pm_{yx_{(2)}}\right\}\right] \quad (6.1.17)$$

$$MSE(T_6) = MSE\left(\bar{y}^*\right) + \left[\left(\frac{1}{n} - \frac{1}{n'}\right)\left\{4\left(\pm_x^2 + \pm_v^2\right) + 4\pm_{yx}\right\}\right.$$

$$\left. + \frac{W_2(k-1)}{n}\left\{\left(\pm_{x_{(2)}}^2 + \pm_{v_{(2)}}^2\right) + 2\pm_{yx_{(2)}}\right\}\right] \quad (6.1.18)$$

$$MSE(T_7) = MSE\left(\bar{y}^*\right) + \left[\left(\frac{1}{n} - \frac{1}{n'}\right)\left\{\sim_2^2\left(\pm_x^2 + \pm_v^2\right) - 2\sim_2\pm_{yx}\right\}\right.$$

$$\left. + \frac{W_2(k-1)}{n}\left\{\sim_1^2\left(\pm_{x_{(2)}}^2 + \pm_{v_{(2)}}^2\right) - 2\sim_1\pm_{yx_{(2)}}\right\}\right] \quad (6.1.19)$$

$$MSE(T_8) = MSE\left(\bar{y}^*\right) + \left[\left(\frac{1}{n} - \frac{1}{n'}\right)\left\{d_2^2\left(\pm_x^2 + \pm_v^2\right) - 2d_2^2\pm_{yx}\right\}\right.$$

$$\left. + \frac{W_2(k-1)}{n}\left\{d_1^2\left(\pm_{x_{(2)}}^2 + \pm_{v_{(2)}}^2\right) - 2d_1\pm_{yx_{(2)}}\right\}\right] \quad (6.1.20)$$

$$MSE(T_9) = MSE\left(\bar{y}^*\right) + \left[\left(\frac{1}{n} - \frac{1}{n'}\right)\left\{d_3^2\left(\pm_x^2 + \pm_v^2\right) + 2d_3\pm_{yx}\right\}\right.$$

$$\left. + \frac{W_2(k-1)}{n}\left\{d_3^2\left(\pm_{x_{(2)}}^2 + \pm_{v_{(2)}}^2\right) + 2d_3\pm_{yx_{(2)}}\right\}\right] \quad (6.1.21)$$

$$MSE(T_{10}) = MSE\left(\bar{y}^*\right) + \left[\left(\frac{1}{n} - \frac{1}{n'}\right)\left\{d_4^2\left(\pm_x^2 + \pm_v^2\right) + 2d_4\pm_{yx}\right\}\right] \quad (6.1.22)$$

The optimum values which minimizes the MSE is

$$\sim_1 = \frac{\pm_{yx_{(2)}}}{\left(\pm^2_{x_{(2)}} + \pm^2_{x_{(2)}}\right)} \tag{6.1.23}$$

$$\sim_2 = \frac{\pm_{yx}}{\left(\pm^2_{x} + \pm^2_{v}\right)} \tag{6.1.24}$$

$$d_1 = \frac{\pm_{yx_{(2)}}}{\left(\pm^2_{x_{(2)}} + \pm^2_{x_{(2)}}\right)} \tag{6.1.25}$$

$$d_2 = \frac{\pm_{yx}}{\left(\pm^2_{x} + \pm^2_{v}\right)} \tag{6.1.26}$$

$$d_3 = -\frac{\left[\left(\frac{1}{n} - \frac{1}{n'}\right)\pm_{yx} + \frac{W_2(k-1)}{n}\pm_{yx_{(2)}}\right]}{\left[\left(\frac{1}{n} - \frac{1}{n'}\right)\left(\pm^2_x + \pm^2_v\right) + \frac{W_2(k-1)}{n}\left(\pm^2_{x_{(2)}} + \pm^2_{v_{(2)}}\right)\right]} \tag{6.1.27}$$

$$d_4 = -\frac{\pm_{yx}}{\left(\pm^2_{x} + \pm^2_{v}\right)} \tag{6.1.28}$$

and minimum MSE's are found to be

$$MSE(T_7)_{min} = MSE\left(\bar{y}^*\right) - \left[\left(\frac{1}{n} - \frac{1}{n'}\right)\frac{\left(\pm_{yx}\right)^2}{\left(\pm^2_x + \pm^2_v\right)} + \frac{W_2\left(k-1\right)}{n}\frac{\left(\pm_{yx_{(2)}}\right)^2}{\left(\pm^2_{x_{(2)}} + \pm^2_{v_{(2)}}\right)}\right] \tag{6.1.29}$$

$$MSE(T_8)_{min} = MSE\left(\bar{y}^*\right) - \left[\left(\frac{1}{n} - \frac{1}{n'}\right)\frac{\left(\pm_{yx}\right)^2}{\left(\pm^2_x + \pm^2_v\right)} + \frac{W_2\left(k-1\right)}{n}\frac{\left(\pm_{yx_{(2)}}\right)^2}{\left(\pm^2_{x_{(2)}} + \pm^2_{v_{(2)}}\right)}\right] \tag{6.1.30}$$

$$MSE(T_9)_{min} = MSE\left(\bar{y}^*\right) - \left[\frac{\left\{\left(\frac{1}{n} - \frac{1}{n'}\right)\pm_{yx} + \frac{k'}{n}\pm_{yx_{(2)}}\right\}^2}{\left\{\left(\frac{1}{n} - \frac{1}{n'}\right)\left(\pm^2_x + \pm^2_v\right) + \frac{k'}{n}\left(\pm^2_{x_{(2)}} + \pm^2_{v_{(2)}}\right)\right\}}\right] \tag{6.1.31}$$

$$MSE(T_{10})_{min} = MSE\left(\bar{y}^*\right) - \left[\left(\frac{1}{n} - \frac{1}{n'}\right)\frac{\left(\pm_{yx}\right)^2}{\left(\pm_x^2 + \pm_v^2\right)}\right] \qquad (6.1.32)$$

$$MSE_{min}\left(T_7\right) = MSE_{min}\left(T_8\right)$$

$$= MSE\left(\bar{y}^*\right) - \left[\left(\frac{1}{n} - \frac{1}{n'}\right)\frac{\left(\pm_{yx}\right)^2}{\left(\pm_x^2 + \pm_v^2\right)} + \frac{W_2\left(k-1\right)}{n}\frac{\left(\pm_{yx_{(2)}}\right)^2}{\left(\pm_{x_{(2)}}^2 + \pm_{v_{(2)}}^2\right)}\right] \qquad (6.1.33)$$

It is noticeable to mention here that Singh and Kumar (2010) concluded that T_7 and T_8 have the same minimum mean square error as seen from (6.1.29) and (6.1.30) and were reported to be better than the other double sampling estimators T_1 to T_{10} under optimizing conditions; see Singh and Kumar (2010).

Our stimulus behind this chapter is the fact that if parameter of the supplementary variable is not known and the supplementary information can be collected without any non-response, then, we can simply make use of the estimates based on the sample and the subsequent sub-sample following the technique of Hansen and Hurwitz, which itself is double sampling see Lohr (1999). Okafor and Lee (2000) among other have incurred an extra cost due to a large first phase preliminary sample of size n' which will ultimately lead to us to some gain in precision but only by extra expenditure thereby adding to the overall cost of the survey. Therefore, a trade-off between precision and cost is required and we are trying to address this issue in this chapter.

6.2 ADAPTED ESTIMATORS

The primary objective of this chapter is to propose cost efficient modified ratio, product and difference type estimators without using conventional double sampling procedures when the supplementary information about the population mean $\bar{X}$ is not known. As mentioned in the previous section we assume that there is complete sample information on the auxiliary character x for the selected units. In this chapter we propose to estimate $\bar{X}$ by $\bar{x}$ instead of $\bar{x}'$ so as to save the cost and in place of the sub-sample mean of the auxiliary character x we are using the structure of Hansen Hurwitz (HH) estimator $\bar{x}^*$, although we have complete sample information on x.These sampling strategies are put to test against double sampling estimators as envisaged by various authors including Khare and Srivastava (1993, 1995), Okafor and Lee (2000), Singh and Kumar (2010).

We propose to study the following adaptations of commonly used estimators under measurement errors

$$t_1 = \frac{\bar{y}^*}{\bar{x}^*}\bar{x} \tag{6.2.1}$$

$$t_2 = \frac{\bar{y}^*}{\bar{x}}\bar{x}^* \tag{6.2.2}$$

$$t_3 = \bar{y}^*\left(\frac{\bar{x}^*}{\bar{x}}\right)^{\sim} \tag{6.2.3}$$

$$t_4 = \bar{y}^* + d\left(\bar{x}^* - \bar{x}\right) \tag{6.2.4}$$

where $\sim$, d are characterizing scalars chosen suitably.

6.3 BIAS AND MEAN SQUARE ERROR OF THE PROPOSED ESTIMATORS

we define the following error terms

$\bar{y}^* = \bar{Y} + \pounds_0^*, \qquad \bar{x}^* = \bar{X} + \pounds_1^* \qquad \bar{x} = \bar{X} + \pounds_1$

such that

$E(\pounds_0^*) = E(\pounds_1^*) = E(\pounds_1) = 0$

and

$$E(\pounds_0^{*2}) = \frac{1}{n}\left(\pm_y^2 + \pm_u^2\right) + \frac{W_2(k-1)}{n}\left(\pm_{y_{(2)}}^2 + \pm_{u_{(2)}}^2\right)$$

$$E(\pounds_1^{*2}) = \frac{1}{n}\left(\pm_x^2 + \pm_v^2\right) + \frac{W_2(k-1)}{n}\left(\pm_{x_{(2)}}^2 + \pm_{v_{(2)}}^2\right)$$

$$E(\pounds_1^2) = \frac{1}{n}\left(\pm_x^2 + \pm_v^2\right)$$

$$E(\pounds_0^*\pounds_1^*) = \mathrm{cov}(\bar{y}^*, \bar{x}^*) = \frac{1}{n}\pm_{xy} + \frac{W_2(k-1)}{n}\pm_{xy_{(2)}}$$

$$E(\pounds_0^*\pounds_1) = \mathrm{cov}(\bar{y}^*, \bar{x}) = \frac{1}{n}\pm_{xy}$$

$$E(\pounds_1^*\pounds_1) = \mathrm{cov}(\bar{x}^*, \bar{x}) = \frac{1}{n}\left(\pm_x^2 + \pm_v^2\right)$$

The biases of the proposed estimators, except the last two as they are obviously unbiased, are given by

$$Bias(t_1) = \frac{(k-1)W_2}{n}\left[\left(\pm_{x_{(2)}}^2 + \pm_{v_{(2)}}^2\right) - \pm_{yx_{(2)}}\right] \qquad (6.3.1)$$

$$Bias(t_2) = \frac{(k-1)W_2}{n}\left[\pm_{yx_{(2)}}\right] \qquad (6.3.2)$$

$$Bias(t_3) = \frac{(k-1)W_2}{n}\left[\frac{\sim^2}{2}\left(\pm_{x_{(2)}}^2 + \pm_{v_{(2)}}^2\right) - \sim\left(\frac{1}{2}\left(\pm_{x_{(2)}}^2 + \pm_{v_{(2)}}^2\right) - \pm_{yx_{(2)}}\right)\right] \qquad (6.3.3)$$

The MSE's of the proposed estimators are given by

$$MSE\left(\bar{y}^*\right) = \left[\frac{(N-n)}{Nn}\left(\pm_y^2 + \pm_u^2\right) + \frac{(k-1)W_2}{n}\left(\pm_{y_{(2)}}^2 + \pm_{u_{(2)}}^2\right)\right] \qquad (6.3.4)$$

$$MSE(t_1) = MSE\left(\bar{y}^*\right) + \frac{(k-1)W_2}{n}\left[\left(\pm^2_{x_{(2)}} + \pm^2_{v_{(2)}}\right) - 2\pm_{yx_{(2)}}\right] \quad (6.3.5)$$

$$MSE(t_2) = MSE\left(\bar{y}^*\right) + \frac{(k-1)W_2}{n}\left[\left(\pm^2_{x_{(2)}} + \pm^2_{v_{(2)}}\right) + 2\pm_{yx_{(2)}}\right] \quad (6.3.6)$$

$$MSE(t_3) = MSE\left(\bar{y}^*\right) + \frac{(k-1)W_2}{n}\left[\sim^2\left(\pm^2_{x_{(2)}} + \pm^2_{v_{(2)}}\right) + 2\sim\pm_{yx_{(2)}}\right] \quad (6.3.7)$$

$$MSE(t_4) = MSE\left(\bar{y}^*\right) + \frac{(k-1)W_2}{n}\left[d^2\left(\pm^2_{x_{(2)}} + \pm^2_{v_{(2)}}\right) + 2d\pm_{yx_{(2)}}\right] \quad (6.3.8)$$

The optimum values minimizing the MSE's are

$$\sim = -\frac{\pm_{yx_{(2)}}}{\left(\pm^2_{x_{(2)}} + \pm^2_{v_{(2)}}\right)} \quad (6.3.9)$$

$$d = -\frac{\pm_{yx_{(2)}}}{\left(\pm^2_{x_{(2)}} + \pm^2_{v_{(2)}}\right)} \quad (6.3.10)$$

and minimum MSE's are found to be

$$MSE(t_3)_{min} = MSE\left(\bar{y}^*\right) - \frac{(k-1)W_2}{n}\frac{\left(\pm_{yx_{(2)}}\right)^2}{\left(\pm^2_{x_{(2)}} + \pm^2_{v_{(2)}}\right)} \quad (6.3.11)$$

$$MSE(t_4)_{min} = MSE\left(\bar{y}^*\right) - \frac{(k-1)W_2}{n}\frac{\left(\pm_{yx_{(2)}}\right)^2}{\left(\pm^2_{x_{(2)}} + \pm^2_{v_{(2)}}\right)} \quad (6.3.12)$$

which is same for the estimators t_3 and t_4. Hence, it can be easily seen that the proposed generalized estimators are always better than the HH estimator under the optimum conditions (6.3.9) and (6.3.10) but less efficient than T_7 and T_8 under the optimum values of scalars involved therein.

6.4 DETERMINATION OF OPTIMUM VALUE OF *n* AND *k*

CASE I: For Fixed Precision

Let us consider a cost function for t_3 as,

$$C = cn + c_1 n_1 + c_2 r$$

where c is the cost per unit of the first attempt with the sample, $n\,;c_1$ is the cost per unit for processing the respondent data at the first attempt in n_1 and c_2 is the cost per unit associated with the subsample, r of n_2.

Since the values of n_1 and r are not known until the first attempt is made, so the expected cost will be used in planning the survey. The expected values of n_1 and r are respectively $W_1 n$ and $\frac{W_2 n}{k}$. Thus the expected cost is given by

$$E(C) = C^* = n\left[c + c_1 W_1 + \frac{c_2 W_2}{k}\right] \tag{6.4.1}$$

Theorem 6.4.1

The optimum values of n and k that minimize the cost for a fixed variance are given by

$$k_{opt} = \sqrt{\frac{c_2\left(U_1 - W_2 U_2\right)}{\left(c + c_1 W_1\right) U_2}} \tag{6.4.2}$$

$$n_{opt} = \frac{\left\{U_1 + \left(k_{opt} - 1\right) W_2 U_2\right\}}{\left\{\frac{V_0}{\bar{Y}^2} + \frac{U_1}{N}\right\}} \tag{6.4.3}$$

where $U_1 = \left\{\pm_y^2 + \pm_y^2\right\}$ and $U_2 = \left\{\left(\pm_{y(2)}^2 + \pm_{u(2)}^2\right) + \sim^2\left(\pm_{x(2)}^2 + \pm_{v(2)}^2\right) + 2\sim\pm_{yx(2)}\right\}$.

The minimum cost for fixed variance V_0 is given by

$$C^* = \frac{1}{V_0}\left[\left(\pm_y^2 + \pm_u^2\right) + (k_{opt} - 1) W_2 \left\{\left(\pm_{y(2)}^2 + \pm_{u(2)}^2\right) + \sim^2\left(\pm_{x(2)}^2 + \pm_{v(2)}^2\right)\right.\right.$$

$$+2\sim\pm_{yx_{(2)}}\Big\}\Bigg]\left[c+c_1W_1+\frac{c_2W_2}{k_{opt}}\right] \qquad (6.4.4)$$

From (6.3.5) to (6.3.8) it is clear that for $\sim\, = 0, -1,\, 1, d$ in the mean square error of t_3 we can get the mean square error of $\bar{y}^*$, t_1, t_2 and t_4 respectively and therefore a similar analysis for each one of the adapted estimator can be conducted.

CASE II: For Fixed Cost

Let C_0 be the total cost (fixed) of the survey apart from overhead cost. The expected total cost of the survey apart from the overhead cost is given by

$$C = n\left(c + c_1W_1 + \frac{c_2W_2}{k}\right) \qquad (6.4.5)$$

where c is the cost per unit of the first attempt with the sample, n; c_1 is the cost per unit for processing the respondent data at the first attempt in n_1 and c_2 is the cost per unit associated with the sub sample r of n_2.The MSE of t_3 can be expressed as,

$$MSE\left(t_3\right) = \bar{Y}^2\left[\frac{\left(U_1 - W_2U_2\right)}{n} + \frac{kW_2U_2}{n} - \frac{U_1}{N}\right] \qquad (6.4.6)$$

where $U_1 = \left\{\pm_y^2 + \pm_y^2\right\}$ and $U_2 = \left\{\left(\pm_{y_{(2)}}^2 + \pm_{u_{(2)}}^2\right) + \sim^2\left(\pm_{x_{(2)}}^2 + \pm_{v_{(2)}}^2\right) + 2\sim\pm_{yx_{(2)}}\right\}$.

Theorem 6.4.2

The optimum values of n and k that minimize the $MSE(t_3)$ for a fixed cost $(C^* < C_0)$ are given by

$$k_{opt} = \sqrt{\frac{c_2\left(U_1 - W_2U_2\right)}{\left(c + c_1W_1\right)U_2}} \qquad (6.4.7)$$

$$n_{opt} = \frac{C_0}{\left\{ c + c_1 W_1 + \frac{c_2 W_2}{k_{opt}} \right\}} \tag{6.4.8}$$

$$MSE\left(t_3\right) = \frac{1}{C}\left[\left(\pm_y^2 + \pm_u^2\right) + (k_{opt} - 1)W_2\left\{\left(\pm_{y_{(2)}}^2 + \pm_{u_{(2)}}^2\right) + \sim^2\left(\pm_{x_{(2)}}^2 + \pm_{v_{(2)}}^2\right)\right.\right.$$

$$\left.\left. + 2\sim\pm_{yx_{(2)}}\right\}\right]\left[c + c_1 W_1 + \frac{c_2 W_2}{k_{opt}}\right] \tag{6.4.9}$$

Therefore, for the optimum values of n and k, the minimum mean square error of t_3 for fixed cost ($C^* < C_0$) is given as (6.4.9).

6.5 AN EMPIRICAL STUDY

The present data belong to the data on physical growth of upper socio-economic group of 95 school going children of Varanasi under an ICMR study, Department of Pediatrics, BHU during 1983-84 has been taken under study, Khare and Sinha (2007). The first 25% (i.e. 24 children) units have been considered as non-response units. The values of parameters related to the study characters y (weight of children in kg), the auxiliary character x (chest circumference of the children in cm) have been given as follows:

$\bar{Y}_2 = 19.4968;\ \bar{X} = 55.8611;\ \pm_y = 3.0435;\ \pm_x = 3.2735;\ \pm_{y_{(2)}} = 02.3552;\ \pm_{x_{(2)}} = 2.5137;$

$\pm_{yx} = 8.428611;\ \pm_{yx_{(2)}} = 4.315874.$

The problem considered is to estimate the weight of the male children aged 6-7 years using chest circumference as the auxiliary character.

Table 6.5.1 *Expected cost and cost efficiency*

Estimator	% ME	k_{opt}	n	n'	Expected Cost	Cost Efficiency
$\overline{y}^*$	0%	1.6445	24.5947	-	1148.1830	1.0000
	1%	1.6445	27.0205	-	1098.6300	0.9900
	5%	1.6445	19.6512	-	1205.5930	0.9524
	10%	1.6445	31.0631	-	1263.0002	0.9091
	15%	1.6445	32.4751	-	1320.4110	0.8695
	20%	1.6445	33.8871	-	1377.820	0.8333
t_1	% ME	k_{opt}	n	n'	Expected Cost	Cost Efficiency
	0%	2.2063	28.5272	-	1092.0424	1.0505
	1%	2.1954	27.2958	-	1046.697	1.0392
	5%	2.1552	29.9514	-	1152.383	0.9963
	10%	2.1115	31.3730	-	1211.666	0.9476
	15%	2.0734	32.7924	-	1270.792	0.9035
	20%	2.0401	34.2100	-	1329.814	0.8634
t_2	% ME	k_{opt}	n	n'	Expected Cost	Cost Efficiency
	0%	0.4998	17.2517	-	1065.674	1.0774
	1%	0.5051	16.6528	-	1023.317	1.0629
	5%	0.5258	18.8739	-	1137.566	1.0093
	10%	0.5499	204756	-	1208.164	0.9503
	15%	0.5725	22.0600	-	1277.676	0.8986
	20%	0.5937	23.6297	-	1346.261	0.8529
t_3	% ME	k_{opt}	n	n'	Cost	Cost Efficiency
	0%	2.5137	26.6865	-	1067.6250	1.0754
	1%	2.4840	28.7824	-	1080.6160	1.0625
	5%	2.3812	29.9408	-	1131.996	1.0143
	10%	2.2804	31.3778	-	1195.155	0.9607
	15%	2.2012	32.8062	-	1257.405	0.9131
	20%	2.1374	34.2285	-	1318.955	0.8705
t_4	% ME	k_{opt}	n	n'	Expected Cost	Cost Efficiency
	0%	2.5137	26.6865	-	1067.6250	1.0754
	1%	2.4840	28.7824	-	1080.6160	1.0625
	5%	2.3812	29.9408	-	1131.996	1.0143
	10%	2.2804	31.3778	-	1195.155	0.9607
	15%	2.2012	32.8062	-	1257.405	0.9131
	20%	2.1374	34.2285	-	1318.955	0.8705

Estimator	% ME	k_{opt}	n	n'	Expected Cost	Cost Efficiency
T_1	0%	1.4912	21.3565	37.6974	1340.953	0.8562
	1%	1.4942	21.7521	37.9019	1359.415	0.8446
	5%	1.5053	23.3229	38.6823	1432.356	0.8016
	10%	1.5171	25.2623	39.5806	1521.664	0.7545
	15%	1.5271	27.1779	40.4025	1609.122	0.7135
	20%	1.5357	29.0724	41.1557	1697.924	0.6774
T_3	% ME	k_{opt}	n	n'	Expected Cost	Cost Efficiency
	0%	1.0628	19.9108	38.057	1366.704	0.8401
	1%	1.07219	20.3264	38.2659	1385.657	0.8286
	5%	1.1072	21.9708	39.0576	1460.283	0.7863
	10%	1.1461	23.9912	39.9628	1551.187	0.7402
	15%	1.1805	25.9781	40.7858	1639.805	0.7002
	20%	1.2111	27.9360	41.5365	1726.433	0.7764
T_5	% ME	k_{opt}	n	n'	Expected Cost	Cost Efficiency
	0%	1.0427	13.6709	41.2635	1123.736	1.0217
	1%	1.0677	14.3120	41.6246	1152.685	0.9961
	5%	1.1542	16.7899	42.8599	1262.733	0.9093
	10%	1.2400	19.7347	44.0244	1390.024	0.8260
	15%	1.3087	225510	44.5879	1508.540	0.7611
	20%	1.8635	25.2655	45.4202	1619.988	0.7087
T_7	% ME	k_{opt}	n	n'	Expected Cost	Cost Efficiency
	0%	1.2047	13.1686	39.0374	1048.192	1.0377
	1%	1.2259	14.5716	41.5669	1137.136	1.0097
	5%	1.2947	17.1215	42.7793	1252.140	0.9169
	10%	1.3561	20.0676	43.9132	1382.226	0.8307
	15%	1.4008	22.8108	44.7573	1501.143	0.7649
	20%	1.4346	25.3957	45.3982	1611.594	0.7124
T_9	% ME	k_{opt}	n	n'	Expected Cost	Cost Efficiency
	0%	1.2046	13.9002	41.2062	1106.424	1.0340
	1%	1.2277	14.6684	41.5553	1140.9660	1.0063
	5%	1.2958	17.2090	42.7512	1255.408	0.9146
	10%	1.3567	20.1461	43.87050	1384.959	0.8290
	15%	1.4011	22.8820	44.7041	1503.458	0.7637
	20%	1.4348	25.4608	45.3371	1613.567	0.7116

Estimator	% ME	k_{opt}	n	n'	Expected Cost	Cost Efficiency
T_{10}	0%	0.6510	10.2888	38.6349	1026.685	1.0595
	1%	0.6846	11.6723	41.2631	1120.574	1.0246
	5%	0.7974	14.6572	42.7939	1252.998	0.9163
	10%	0.9053	17.9767	44.1272	1395.733	0.8226
	15%	0.9899	209887	45.0692	1522.139	0.7573
	20%	1.0586	23.7788	45.7578	1637.226	0.7013
T_8	% ME	k_{opt}	n	n'	Expected Cost	Cost Efficiency
	0%	1.2047	13.1686	39.0374	1048.192	1.0377
	1%	1.2259	14.5716	41.5669	1137.1360	1.0097
	5%	1.2947	17.1215	42.7793	1252.140	0.9169
	10%	1.3562	200676	43.9132	1382.226	0.8307
	15%	1.4008	22.8108	44.7573	1501.143	0.7649
	20%	1.4346	25.3957	45.3982	1611.594	0.7125

Table 6.5.2 *PRE & MSE under measurement error & non-response*

↓ Estimators		$1/k$			
	↓ME %	1/2	1/3	1/4	1/5
$\bar{y}^*$	0%	0.3125(100)	0.3525(100)	0.3926(100)	0.4326(100)
	1%	0.3156(99)	0.3560(99)	0.3965(99)	0.4369(99)
	5%	0.3281(95)	0.3401(95)	0.4122(95)	0.4542(95)
	10%	0.3437(91)	0.3878(91)	0.4318(91)	0.4759(91)
	15%	0.3593(87)	0.4054(87)	0.4514(87)	0.4975(87)
	20%	0.3750(83)	0.4230(83)	0.4711(83)	0.5191(83)
t_1	↓ME %	1/2	1/3	1/4	1/5
	0%	0.2963(105)	0.3201(110)	0.3440(114)	0.3678(118)
	1%	0.2995(104)	0.3268(109)	0.3481(113)	0.3724(116)
	5%	0.3122(100)	0.3383(104)	0.3644(105)	0.3906(111)
	10%	0.3281(95)	0.3565(99)	0.3849(102)	0.4133(105)
	15%	0.3440(91)	0.3747(94)	0.4054(97)	0.4361(99)
	20%	0.3599(87)	0.3929(90)	0.4258(92)	0.4588(94)
t_2	↓ME %	1/2	1/3	1/4	1/5
	0%	0.4468(70)	0.6211(57)	0.7954(49)	0.9698(45)
	1%	0.4499(69)	0.6247(56)	0.7995(49)	0.9745(44)
	5%	0.4627(68)	0.6393(55)	0.8169(48)	0.9925(44)
	10%	0.4786(65)	0.6575(54)	0.8364(47)	1.0153(43)
	15%	0.4945(63)	0.6757(52)	0.8568(46)	1.0379(42)
	20%	0.5104(61)	0.6938(51)	0.8773(45)	1.0607(41)
t_3	↓ME %	1/2	1/3	1/4	1/5
	0%	0.2912(107)	0.3100(114)	0.3287(119)	0.3475(125)
	1%	0.2945(106)	0.3139(112)	0.3333(118)	0.3526(123)
	5%	0.3078(101)	0.3296(107)	0.3514(112)	0.3732(116)
	10%	0.3244(96)	0.3491(101)	0.3738(105)	0.3985(109)
	15%	0.3408(92)	0.3684(96)	0.3959(99)	0.4235(102)
	20%	0.3572(87)	0.3876(91)	0.4179(94)	0.4482(97)
t_4	↓ME %	1/2	1/3	1/4	1/5
	0%	0.2912(107)	0.3100(114)	0.3287(119)	0.3475(125)
	1%	0.2945(106)	0.3139(112)	0.3333(118)	0.3526(123)
	5%	0.3078(101)	0.3296(107)	0.3514(112)	0.3732(116)
	10%	0.3244(96)	0.3491(101)	0.3738(105)	0.3985(109)
	15%	0.3408(92)	0.3684(96)	0.3959(99)	0.4235(102)
	20%	0.3572(87)	0.3876(91)	0.4179(94)	0.4482(97)

	↓ME %	1/2	1/3	1/4	1/5
T_1	0%	0.2270(138)	0.2509(141)	0.2747(143)	0.2986(145)
	1%	0.2304(136)	0.2547(138)	0.2790(141)	0.2033(143)
	5%	0.2439(128)	0.2701(131)	0.2962(133)	0.3223(134)
	10%	0.2608(120)	0.2892(122)	0.3176(124)	0.3460(125)
	15%	0.2777(112)	0.3084(114)	0.3391(116)	0.3698(117)
	20%	0.2946(106)	0.3276(108)	0.3605(109)	0.3935(110)
T_3	↓ME %	1/2	1/3	1/4	1/5
	0%	0.2432(128)	0.2833(124)	0.3233(121)	0.3633(119)
	1%	0.2465(127)	0.2870(123)	0.3274(120)	0.3679(118)
	5%	0.2598(120)	0.3019(117)	0.3439(114)	0.3860(112)
	10%	0.2764(113)	0.3205(110)	0.3645(108)	0.4086(106)
	15%	0.2931(107)	0.3391(104)	0.3851(102)	0.4312(100)
	20%	0.3097(101)	0.3577(99)	0.4058(97)	0.4538(95)
T_5	↓ME %	1/2	1/3	1/4	1/5
	0%	0.1973(158)	0.2211(159)	0.2450(160)	0.2688(161)
	1%	0.2012(155)	0.2255(156)	0.2499(157)	0.2742(158)
	5%	0.2171(144)	0.2433(145)	0.2694(146)	0.2955(146)
	10%	0.2370(132)	0.2654(133)	0.2938(134)	0.3222(134)
	15%	0.2568(122)	0.2875(123)	0.3182(123)	0.3489(124)
	20%	0.2767(113)	0.3096(114)	0.3426(115)	0.3758(115)
T_7	↓ME %	1/2	1/3	1/4	1/5
	0%	0.1909(164)	0.2097(168)	0.2284(172)	0.2472(175)
	1%	0.1952(160)	0.2146(164)	0.2340(168)	0.2534(172)
	5%	0.2123(147)	0.2341(151)	0.2559(153)	0.2777(156)
	10%	0.2332(134)	0.2579(137)	0.2834(139)	0.3073(141)
	15%	0.2536(123)	0.2812(125)	0.3087(127)	0.3363(129)
	20%	0.2737(114)	0.3040(116)	0.3343(117)	0.3646(119)
T_9	↓ME %	1/2	1/3	1/4	1/5
	0%	0.1914(163)	0.2103(168)	0.2292(172)	0.2481(174)
	1%	0.1958(160)	0.2153(164)	0.2348(167)	0.2543(170)
	5%	0.2128(147)	0.2347(150)	0.2566(153)	0.2785(155)
	10%	0.2337(134)	0.2585(136)	0.2833(139)	0.3082(140)
	15%	0.2541(123)	0.2818(125)	0.3094(127)	0.3372(128)
	20%	0.2741(114)	0.3045(116)	0.3350(117)	0.3654(118)

	↓ME %	1/2	1/3	1/4	1/5
T_{10}	0%	0.2122(147)	0.2522(140)	0.2923(134)	0.3323(130)
	1%	0.2163(144)	0.2568(137)	0.2972(132)	0.3376(128)
	5%	0.2326(134)	0.2746(128)	0.3167(124)	0.3587(121)
	10%	0.2526(124)	0.2966(119)	0.3406(115)	0.3847(112)
	15%	0.2721(114)	0.3182(111)	0.3642(108)	0.4103(105)
	20%	0.2914(107)	0.3395(104)	0.3875(101)	0.4355(99)
T_8	↓ME %	1/2	1/3	1/4	1/5
	0%	0.1909(164)	0.2097(168)	0.2284(172)	0.2472(175)
	1%	0.1952(160)	0.2146(164)	0.2340(168)	0.2534(172)
	5%	0.2123(147)	0.2341(151)	0.2559(153)	0.2777(156)
	10%	0.2332(134)	0.2579(137)	0.2826(139)	0.3073(141)
	15%	0.2536(123)	0.2812(125)	0.3087(127)	0.3363(129)
	20%	0.2737(114)	0.3040(116)	0.3343(117)	0.3646(119)

Table 6.5.3 *PRE(MSE) without measurement error and with non-response error for optimum value of k*

Estimators	MSE
$\bar{y}^*$	0.4133(100)
t_1	0.3945(105)
t_2	0.3836(108)
t_3	0.3843(108)
t_4	0.3843(108)
T_1	0.4827(86)
T_3	0.4920(84)
T_5	0.4045(102)
T_7	0.3983(104)
T_9	0.3998(103)
T_{10}	0.3901(106)
T_8	0.3983(104)

Table 6.5.4 *PRE(MSE) without measurement error and with non-response error*

Estimators	1/2	1/3	1/4	1/5
$\bar{y}^*$	0.3125(100)	0.3525(100)	0.3925(100)	0.4326(100)
t_1	0.2963(105)	0.3201(110)	0.3440(114)	0.3678(118)
t_2	0.4468(70)	0.6211(57)	0.7954(49)	0.9698(45)
t_3	0.2912(107)	0.3099(114)	0.3287(119)	0.3475(124)
t_4	0.2912(107)	0.3099(114)	0.3287(119)	0.3475(124)
T_1	0.2270(138)	0.2509(141)	0.2747(143)	0.2986(145)
T_3	0.2432(128)	0.2833(124)	0.3233(121)	0.3633(119)
T_5	0.1973(118)	0.2211(159)	0.2450(160)	0.2688(161)
T_7	0.1909(164)	0.2097(168)	0.2284(172)	0.2472(175)
T_9	0.1914(163)	0.2103(168)	0.2292(171)	0.2481(174)
T_{10}	0.2122(147)	0.2522(140)	0.2923(134)	0.3323(130)
T_8	0.1909(164)	0.2097(168)	0.2284(172)	0.2472(175)

6.6 CONCLUDING REMARKS

1. It is evident from the expressions (6.1.13) to (6.1.22) and (6.3.4) to (6.3.8) of MSE's of the estimators that the measurement errors seem to have inflated the MSE of these estimators and thereby decreasing the efficiency.

2. The expressions (6.1.13) to (6.1.22) and (6.3.4) to (6.3.8) of MSE can be broken into 4 major components owing to non-response and measurement error are given below:

$$MSE = A + B + C + D$$

where A = Component of MSE due to sampling error without measurement error and non-response,

B = Component of MSE due to sampling error with measurement error and without non-response,

C = Component of MSE due to sampling error without measurement error and with non-response, and

D = Component of MSE due to sampling error with measurement error and without non-response. For Example: Consider the expression of MSE of t_4 given by

$$MSE(t_4) = \underbrace{\frac{1}{n}\pm_y^2}_{A} + \underbrace{\frac{1}{n}\pm_u^2}_{B} + \underbrace{\frac{W_2(k-1)}{n}\left(\pm_{y(2)}^2 + d^2\pm_{x(2)}^2 + 2d_1\pm_{xy(2)}\right)}_{C} + \underbrace{\frac{W_2(k-1)}{n}\left(\pm_{u(2)}^2 + d^2\pm_{v(2)}^2\right)}_{D}$$

3. If the measurement error is absent then we get the expression of the MSE of conventional estimators under non-response from the results of this study thereby the present study provides a more general and pragmatic approach for the estimation of population mean. For example: When $u_i = 0 = v_i$, for each i, then $\sigma_u^2 = \sigma_v^2 = \sigma_{u_2}^2 = \sigma_{v_2}^2 = 0$ and we get

$$MSE(t_4) = \underbrace{\frac{1}{n}\sigma_y^2}_{A} + \underbrace{\frac{W_2(k-1)}{n}\left(\sigma_{y(2)}^2 + d^2\sigma_{x(2)}^2 + 2d_1\sigma_{xy(2)}\right)}_{C}$$

so that only components A and C are left, which is same expression as proposed by Singh and Kumar (2010) while proposing t_4.

4. If the measurement error is absent then we get the expression of the optimum values of the characterizing scalars and minimum MSE of conventional estimators under non-response from the results of this study thereby the present study. For example: When $u_i = 0 = v_i$, for each i, then $\sigma_u^2 = \sigma_v^2 = \sigma_{u_2}^2 = \sigma_{v_2}^2 = 0$ and we get optimum value of d as

$$d = -\sigma_{xy(2)} / \sigma_{x(2)}^2$$

and the minimum mean square error of t_4 as

$$MSE(t_4)_{\min} = \frac{1}{n}\sigma_y^2 + \frac{W_2(k-1)}{n}\left(1 - \rho_{(2)}^2\right)\sigma_{y(2)}^2$$

which is same expression of minimum MSE as proposed by Bhushan and Naqvi (2015) while proposing t_4.

5. The measurement error seems to have affected all the estimators but the cost efficient estimators t_3 and t_4 perform far better than the remaining

double sampling estimators. For example: This is w.r.t. the empirical results in table 6.5.1 where t_4 utilized the auxiliary information optimally and outperformed all the remaining estimators when the cost was also taken into account. The expected cost incurred in the estimators t_3 and t_4 is less than as compared to the expected cost incurred for all the other proposed estimators and the conventional double sampling estimators proposed by Singh and Kumar (2010), Tabassum and Khan (2004) and Okafor and Lee (2000) considered under optimum conditions.

6. The measurement error seems to have affected the double sampling estimators more where the auxiliary information was properly utilized than the cost efficient estimators under optimal conditions. For example: This is w.r.t. the empirical results in table 6.5.1 where the double sampling estimator T_8 lost 32% cost efficiency which is far more in comparison to t_3, cost efficient estimator the auxiliary information was not properly utilized, lost only 20% cost efficiency or even $\bar{y}*$ which lost 17% cost efficiency.

7. The measurement error seems to have affected the double sampling estimators more in terms of efficiency where the auxiliary information was properly utilized than the suggested cost efficient estimators. For example: This is w.r.t. the empirical results in table 6.5.2 where double sampling estimator T_8 utilized the auxiliary information optimally and has lost 55% efficiency with $1/4$ sub-sampling fraction which is far more in comparison to t_4, cost efficient estimator lost only 25% efficiency with $1/4$ sub-sampling

fraction or even $\bar{y}*$ which lost 17% efficiency with 1/4 sub-sampling fraction.

8. The percent relative efficiency of various estimators were calculated w. r. t. $\bar{y}*$ with optimum sub-sampling fraction $1/k$ in table 6.5.3. The best choice, which is in consonance with the theoretical results, is t_3 and t_4 at all levels of measurement error. Further, the performance of the estimators decreases with the increasing level of measurement error. The cost efficient estimators retain more than 100% efficiency even at 5% level of measurement error while the double sampling estimators retain more than 100% efficiency only till 1% level of measurement error.

9. The gains in efficiency of various estimators were calculated w. r. t. $\bar{y}*$ with no measurement error in table 6.5.4. The best choice in terms of efficiency (not cost efficiency), which is in consonance with the theoretical results, is T_7 and T_8 at all levels of measurement error. The table 6.5.4 also demonstrates that the standard results of non-response estimators can be obtained when there is no measurement error. The comment 5 reiterated here that estimators t_3 and t_4 are still the most cost efficient estimators.

CHAPTER 7

GENERALIZED COST EFFICIENT ESTIMATION IN PRESENCE OF MEASUREMENT ERRORS AND NON-RESPONSE USING AUXILIARY VARIABLE

SUMMARY

In this chapter we have studied a couple of generalized cost efficient classes of estimators under measurement errors and non-response for estimating population mean using auxiliary information. These classes of estimators have been proposed as an alternative to the class of estimators proposed for only non-response by Singh and Kumar (2010) and Singh and Bhushan (2012). The results are derived under measurement error and non-response error simultaneously. These estimators are put to test against Singh and Kumar (2010) and Singh and Bhushan (2012) estimators under the cost efficiency criteria. A comprehensive comparative study is carried out both theoretically as well as empirically to study the effect of presence of measurement error.

7.1 INTRODUCTION

Over the past several decades, statisticians are paying their attention towards the problem of estimation of parameters in the presence of measurement errors. In survey sampling, the properties of data usually presuppose that the observations are the correct measurements on characteristics being studied. However such assumption does not satisfied in many applications and data is contaminated with measurement errors, such as non-response errors, reporting errors and computing errors. These measurement errors make the result invalid. If percentage of measurement errors are very small and we can neglect it, then the statistical inference belongs on data observed continue to remain valid. On the another side, if measurement errors are not appreciably small and negligible, the inferences may not be simply invalid and inaccurate but may often lead to unexpected, undesirable and unfortunate consequences (see Srivastava and Shalabh, 2001). Some important sources of measurement errors in survey data are discussed in Cochran (1968), Shalabh (1997), and Singh and Karpe (2008, 2010) studied some estimators of population mean under measurement errors.

Now we consider a finite population $U=\left(U_1,U_2,...U_N\right)$ of N units. Let Y and X be the study variate and auxiliary variate, respectively. Further we assume that we have a set of n paired observations obtained through simple random sampling procedure on two characteristics X and Y and suppose that x_i, and y_i for the i^{th} sampling units are observed with measurement error instead of their true values (X_i, Y_i). For a simple random sampling scheme, let (x_i, y_i) be observed values instead of the true values (X_i, Y_i) for i^{th} (i=1, 2,...n) unit such as

$$u_i = y_i - Y_i \qquad (6.1.1)$$

$$v_i = x_i - X_i \qquad (6.1.2)$$

where u_i and v_i are combined as measurement errors in y_i and x_i respectively which are stochastic (probabilistic) in nature with mean zero, variances $\pm_u^2$ and $\pm_v^2$ respectively. Further, we assume that u_i's and v_i's are uncorrelated although X_i's Y_i's are correlated.

Further, currently a lot of attention has been paid to the problem of non-response. The problem of non-response being a very undesirable though unavoidable feature of sample surveys vitiates the unbiasedness and reliability of the estimates. Most of the time, the required information may not always be collected from all the selected units in the sample even after call-backs. The inferences based on such incomplete information are generally biased and the results may be misleading to a great extent when the class of respondents differs significantly from the class of non-respondents. Hansen and Hurwitz (1946), in their seminal paper, gave an inventive idea of estimation in presence of non-response using double sampling scheme. They considered the problem of non-response in a mailed questionnaire survey for estimating the population mean by taking a sub-sample from the non-respondent stratum by direct interview and an estimator was proposed as convex combination of the information available from both the response stratum as well as from the non-response stratum.

It is a very well-known practice among survey statisticians to incorporate some auxiliary variable which closely related to study variable for drawing accurate inferences. Assuming the population mean $\bar{X}$ of the auxiliary variable to be known, the problem of estimation of population mean in presence of deterministic non-response has been considered initially by Cochran (1977), Rao (1986, 1987), Khare and Srivastava (1993, 1997), Singh and Kumar (2008) among others. Further, in absence of such auxiliary information we generally resort to double sampling. In case of non-response, double sampling was suggested by Okafor and Lee (2000), Tabassum and Khan (2004) and revisited recently by Singh and Kumar (2010) and Singh and Bhushan (2012) among others if such auxiliary information is absent. When population mean $\bar{X}$ of the auxiliary variable is not known then Okafor and Lee (2000) proposed to use the sample mean $\bar{x}'$ obtained from a large first phase sample of size n' drawn from N units by SRSWOR. It is important to mention here that it is assumed that all the first phase sample units supplied the auxiliary information; see Singh and Kumar (2010) i.e. there is no non-response as far as the auxiliary variable is concerned. Subsequently, a second phase sample of size n $(n<n')$ is drawn from the n' by SRSWOR and study variable y is measured on the selected units. Further, from the n second phase sample units, let n_1 units respond and n_2 units refuse to respond to the call of interviewer. So, we use Hansen Hurwitz (1946) sampling strategy to sub-sample r units from n_2 non-responding units such that $r = n_2/k$, $k>1$. It is again implicitly assumed that these r units respond to the second call. An example was cited by Okafor and Lee (2000) and then by Tabassum and Khan (2004).

Throughout, in this chapter, we have assumed the deterministic set up of non-response exactly on the similar lines as that of Okafor and Lee (2000) and assume that the whole population (denoted by U) is stratified into two strata: one is the respondent stratum (denoted by U_1) of N_1 units, which would respond on the first call at the second phase and the other is non-respondent stratum (denoted by U_2) of N_2 units, which would not respond on the first call but would respond on the second call. Also, let the first and second phase samples be denoted by s and s' respectively, and let $s_1 = s \cap U_1$ and $s_2 = s \cap U_2$. Further, the sub-sample of s_2 will be denoted by s_{2m}. Further, as a tradition, we denote the population parameters by uppercase letters and the sample statistics are denoted by lowercase letters.

The Okafor and Lee (2000) ratio estimator, revisited by, Tabassum and Khan (2004), is given by

$$T_1 = \frac{\bar{y}^*}{\bar{x}^*}\bar{x}' \tag{7.1.3}$$

Further, perusing this idea Singh and Kumar (2010) proposed some more estimators given by

$$T_2 = \frac{\bar{y}^*}{\bar{x}'}\bar{x}^* \tag{7.1.4}$$

$$T_3 = \frac{\bar{y}^*}{\bar{x}}\bar{x}' \tag{7.1.5}$$

$$T_4 = \frac{\bar{y}^*}{\bar{x}'}\bar{x} \tag{7.1.6}$$

$$T_{11} = \bar{y}^*\left(\frac{\bar{x}'}{\bar{x}}\right)^{\sim} \tag{7.1.7}$$

$$T_{12} = \bar{y}^*\left(\frac{\bar{x}^*}{\bar{x}'}\right)^{\sim} \tag{7.1.8}$$

$$T_7 = \bar{y}^*\left(\frac{\bar{x}}{\bar{x}^*}\right)^{\sim_1}\left(\frac{\bar{x}'}{\bar{x}}\right)^{\sim_2} \tag{7.1.9}$$

$$T_8 = \bar{y}^* + d_1\left(\bar{x} - \bar{x}^*\right) + d_2\left(\bar{x}' - \bar{x}\right) \tag{7.1.10}$$

where $\bar{x}$ is the sample mean of the auxiliary character x based on n units; $\bar{x}'$ is the sample mean of x based on n' units and $\bar{y}^*$ is the mean of the study character y where $\bar{y}^* = \frac{n_1}{n}\bar{y}_{(1)} + \frac{n_2}{n}\bar{y}^*_{(2)}$ and $\bar{x}^* = \frac{n_1}{n}\bar{x}_{(1)} + \frac{n_2}{n}\bar{x}^*_{(2)}$ $\left(\text{where } w_1 = \frac{n_1}{n} \text{ and } w_2 = \frac{n_2}{n}\right)$ with $\left(\bar{x}_{(1)}, \bar{y}_{(1)}\right)$ and $\left(\bar{x}^*_{(2)}, \bar{y}^*_{(2)}\right)$ being the sample means based on n_1 units and sub-sample means based on r units of the (x, y) respectively; $\sim$, $\sim_1$, $\sim_2$, d_1 and d_2 are the characterising scalars to be chosen suitably.

It is important to note that $\bar{y}^*$ is the HH estimator; T_1 is the Okafor and Lee (2000) ratio estimator; T_2, T_3, T_4 and T_5 are studied by Singh and Kumar (2010) and found that T_7 performs better that all other alternatives under optimum conditions. Further, T_{11} and T_{12} are included by us to make our study comprehensive. The idea behind construction of T_7 by Singh and Kumar (2010) was to chain T_{11} and T_{12} so as to ensue from maximum gain in efficiency.

The estimators T_1 ,T_2, T_3, T_4, T_{11} and T_{12} along with some other difference type estimators were generalized by Singh and Bhushan (2012) by

$$t_h = h\left(\bar{y}^*, \bar{x}^*, \bar{x}'\right) \tag{7.1.11}$$

$$t_H = H\left(\bar{y}^*, \bar{x}, \bar{x}'\right) \tag{7.1.12}$$

where $h(.)$ and $H(.)$ being bounded functions satisfy the following regularity conditions such that

(i) $h(\mathbf{P}) = \bar{Y}$ and $H(\mathbf{P}) = \bar{Y}$ (7.1.13)

(ii) first order partial derivative with respect to $\bar{y}^*$ at $\mathbf{P} \equiv \left(\bar{Y}, \bar{X}, \bar{X}\right)$ is unity, that is,

$$h_0 = 1 \text{ and } H_0 = 1 \quad (7.1.14)$$

(iii) $h_{00} = 0$ and $H_{00} = 0$ (7.1.15)

(iv) first order partial derivative of $h\left(\bar{y}^*, \bar{x}^*, \bar{x}'\right)$ and $H\left(\bar{y}^*, \bar{x}, \bar{x}'\right)$ with respect to $\bar{x}^*$ and $\bar{x}$ respectively at $\mathbf{P} \equiv \left(\bar{Y}, \bar{X}, \bar{X}\right)$ satisfy

$$h_1 = -h_2 \text{ and } H_1 = -H_2 \quad (7.1.16)$$

(v) $h_{01} = -h_{02}$ and $H_{01} = -H_{02}$ (7.1.17)

These conditions are similar to that Diana and Tommasi (2003).

Though the idea of Singh and Kumar (2010), motivated by Singh and Kumar (2008), was ingenious in the construction of estimator thereby increasing efficiency but it still incurred extra cost due to “double sampling”. The real motivation behind this chapter is the fact that if auxiliary parameter is not known and the auxiliary information can be collected without any non-response, then, we can simply make use of the estimates based on the sample and the subsequent sub-sample following the philosophy of Hansen and Hurwitz, which is itself double sampling see Lohr (1999) instead of using “double sampling” as envisaged by Okafor and Lee (2000).

The following cost efficient adaptations of commonly used estimators under non-response were proposed by Bhushan and Naqvi (2015)

$$t_1 = \frac{\bar{y}^*}{\bar{x}^*}\bar{x} \quad (7.1.18)$$

$$t_2 = \frac{\bar{y}^*}{\bar{x}}\bar{x}^* \quad (7.1.19)$$

$$t_3 = \bar{y}^*\left(\frac{\bar{x}^*}{\bar{x}}\right)^{\sim} \quad (7.1.20)$$

$$t_4 = \bar{y}^* + d\left(\bar{x}^* - \bar{x}\right) \qquad (7.1.21)$$

where ~ , d are characterizing scalars chosen suitably. In this chapter, we have tried to study their generalized versions under measurement errors.

7.2 SUGGESTED CLASSES

Our main objective here is to propose couple of generalised cost efficient classes of estimators without using conventional double sampling procedures when the auxiliary information about the population mean is not known. These sampling strategies are put to test against "double sampling" estimators as envisaged by various authors including Khare and Srivastava (1993, 1995), Okafor and Lee (2000), Singh and Kumar (2010), Singh and Bhushan (2012)among others.

In this chapter, following Diana and Tommasi (2003), we propose to use the HH technique for sub-sampling non-respondents and we propose to use the following classes of estimators given by

$$\bar{y}_g^* = \bar{y}^* g\left(\bar{x}^*, \bar{x}\right) \qquad (7.2.1)$$

$$\bar{y}_G^* = G\left(\bar{y}^*, \bar{x}^*, \bar{x}\right) \qquad (7.2.2)$$

where $g(.)$ and $G(.)$ are the bounded functions satisfying the following regularity conditions such that

(i) $g(\mathbf{Q}) = 1$ and $G(\mathbf{P}) = \bar{Y}$ (3.2.3)

(ii) first order partial derivative of $G\left(\bar{y}^*, \bar{x}, \bar{x}\right)$ with respect to $\bar{y}^*$ at $\mathbf{P} \equiv \left(\bar{Y}, \bar{X}, \bar{X}\right)$ is unity, that is,

$$G_0 = 1 \qquad (7.2.4)$$

(iii) first order partial derivatives of $g\left(\bar{x}^{*},\bar{x}\right)$ at $\mathbf{Q}\equiv\left(\bar{X},\bar{X}\right)$ and of $G\left(\bar{y}^{*},\bar{x},\bar{x}\right)$ at $\mathbf{P}\equiv\left(\bar{Y},\bar{X},\bar{X}\right)$ with respect to $\bar{x}^{*}$ and $\bar{x}$ respectively satisfy

$$g_1=-g_2 \text{ and } G_1=-G_2 \tag{7.2.5}$$

(iv) second order partial derivatives of $G\left(\bar{y}^{*},\bar{x},\bar{x}\right)$ at $\mathbf{P}\equiv\left(\bar{Y},\bar{X},\bar{X}\right)$ with respect to $\left(\bar{y}^{*},\bar{x}^{*}\right)$ and $\left(\bar{y}^{*},\bar{x}\right)$ respectively satisfy

$$G_{01}=-G_{02} \tag{7.2.6}$$

It may be noted here that $\bar{y}_G^{*}$ is an extended class of estimator and $\bar{y}_g^{*}$ is also a subclass of this wider class.

Theorem 7.2.1: To the first order of approximation

(i) The bias of the proposed class of estimators is given by

$$Bias\left(\bar{y}_G^{*}\right)=\left[\frac{(N-n)}{Nn}\left\{\frac{1}{2}\left(G_{11}+G_{22}+2G_{12}\right)\left(\pm_x^2+\pm_v^2\right)\right\}+\frac{(k-1)W_2}{n}\left\{\pm_{yx_{(2)}}G_{01}\right.\right.$$
$$\left.\left.+\frac{\bar{X}^2}{2}C_{x_{(2)}}^2G_{11}\right\}\right] \tag{7.2.7}$$

(ii) the MSE's of the proposed class of estimators is given by

$$MSE\left(\bar{y}_G^{*}\right)=MSE\left(\bar{y}^{*}\right)+\frac{(k-1)W_2}{n}\left[\left(\pm_{x_{(2)}}^2+\pm_{v_{(2)}}^2\right)G_1^2+2\pm_{yx_{(2)}}G_1\right] \tag{7.2.8}$$

Corollary 7.2.2: To the first order of approximation

(i) The bias of the proposed class of estimators is given by

$$Bias\left(\bar{y}_g^{*}\right)=\left[\frac{(N-n)}{Nn}\frac{1}{2}\left(g_{11}+g_{22}+2g_{12}\right)\left(\pm_x^2+\pm_v^2\right)\right.$$
$$\left.+\frac{(k-1)W_2}{n}\left\{\pm_{yx_{(2)}}g_1+\frac{1}{2}\left(\pm_{x_{(2)}}^2+\pm_{v_{(2)}}^2\right)g_2\right\}\right] \tag{7.2.9}$$

(ii) the MSE's of the proposed class of estimators is given by

$$MSE\left(\bar{y}_g^*\right) = MSE\left(\bar{y}^*\right) + \frac{(k-1)W_2}{n}\left[\bar{Y}^2\left(\pm_{x_{(2)}}^2 + \pm_{v_{(2)}}^2\right)g_1^2 + 2\bar{Y}\pm_{yx_{(2)}} g_1\right] \qquad (7.2.10)$$

Theorem 7.2.3: To the first order of approximation optimum value of the derivatives g and G are given as

$$g_{(opt)} = -\frac{\pm_{yx_{(2)}}}{\bar{Y}\left(\pm_{x_{(2)}}^2 + \pm_{v_{(2)}}^2\right)} = -d \qquad (7.2.11)$$

$$G_{1_{(opt)}} = -\frac{\pm_{yx_{(2)}}}{\left(\pm_{x_{(2)}}^2 + \pm_{v_{(2)}}^2\right)} = -D \qquad (7.2.12)$$

and minimum MSE's are

$$MSE\left(\bar{y}_g^*\right)_{\min} = MSE\left(\bar{y}^*\right) - \frac{(k-1)W_2}{n}\frac{\pm_{yx_{(2)}}^2}{\left(\pm_{x_{(2)}}^2 + \pm_{v_{(2)}}^2\right)} \qquad (7.2.13)$$

$$MSE\left(\bar{y}_G^*\right)_{\min} = MSE\left(\bar{y}^*\right) - \frac{(k-1)W_2}{n}\frac{\pm_{yx_{(2)}}^2}{\left(\pm_{x_{(2)}}^2 + \pm_{v_{(2)}}^2\right)} \qquad (7.2.14)$$

It may be noted that the same inequality holds for the class $\bar{y}_G^*$ and the minimum MSE of the two classes of estimators is same at that of (7.2.14). It can be easily seen that the proposed generalized estimators are always better than the HH estimator under optimum conditions.

7.3 DETERMINATION OF OPTIMUM VALUE OF *n* AND *k*

Let us consider a cost function for $\bar{y}_G^*$ as

$$C = cn + c_1 n_1 + c_2 r \qquad (7.3.1)$$

where c is the cost per unit of the first attempt with the sample, n; c_1 is the cost per unit for processing the respondent data at the first attempt in n_1 and c_2 is the cost per unit associated with the subsample, r of n_2.

Since the values of n_1 and r is not known until the first attempt is made, so the expected cost will be used in planning the survey. The expected values of n_1 and r are $W_1 n$ and $\frac{W_2 n}{k}$. Thus the expected cost is given by

$$E\left(C\right) = C^* = n\left[c + c_1 W_1 + \frac{c_2 W_2}{k}\right] \tag{7.3.2}$$

CASEI: For Fixed Precision

Theorem 7.3.1: The optimum values of n and k that minimize the cost for fixed variance V_0 is given by

$$k_{opt} = \sqrt{\frac{c_2\left(U_1 - W_2 U_2\right)}{\left(c + c_1 W_1\right) U_2}} \tag{7.3.1.1}$$

and

$$n_{opt} = \frac{\left\{U_1 + \left(k_{opt} - 1\right) W_2 U_2\right\}}{\left\{\frac{V_0}{\bar{Y}^2} + \frac{U_1}{N}\right\}} \tag{7.3.1.2}$$

The minimum cost is given by

$$C^* = \frac{1}{V_0}\left[\left(\pm_y^2 + \pm_u^2\right) + (k_{opt} - 1) W_2 \left\{\left(\pm_{y(2)}^2 + \pm_{u(2)}^2\right) + \left(\pm_{x(2)}^2 + \pm_{v(2)}^2\right) G_1^2 + 2\pm_{yx(2)} G_1\right\}\right]\left[c + c_1 W_1 + \frac{c_2 W_2}{k_{opt}}\right] \tag{7.3.1.3}$$

CASE II: For Fixed Cost

Let C_0 be the total cost (fixed) of the survey apart from overhead cost and we wish to determine minimum MSE for a fixed cost such that $C^* < C_0$.

The MSE of $\bar{y}_G^*$ can be expressed as

$$MSE\left(\bar{y}_G^*\right) = \bar{Y}^2\left[\frac{U_1}{n} + \frac{k W_2 U_2}{n} - \frac{W_2 U_2}{n} - \frac{U_1}{N}\right] \tag{7.3.6}$$

where $U_1 = \left\{\pm_y^2 + \pm_u^2\right\}$ and $U_2 = \left\{\left(\pm_{y(2)}^2 + \pm_{u(2)}^2\right) + \left(\pm_{x(2)}^2 + \pm_{v(2)}^2\right)G_1^2 + 2\pm_{yx(2)} G_1\right\}$

Theorem 7.3.2: The optimum values of n and k that minimize the $MSE(\bar{y}_G^*)$ for a fixed cost $(C^* < C_0)$ are given by

$$k_{opt} = \sqrt{\frac{c_2\left(U_1 - W_2 U_2\right)}{\left(c + c_1 W_1\right) U_2}} \qquad (7.3.2.1)$$

and

$$n_{opt} = \frac{C_0}{\left\{c + c_1 W_1 + \frac{c_2 W_2}{k_{opt}}\right\}} \qquad (7.3.2.2)$$

The minimum mean square error of $\bar{y}_G^*$ for fixed cost $(C^* < C_0)$ is given by

$$C^* = \frac{1}{C_0}\left[\left(\pm_y^2 + \pm_u^2\right) + (k_{opt} - 1)W_2\left\{\left(\pm_{y(2)}^2 + \pm_{u(2)}^2\right) + \left(\pm_{x(2)}^2 + \pm_{v(2)}^2\right)G_1^2 + 2\pm_{yx(2)} G_1\right\}\right]\left[c + c_1 W_1 + \frac{c_2 W_2}{k_{opt}}\right]$$

(7.3.2.4)

A similar result for $\bar{y}_g^*$ can also be easily obtained.

7.4 AN EMPIRICAL STUDY

The present data belong to the data on physical growth of upper socio-economic group of 95 school going children of Varanasi under an ICMR study, Department of Pediatrics, BHU during 1983-84 has been taken under study, Khare and Sinha (2007). The first 25% (i.e. 24 children) units have been considered as non-response units. The values of parameters related to the study characters y (weight of children in kg), the auxiliary character x (chest circumference of the children in cm) have been given as follows:

$\bar{Y}_2 = 19.4968;\ \bar{X} = 55.8611;\ \pm_y = 3.0435;\ \pm_x = 3.2735;\ \pm_{y(2)} = 02.3552;\ \pm_{x(2)} = 2.5137;$

$\pm_{yx} = 8.428611;\ \pm_{yx(2)} = 4.315874.$

The problem considered is to estimate the weight of the male children aged 6-7 years using chest circumference as the auxiliary character.

Table 7.4.1 *Expected cost and cost efficiency*

Estimator	% ME	k_{opt}	n	n'	Cost	Cost Efficiency
$\overline{y}^*$	0%	1.6445	24.5947	-	1148.1830	1.0000
	1%	1.6445	27.0205	-	1098.6300	0.9900
	5%	1.6445	19.6512	-	1205.5930	0.9524
	10%	1.6445	31.0631	-	1263.0002	0.9091
	15%	1.6445	32.4751	-	1320.4110	0.8695
	20%	1.6445	33.8871	-	1377.820	0.8333
t_1	% ME	k_{opt}	n	n'	Cost	Cost Efficiency
	0%	2.2063	28.5272	-	1092.0424	1.0505
	1%	2.1954	27.2958	-	1046.697	1.0392
	5%	2.1552	29.9514	-	1152.383	0.9963
	10%	2.1115	31.3730	-	1211.666	0.9476
	15%	2.0734	32.7924	-	1270.792	0.9035
	20%	2.0401	34.2100	-	1329.814	0.8634
t_2	% ME	k_{opt}	n	n'	Cost	Cost Efficiency
	0%	0.4998	17.2517	-	1065.674	1.0774
	1%	0.5051	16.6528	-	1023.317	1.0629
	5%	0.5258	18.8739	-	1137.566	1.0093
	10%	0.5499	204756	-	1208.164	0.9503
	15%	0.5725	22.0600	-	1277.676	0.8986
	20%	0.5937	23.6297	-	1346.261	0.8529
t_3	% ME	k_{opt}	n	n'	Cost	Cost Efficiency
	0%	2.5137	26.6865	-	1067.6250	1.0754
	1%	2.4840	28.7824	-	1080.6160	1.0625
	5%	2.3812	29.9408	-	1131.996	1.0143
	10%	2.2804	31.3778	-	1195.155	0.9607
	15%	2.2012	32.8062	-	1257.405	0.9131
	20%	2.1374	34.2285	-	1318.955	0.8705
t_4	% ME	k_{opt}	n	n'	Cost	Cost Efficiency
	0%	2.5137	26.6865	-	1067.6250	1.0754
	1%	2.4840	28.7824	-	1080.6160	1.0625
	5%	2.3812	29.9408	-	1131.996	1.0143
	10%	2.2804	31.3778	-	1195.155	0.9607
	15%	2.2012	32.8062	-	1257.405	0.9131
	20%	2.1374	34.2285	-	1318.955	0.8705

Estimator	% ME	k_{opt}	n	n'	Cost	Cost Efficiency
T_1	0%	1.4912	21.3565	37.6974	1340.953	0.8562
	1%	1.4942	21.7521	37.9019	1359.415	0.8446
	5%	1.5053	23.3229	38.6823	1432.356	0.8016
	10%	1.5171	25.2623	39.5806	1521.664	0.7545
	15%	1.5271	27.1779	40.4025	1609.122	0.7135
	20%	1.5357	29.0724	41.1557	1697.924	0.6774
T_3	% ME	k_{opt}	n	n'	Cost	Cost Efficiency
	0%	1.0628	19.9108	38.057	1366.704	0.8401
	1%	1.07219	20.3264	38.2659	1385.657	0.8286
	5%	1.1072	21.9708	39.0576	1460.283	0.7863
	10%	1.1461	23.9912	39.9628	1551.187	0.7402
	15%	1.1805	25.9781	40.7858	1639.805	0.7002
	20%	1.2111	27.9360	41.5365	1726.433	0.7764
T_5	% ME	k_{opt}	n	n'	Cost	Cost Efficiency
	0%	1.0427	13.6709	41.2635	1123.736	1.0217
	1%	1.0677	14.3120	41.6246	1152.685	0.9961
	5%	1.1542	16.7899	42.8599	1262.733	0.9093
	10%	1.2400	19.7347	44.0244	1390.024	0.8260
	15%	1.3087	225510	44.5879	1508.540	0.7611
	20%	1.8635	25.2655	45.4202	1619.988	0.7087
T_7	% ME	k_{opt}	n	n'	Cost	Cost Efficiency
	0%	1.2047	13.1686	39.0374	1048.192	1.0377
	1%	1.2259	14.5716	41.5669	1137.136	1.0097
	5%	1.2947	17.1215	42.7793	1252.140	0.9169
	10%	1.3561	20.0676	43.9132	1382.226	0.8307
	15%	1.4008	22.8108	44.7573	1501.143	0.7649
	20%	1.4346	25.3957	45.3982	1611.594	0.7124
T_9	% ME	k_{opt}	n	n'	Cost	Cost Efficiency
	0%	1.2046	13.9002	41.2062	1106.424	1.0340
	1%	1.2277	14.6684	41.5553	1140.9660	1.0063
	5%	1.2958	17.2090	42.7512	1255.408	0.9146
	10%	1.3567	20.1461	43.87050	1384.959	0.8290
	15%	1.4011	22.8820	44.7041	1503.458	0.7637
	20%	1.4348	25.4608	45.3371	1613.567	0.7116

Estimator	% ME	k_{opt}	n	n'	Cost	Cost Efficiency
T_{10}	0%	0.6510	10.2888	38.6349	1026.685	1.0595
	1%	0.6846	11.6723	41.2631	1120.574	1.0246
	5%	0.7974	14.6572	42.7939	1252.998	0.9163
	10%	0.9053	17.9767	44.1272	1395.733	0.8226
	15%	0.9899	209887	45.0692	1522.139	0.7573
	20%	1.0586	23.7788	45.7578	1637.226	0.7013
T_8	% ME	k_{opt}	n	n'	Cost	Cost Efficiency
	0%	1.2047	13.1686	39.0374	1048.192	1.0377
	1%	1.2259	14.5716	41.5669	1137.1360	1.0097
	5%	1.2947	17.1215	42.7793	1252.140	0.9169
	10%	1.3562	200676	43.9132	1382.226	0.8307
	15%	1.4008	22.8108	44.7573	1501.143	0.7649
	20%	1.4346	25.3957	45.3982	1611.594	0.7125
$\bar{y}_g^*$	% ME	k_{opt}	n	n'	Cost	Cost Efficiency
	0%	2.5137	26.6865	-	1067.6250	1.0754
	1%	2.4840	28.7824	-	1080.6160	1.0625
	5%	2.3812	29.9408	-	1131.996	1.0143
	10%	2.2804	31.3778	-	1195.155	0.9607
	15%	2.2012	32.8062	-	1257.405	0.9131
	20%	2.1374	34.2285	-	1318.955	0.8705
$\bar{y}_G^*$	% ME	k_{opt}	n	n'	Cost	Cost Efficiency
	0%	2.5137	26.6865	-	1067.6250	1.0754
	1%	2.4840	28.7824	-	1080.6160	1.0625
	5%	2.3812	29.9408	-	1131.996	1.0143
	10%	2.2804	31.3778	-	1195.155	0.9607
	15%	2.2012	32.8062	-	1257.405	0.9131
	20%	2.1374	34.2285	-	1318.955	0.8705

Table 7.4.2 *PRE & MSE under measurement error & non-response*

↓ Estimators		$1/k$			
	↓ME %	1/2	1/3	1/4	1/5
$\bar{y}^*$	0%	0.2904(100)	0.3304(100)	0.3705(100)	0.4105(100)
	1%	0.2933(99)	0.3337(99)	0.3742(99)	0.4146(99)
	5%	0.3049(95)	0.3469(95)	0.3889(95)	0.4310(95)
	10%	0.3194(91)	0.3635(91)	0.4075(91)	0.4516(91)
	15%	0.3339(87)	0.3799(87)	0.4260(87)	0.4721(87)
	20%	0.3485(83)	0.3965(83)	0.4446(83)	0.4926(83)
t_1	↓ME %	1/2	1/3	1/4	1/5
	0%	0.2742(106)	0.2980(111)	0.3219(115)	0.3457(119)
	1%	0.2771(105)	0.3014(110)	0.3257(114)	0.3501(117)
	5%	0.2889(100)	0.3151(105)	0.3413(109)	0.3674(112)
	10%	0.3038(96)	0.3322(99)	0.3606(103)	0.3890(106)
	15%	0.3186(91)	0.3493(95)	0.3799(98)	0.4106(100)
	20%	0.3334(87)	0.3664(90)	0.3993(93)	0.4323(95)
t_2	↓ME %	1/2	1/3	1/4	1/5
	0%	0.4247(68)	0.5990(55)	0.7734(48)	0.9477(43)
	1%	0.4276(68)	0.6024(55)	0.7772(48)	0.9520(43)
	5%	0.4395(66)	0.6161(54)	0.7927(47)	0.9693(42)
	10%	0.4543(64)	0.6332(52)	0.8121(46)	0.9909(41)
	15%	0.4691(62)	0.6502(51)	0.8314(45)	1.0126(41)
	20%	0.4839(60)	0.6673(50)	0.8508(44)	1.034(40)
t_3	↓ME %	1/2	1/3	1/4	1/5
	0%	0.2691(108)	0.2879(115)	0.3066(121)	0.3254(126)
	1%	0.2722(107)	0.2916(113)	0.3109(119)	0.3303(124)
	5%	0.2846(102)	0.3064(108)	0.3282(113)	0.3499(117)
	10%	0.3001(97)	0.3248(102)	0.3495(106)	0.3742(110)
	15%	03154(92)	0.3429(96)	0.3705(100)	0.3981(103)
	20%	0.3307(88)	0.3610(92)	0.3914(95)	0.4217(97)
t_4	↓ME %	1/2	1/3	1/4	1/5
	0%	0.2691(108)	0.2879(115)	0.3066(121)	0.3254(126)
	1%	0.2722(107)	0.2916(113)	0.3109(119)	0.3303(124)
	5%	0.2846(102)	0.3064(108)	0.3282(113)	0.3499(117)
	10%	0.3001(97)	0.3248(102)	0.3495(106)	0.3742(110)
	15%	03154(92)	0.3429(96)	0.3705(100)	0.3981(103)
	20%	0.3307(88)	0.3610(92)	0.3914(95)	0.4217(97)

	↓ME %	1/2	1/3	1/4	1/5
T_1	0%	0.2159(135)	0.2397(138)	0.2636(141)	0.2874(143)
	1%	0.2189(133)	0.2432(136)	0.2676(138)	0.2919(141)
	5%	0.2315(125)	0.2576(128)	0.2838(131)	0.3099(132)
	10%	02471(118)	0.2755(120)	0.3039(122)	0.3323(124)
	15%	0.2627(110)	0.2934(113)	0.3241(114)	0.3548(116)
	20%	0.2784(104)	0.3113(106)	0.3443(108)	0.3773(109)
T_3	↓ME %	1/2	1/3	1/4	1/5
	0%	0.2321(125)	0.2721(121)	0.3121(119)	0.3522(117)
	1%	0.2351(124)	0.2756(120)	0.3160(117)	0.3564(115)
	5%	0.2474(117)	0.2894(114)	0.3315(112)	0.3735(110)
	10%	0.2628(111)	0.3068(108)	0.3508(106)	0.3949(104)
	15%	0.2781(104)	0.3241(102)	0.3702(100)	0.4162(99)
	20%	0.2935(99)	0.3415(97)	0.3895(95)	0.4376(94)
T_5	↓ME %	½	1/3	1/4	1/5
	0%	0.1908(152)	0.2146(154)	0.2385(155)	0.2623(156)
	1%	0.1944(149)	0.2187(151)	0.2430(152)	0.2673(154)
	5%	0.2089(139)	0.2350(141)	0.2612(142)	0.2873(143)
	10%	0.2270(128)	0.2555(129)	0.2839(130)	0.3123(131)
	15%	0.2452(118)	0.2758(120)	0.3065(121)	0.3372(122)
	20%	0.2633(110)	0.2962(111)	0.3292(112)	0.3622(111)
T_7	↓ME %	1/2	1/3	1/4	1/5
	0%	0.1846(157)	0.2034(162)	0.2222(167)	0.2409(170)
	1%	0.1886(154)	0.2080(159)	0.2273(163)	0.2467(166)
	5%	0.2042(142)	0.2260(146)	0.2477(150)	0.2695(152)
	10%	0.2233(130)	0.2480(133)	0.2727(136)	0.2974(138)
	15%	0.2419(120)	0.2695(123)	0.2971(125)	0.3246(126)
	20%	0.2603(111)	0.2906(114)	0.3209(115)	0.3513(117)
T_9	↓ME %	½	1/3	1/4	1/5
	0%	0.1851(157)	0.2040(162)	0.2229(166)	0.2418(170)
	1%	0.1891(154)	0.2086(158)	0.2281(162)	0.2475(166)
	5%	0.2047(142)	0.2266(146)	0.2484(149)	0.2703(152)
	10%	0.2238(130)	0.2486(133)	0.2734(136)	0.2982(138)
	15%	0.2424(120)	0.2701(122)	0.2977(126)	0.3254(126)
	20%	0.2607(111)	0.2912(113)	0.3216(115)	0.3519(116)

	↓ME %	1/2	1/3	1/4	1/5
T_{10}	0%	0.2059(141)	0.2460(134)	0.2860(130)	0.3260(126)
	1%	0.2096(139)	0.2501(132)	0.2905(128)	0.3309(124)
	5%	0.2245(129)	0.2665(124)	0.3085(120)	0.3506(117)
	10%	0.2426(120)	0.2867(115)	0.3307(112)	0.3748(110)
	15%	0.2605(111)	0.3065(108)	0.3526(105)	0.3986(103)
	20%	0.2781(104)	0.3261(101)	0.3742(99)	0.4222(97)
T_8	↓ME %	1/2	1/3	1/4	1/5
	0%	0.1846(157)	0.2034(162)	0.2222(167)	0.2409(170)
	1%	0.1886(154)	0.2080(159)	0.2273(163)	0.2467(166)
	5%	0.2042(142)	0.2259(146)	0.2477(150)	0.2695(152)
	10%	0.2233(130)	0.2479(133)	0.2727(136)	0.2974(138)
	15%	0.2419(120)	0.2695(123)	0.2971(125)	0.3246(126)
	20%	0.2603(111)	0.2906(113)	0.3209(115)	0.3513(117)
$\overline{y}_g^*$	↓ME %	1/2	1/3	1/4	1/5
	0%	0.2691(108)	0.2879(115)	0.3066(121)	0.3254(126)
	1%	0.2722(107)	0.2916(113)	0.3109(119)	0.3303(124)
	5%	0.2846(102)	0.3064(108)	0.3282(113)	0.3499(117)
	10%	0.3001(97)	0.3248(102)	0.3495(106)	0.3742(110)
	15%	03154(92)	0.3429(96)	0.3705(100)	0.3981(103)
	20%	0.3307(88)	0.3610(92)	0.3914(95)	0.4217(97)
$\overline{y}_G^*$	↓ME %	1/2	1/3	1/4	1/5
	0%	0.2691(108)	0.2879(115)	0.3066(121)	0.3254(126)
	1%	0.2722(107)	0.2916(113)	0.3109(119)	0.3303(124)
	5%	0.2846(102)	0.3064(108)	0.3282(113)	0.3499(117)
	10%	0.3001(97)	0.3248(102)	0.3495(106)	0.3742(110)
	15%	03154(92)	0.3429(96)	0.3705(100)	0.3981(103)
	20%	0.3307(88)	0.3610(92)	0.3914(95)	0.4217(97)

Table 7.4.5 Optimum MSE(PRE) at different label of measurement error.

Estimators ↓	MSE(PRE)					
Per cent of ME →	0%	1%	5%	10%	15%	20%
$\bar{y}^*$	0.3758(100)	0.3795(99)	0.3946(95)	0.4133(91)	0.4321(87)	0.4509(83)
t_1	0.3577(105)	0.3616(104)	0.3771(100)	0.3965(95)	0.4159(90)	0.4352(86)
t_2	0.3488(108)	0.3535(106)	0.3723(101)	0.3954(95)	0.4181(90)	0.4406(85)
t_3	0.3494(108)	0.3536(106)	0.3705(101)	0.3911(96)	0.4115(91)	0.4317(87)
t_4	0.3494(108)	0.3536(106)	0.3705(101)	0.3911(96)	0.4115(91)	0.4317(87)
T_1	0.4388(86)	0.4449(84)	0.4688(80)	0.4979(75)	0.5266(71)	0.5547(68)
T_3	0.4473(84)	0.4535(83)	0.4779(79)	0.5077(74)	0.5367(70)	0.5650(67)
T_5	0.3678(102)	0.3772(100)	0.4132(91)	0.4549(83)	0.4937(76)	0.5302(71)
T_7	0.3621(104)	0.3722(101)	0.4098(92)	0.4524(83)	0.4913(76)	0.5274(71)
T_9	0.3636(103)	0.3736(101)	0.4110 (91)	0.4534(83)	0.4921(76)	0.5282(71)
T_{10}	0.3546(106)	0.3667(102)	0.4101(92)	0.4568(82)	0.4982(75)	0.5358(70)
T_8	0.3621(104)	0.3721(101)	0.4098(92)	0.4524(83)	0.4913(76)	0.5274(71)
$\bar{y}_g^*$	0.3494(108)	0.3536(106)	0.3705(101)	0.3911(96)	0.411591)	0.4317(87)
$\bar{y}_G^*$	0.3494(108)	0.3536(106)	0.3705(101)	0.3911(96)	0.411591)	0.4317(87)

Table 7.4.3 MSE(PRE) without measurement error.

Estimators	1/2	1/3	1/4	1/5
$\bar{y}^*$	0.2904(100)	0.3304(100)	0.3705(100)	0.4105(100)
t_1	0.2742(106)	0.2980(110)	0.3219(114)	0.3457(118)
t_2	0.4247(68)	0.5990(55)	0.7733(48)	0.9477(43)
t_3	0.2690(108)	0.2879(115)	0.3066(121)	0.3254(126)
t_4	0.2690(108)	0.2879(115)	0.3066(121)	0.3254(126)
T_1	0.2159(134)	0.2397(138)	0.2636(141)	0.2874(143)
T_3	0.2320(125)	0.2721(121)	0.3121(119)	0.3522(117)
T_5	0.1908(152)	0.2146(154)	0.2385(155)	0.2623(156)
T_7	0.1846(157)	0.2034(162)	0.2221(167)	0.2409(170)
T_9	0.1851(157)	0.2040(162)	0.2229(166)	0.2418(170)
T_{10}	0.2059(141)	0.2459(134)	0.2859(130)	0.3260(126)
T_8	0.1846(157)	0.2034(162)	0.2222(167)	0.2409(170)
T_g	0.1846(157)	0.2034(162)	0.2222(167)	0.2409(170)
T_G	0.1846(157)	0.2034(162)	0.2222(167)	0.2409(170)
$\bar{y}_g^*$	0.2690(108)	0.2879(115)	0.3066(121)	0.3254(126)
$\bar{y}_G^*$	0.2690(108)	0.2879(115)	0.3066(121)	0.3254(126)

7.5 CONCLUDING REMARKS

1. It is evident from the expressions (7.2.8) and (7.2.10) of MSE's of the estimators that the measurement errors seem to have inflated the MSE of these estimators and thereby decreasing the efficiency.

2. The expressions (7.2.8) and (7.2.10) of MSE can be broken into 4 major components owing to non-response and measurement error are given below:

$$MSE = A + B + C + D$$

where A = Component of MSE due to sampling error without measurement error and non-response,

B = Component of MSE due to sampling error with measurement error and without non-response,

C = Component of MSE due to sampling error without measurement error and with non-response, and

D = Component of MSE due to sampling error with measurement error and without non-response. For Example: Consider the expression of MSE of $\bar{y}_G^*$ given by

$$MSE(\bar{y}_G^*) = \underbrace{\frac{1}{n}\pm_y^2}_{A} + \underbrace{\frac{1}{n}\pm_u^2}_{B} + \underbrace{\frac{W_2(k-1)}{n}\left(\pm_{y_{(2)}}^2 + G_1^2\pm_{x_{(2)}}^2 + 2G_1\pm_{xy_{(2)}}\right)}_{C} + \underbrace{\frac{W_2(k-1)}{n}\left(\pm_{u_{(2)}}^2 + G_1^2\pm_{v_{(2)}}^2\right)}_{D}$$

3. If the measurement error is absent then we get the expression of the MSE of conventional estimators under non-response from the results of this study thereby the present study provides a more general and pragmatic approach for the estimation of population mean. For example: When u_i = 0 = v_i, for each i, then $\pm_u^2 = \pm_v^2 = \pm_{u_2}^2 = \pm_{v_2}^2 = 0$ and we get

$$MSE(\bar{y}_G^*) = \underbrace{\frac{1}{n}\pm_y^2}_{A} + \underbrace{\frac{W_2(k-1)}{n}\left(\pm_{y_{(2)}}^2 + G_1^2 \pm_{x_{(2)}}^2 + 2G_1 \pm_{xy_{(2)}}\right)}_{C}$$

so that only components A and C are left, which is same expression as proposed by Bhushan and Naqvi (2015) while proposing $\bar{y}_G^*$.

4. If the measurement error is absent then we get the expression of the optimum values of the characterizing scalars and minimum MSE of conventional estimators under non-response from the results of this study thereby the present study. For example: When u_i = 0 = v_i, for each i, then $\pm_u^2 = \pm_v^2 = \pm_{u_2}^2 = \pm_{v_2}^2 = 0$ and we get optimum value of G_1 as

$$G_1 = -\pm_{xy_{(2)}} / \pm_{x_{(2)}}^2$$

and the minimum mean square error of $\bar{y}_G^*$ as

$$MSE(\bar{y}_G^*)_{\min} = \frac{1}{n}\pm_y^2 + \frac{W_2(k-1)}{n}\left(1 - \circ_{(2)}^2\right)\pm_{y_{(2)}}^2$$

which is same expression of minimum MSE as proposed by Bhushan and Naqvi (2015) while proposing $\bar{y}_G^*$.

5. The measurement error seems to have affected all the estimators but the cost efficient estimators $\bar{y}_g^*$ and $\bar{y}_G^*$ perform far better than the remaining double sampling estimators. For example: This is w.r.t. the empirical results in table 7.4.1 where $\bar{y}_G^*$ utilized the auxiliary information optimally and outperformed all the remaining estimators when the cost was also taken into account. The expected cost incurred in the estimators $\bar{y}_g^*$ and $\bar{y}_G^*$ is less than as compared to the expected cost incurred for all the other proposed estimators and the conventional double sampling estimators proposed by Singh and Kumar (2010), Tabassum and Khan (2004) and Okafor and Lee (2000) considered under optimum conditions.

6. The measurement error seems to have affected the double sampling estimators more where the auxiliary information was properly utilized than the cost efficient estimators under optimal conditions. For example: This is w.r.t. the empirical results in table 7.4.1 where the double sampling estimator T_G lost 32% cost efficiency which is far more in comparison to $\bar{y}_G^*$, cost efficient estimator the auxiliary information was not properly utilized, lost only 20% cost efficiency or even $\bar{y}*$ which lost 17% cost efficiency.

7. The measurement error seems to have affected the double sampling estimators more in terms of efficiency where the auxiliary information was properly utilized than the suggested cost efficient estimators. For example: This is w.r.t. the empirical results in table 7.4.2 where double sampling estimator T_G utilized the auxiliary information optimally and has lost 53% efficiency with $1/5$ sub-sampling fraction which is far more in comparison

to $\bar{y}_G^*$, cost efficient estimator lost only 23% efficiency with 1/5 sub-sampling fraction or even $\bar{y}*$ which lost 17% efficiency with 1/5 sub-sampling fraction.

8. The percent relative efficiency of various estimators were calculated w. r. t. $\bar{y}*$ with optimum sub-sampling fraction $1/k$ in table 7.4.3. The best choice, which is in consonance with the theoretical results, is $\bar{y}_g^*$ and $\bar{y}_G^*$ at all levels of measurement error. Further, the performance of the estimators decreases with the increasing level of measurement error. The cost efficient estimators retain more than 100% efficiency even at 5% level of measurement error while the double sampling estimators retain more than 100% efficiency only till 1% level of measurement error.

9. The gains in efficiency of various estimators were calculated w. r. t. $\bar{y}*$ with no measurement error in table 7.4.4. The best choice in terms of efficiency (not cost efficiency), which is in consonance with the theoretical results, is T_g and T_G at all levels of measurement error. The table 7.4.4 also demonstrates that the standard results of non-response estimators can be obtained when there is no measurement error. The comment 5 reiterated here that estimators $\bar{y}_g^*$ and $\bar{y}_G^*$ are still the most cost efficient estimators.

CHAPTER 8

SOME GENERALIZED CLASSES OF JACK-KNIFE ESTIMATOR OF POPULATION PARAMETERS USING TWO AUXILIARY VARIABLES UNDER MEASUREMENT ERRORS

SUMMARY

In this chapter, we have proposed three generalized classes of estimators of population mean, ratio and product of two population means using auxiliary information of two variables in presence of measurement errors. Further we have also proposed the corresponding classes of unbiased estimators using the jack-knife version of Quenouille's under measurement errors. The biases and mean square error of the proposed classes are obtained. We have also analyzed the properties of the generalized estimator in presence of measurement errors. Finally, some concluding remarks are made clearly demonstrating that some important classes of estimators are special cases of the proposed study.

8.1 INTRODUCTION

Over the past several decades, statisticians are paying their attention towards the problem of estimation of parameters in the presence of measurement errors. In survey sampling, the properties of data usually presuppose that the observations are the correct measurements on characteristics being studied. However this assumption is not satisfied in many applications and data is contaminated with measurement errors, such as non-response errors, reporting errors and computing errors. These measurement errors make the result invalid, which are meant for no measurement error case. If measurement errors are very small and we can neglect it, then the statistical inference based on data observed continue to remain valid. On the contrary, when they are not appreciably small and negligible, the inferences may not be simply invalid and inaccurate but may often lead to unexpected, undesirable and unfortunate consequences (see Srivastava and Shalabh, 2001). Some important sources of measurement errors in survey data are discussed in Cochran (1968), Shalabh (1997), and Singh and Karpe (2008, 2010) studied some estimators of population mean under measurement errors.

For a simple random sample of size *n*, let $\left(x_{1j}, x_{2j}, y_j\right)$ be the pair of values instead of the true values $\left(X_{1j}, X_{2j}, Y_j\right)$ on the characteristics $\left(X_1, X_2, Y\right)$ respectively. Let the observational or measurement error be defined as

$$u_j = y_j - Y_j \qquad j=1,2,\ldots, n \tag{8.1.1}$$

$$v_{1j} = x_{1j} - X_{1j} \tag{8.1.2}$$

$$v_{2j} = x_{2j} - X_{2j} \tag{8.1.3}$$

8.2 PROPOSED GENERALIZED CLASS OF ESTIMATORS I

The generalized estimator of population mean μ_Y using means μ_1 and μ_2 of auxiliary variables X_1 and X_2 respectively, is given as

$$\bar{y}_a = f\left(\bar{y}, \frac{\bar{x}_1}{\mu_1}, \frac{\bar{x}_2}{\mu_2}\right) \tag{8.2.1}$$

satisfying the regularity condition

(i) $f\left(\mu_Y, 1, 1\right) = \mu_Y$ (8.2.2)

(ii) The function is continuous and bounded in the closed interval of real line **R**.

(iii) The first and second order partial derivatives of the function are exist and are continuous and bounded in **R**.

where $\bar{x}_1$, $\bar{x}_2$ and $\bar{y}$ are the sample mean of X_1, X_2 and Y respectively for a simple random variable of size *n*.

8.3 PROPOSED GENERALIZED CLASS OF ESTIMATORS II

The generalized estimator of population ratio $\hat{R}_g$ using estimated ratio of means (Y_1, Y_2) be $\hat{R} = \frac{\hat{\mu}_{Y_1}}{\hat{\mu}_{Y_2}}$ and μ_1 and μ_2 be the population mean of auxiliary variable X_1 and X_2 respectively, is

$$\hat{R}_g = g\left[\hat{R}, \frac{\bar{x}_1}{\mu_1}, \frac{\bar{x}_2}{\mu_2}\right] \qquad (8.3.1)$$

where $(\bar{x}_1, \bar{x}_2)$ are the sample means of X_1, X_2 and $\hat{R} = \frac{\bar{y}_1}{\bar{y}_2}$ is the estimate of $R = \frac{\mu_{Y_1}}{\mu_{Y_2}}$ respectively for a simple random variable of size *n*. Also, $g(.)$ is the function of $\frac{\bar{y}_1}{\bar{y}_2}$, $\frac{\bar{x}_1}{\mu_1}$ and $\frac{\bar{x}_2}{\mu_2}$ satisfying

(i) $g(R, 1, 1) = R$ (8.3.2)

(ii) The function is continuous and bounded in the closed interval of real line **R**.

(iii) The first and second order partial derivatives of the function are exist and are continuous and bounded in **R**.

8.4 PROPOSED GENERALIZED CLASS OF ESTIMATORS III

The generalized estimator $\hat{P}_h$ product of population means P using product of means of (Y_1, Y_2) be $\hat{P} = \hat{\mu}_{Y_1}\hat{\mu}_{Y_2}$, μ_1 and μ_2 of auxiliary variable X_1 and X_2 respectively, is

$$\hat{P}_h = h\left[\hat{P}, \frac{\overline{x}_1}{«_1}, \frac{\overline{x}_2}{«_2}\right] \qquad (8.4.1)$$

where $(\overline{x}_1, \overline{x}_2)$ are the sample means of X_1, X_2 and $\hat{P}=\overline{y}_1\overline{y}_2$ is the estimate of $P = «_{Y_1}«_{Y_2}$ respectively for a simple random sample of size n. Also, $h(.)$ is the function of $\overline{y}_1\overline{y}_2$, $\frac{\overline{x}_1}{«_1}$ and $\frac{\overline{x}_2}{«_2}$ satisfying

(i) $h(P,1,1) = P$ (8.4.2)

(ii) The function is continuous and bounded in the closed interval of real line **R**.

(iii) The first and second order partial derivatives of the function are exist and are continuous and bounded in **R**.

8.5 BIAS AND MSE OF THE PROPOSED CLASSES OF GENERALIZED ESTIMATORS

For a simple random variable of size *n*, we define the following terms given as

$$\overline{y} = \frac{1}{n}\left[\sum_{j=1}^{n} y_j\right]$$

$$= \frac{1}{n}\left[\sum_{j=1}^{n} u_j + Y_j - \mu_Y + \mu_Y\right]$$

$$= \frac{1}{\sqrt{n}}\left[W_u + W_Y\right] + \mu_Y \qquad (8.5.1)$$

where $W_u = \frac{1}{\sqrt{n}}\sum_{j=1}^{n} u_j$ and $W_Y = \frac{1}{\sqrt{n}}\sum_{j=1}^{n}(y_j - «_Y)$.

similarly,

$$\bar{x}_1 = \frac{1}{\sqrt{n}}\left[W_{v_1} + W_{x_1}\right] + «_1 \qquad (8.5.2)$$

where $W_{v_1} = \frac{1}{\sqrt{n}}\sum_{j=1}^{n} v_{1j}$ and $W_{X_1} = \frac{1}{\sqrt{n}}\sum_{j=1}^{n}(x_j - «_1)$.

and

$$\bar{x}_2 = \frac{1}{\sqrt{n}}\left[W_{v_2} + W_{x_2}\right] + «_2 \qquad (8.5.3)$$

where $W_{v_2} = \frac{1}{\sqrt{n}}\sum_{j=1}^{n} v_{2j}$ and $W_{X_2} = \frac{1}{\sqrt{n}}\sum_{j=1}^{n}(x_j - «_2)$.

Using (8.5.1), (8.5.2) and (8.5.3) in (8.2.1), we have

$$\bar{y}_a = f\left[\bar{y}, \frac{\bar{x}_1}{«_1}, \frac{\bar{x}_2}{«_2}\right]$$

$$= \Bigg[f(\mu_Y, 1, 1) + (\bar{y} - \mu_Y)\frac{\partial f}{\partial u} + \left(\frac{\bar{x}_1}{\mu_1} - 1\right)\frac{\partial f}{\partial v} + \left(\frac{\bar{x}_2}{\mu_2} - 1\right)\frac{\partial f}{\partial w}$$

$$+ \frac{1}{2!}\Bigg\{(\bar{y} - \mu_Y)^2 \frac{\partial^2 f}{\partial u^2} + \left(\frac{\bar{x}_1}{\mu_1} - 1\right)^2 \frac{\partial^2 f}{\partial v^2} + \left(\frac{\bar{x}_2}{\mu_2} - 1\right)^2 \frac{\partial^2 f}{\partial w^2} + 2(\bar{y} - \mu_Y)\left(\frac{\bar{x}_1}{\mu_1} - 1\right)\frac{\partial^2 f}{\partial u \partial v}$$

$$+ 2\left(\frac{\bar{x}_1}{\mu_1} - 1\right)\left(\frac{\bar{x}_2}{\mu_2} - 1\right)\frac{\partial^2 f}{\partial v \partial w} + 2\left(\frac{\bar{x}_2}{\mu_2} - 1\right)(\bar{y} - \mu_Y)\frac{\partial^2 f}{\partial w \partial u}\Bigg\} + \ldots \Bigg]$$

$$\bar{y}_a - «_Y = \Bigg[\frac{(W_u + W_y)}{\sqrt{n}} f_1 + \frac{(W_{v_1} + W_{x_1})}{«_1\sqrt{n}} f_2 + \frac{(W_{v_2} + W_{x_2})}{«_2\sqrt{n}} f_3 + \frac{1}{2!}\Bigg\{\frac{(W_{v_1} + W_{x_1})^2}{«_1^2 n} f_{020}$$

$$+ \frac{(W_{v_2} + W_{x_2})^2}{«_2^2 n} f_{002} + 2\frac{(W_u + W_y)(W_{v_1} + W_{x_1})}{n«_1} f_{110}$$

$$+\frac{\left(W_{v_1}+W_{x_1}\right)\left(W_{v_2}+W_{x_2}\right)}{«_1«_2} f_{011}+2\frac{\left(W_u+W_y\right)\left(W_{v_2}+W_{x_2}\right)}{«_2} f_{101}\Big\}\Big]+\ldots \tag{8.5.4}$$

On taking the expectation of (8.5.4) we get bias of $\bar{y}_a$, given by

$$Bias\left(\bar{y}_a\right)=\frac{1}{2n}\left[\frac{1}{«_1^2}\pm_{x_1}^2 f_{020}+\frac{1}{«_2^2}\pm_{x_2}^2 f_{002}+2\left\{\frac{\pm_{(y,x_1)}}{«_1} f_{100}+\frac{\pm_{(x_1,x_2)} f_{011}}{«_1«_2}+\frac{\pm_{(y,x_2)}}{«_2} f_{101}\right\}\right]$$

$$+\frac{1}{2n}\left[\frac{\pm_{v_1}^2}{«_1^2} f_{020}+\frac{\pm_{v_2}^2}{«_2^2} f_{002}\right] \tag{8.5.5}$$

Define the terms similarly as in (8.5.1), (8.5.2) and (8.5.3) and proceeding similarly, we get the biases of the estimators $\hat{R}_g$ and $\hat{P}_h$ given by

$$Bias\left(\hat{R}_g\right)=\frac{R}{n}\Bigg[\left(\frac{1}{2«_1^2}\pm_{x_1}^2 g_{030}+\frac{1}{2«_2^2}\pm_{x_2}^2 g_{003}+\frac{1}{«_R«_1«_2}\pm_{(x_1,x_2)} g_{011}\right)$$

$$+\frac{1}{«_{Y_1}}\left(\frac{1}{«_1}\pm_{(x_1,y_1)} g_{110}+\frac{1}{«_2}\pm_{(x_2,y_1)} g_{101}\right)$$

$$-\frac{1}{«_{Y_2}}\left(\frac{1}{«_1}\pm_{(x_1,y_2)} g_{110}+\frac{1}{«_2}\pm_{(x_2,y_2)} g_{101}\right)\Bigg]+\frac{R}{n}\left[\frac{1}{2«_1^2}\pm_{v_1}^2 g_{030}+\frac{1}{2«_2^2}\pm_{v_2}^2 g_{003}\right] \tag{8.5.6}$$

and,

$$Bias\left(\hat{P}_h\right)=\frac{P}{n}\Bigg[\Bigg(\frac{1}{2«_1^2}\pm_{x_1}^2 h_{030}+\frac{1}{2«_2^2}\pm_{x_2}^2 h_{003}+\frac{1}{«_P«_1«_2}\pm_{(x_1,x_2)} h_{011}$$

$$+\frac{1}{«_{Y_1}}\left(\frac{1}{«_1}\pm_{(x_1,y_1)} h_{110}+\frac{1}{«_2}\pm_{(x_2,y_1)} h_{101}\right)+\frac{1}{«_{Y_2}}\left(\frac{1}{«_1}\pm_{(x_1,y_2)} h_{110}+\frac{1}{«_2}\pm_{(x_2,y_2)} h_{101}\right)\Bigg)\Bigg]$$

$$+\frac{P}{n}\left[\frac{1}{2«_1^2}\pm_{v_1}^2 h_{030}+\frac{1}{2«_2^2}\pm_{v_2}^2 h_{003}\right] \tag{8.5.7}$$

Now, squaring (8.5.4) on both sides and taking expectation, ignoring the terms of order greater than two, we get the mean square error of $\bar{y}_a$ given by

$$MSE\left(\bar{y}_a\right)=E\left(\bar{y}_a-«_Y\right)^2$$

$$=E\left[\frac{\left(W_u+W_y\right)}{\sqrt{n}} f_1+\frac{\left(W_{v_1}+W_{x_1}\right)}{«_1\sqrt{n}} f_2+\frac{\left(W_{v_2}+W_{x_2}\right)}{«_2\sqrt{n}} f_3\right]^2$$

$$= E\left[\frac{\left(W_u + W_y\right)^2}{n} f_1^2 + \frac{\left(W_{v_1} + W_{x_1}\right)^2}{«_1^2 n} f_2^2 + \frac{\left(W_{v_2} + W_{x_2}\right)^2}{«_2^2 n} f_3^2 \right.$$

$$+2\left\{ \frac{\left(W_u + W_y\right)}{\sqrt{n}} \frac{\left(W_{v_1} + W_{x_1}\right)}{«_1 \sqrt{n}} f_1 f_2 + \frac{\left(W_{v_1} + W_{x_1}\right)}{«_1 \sqrt{n}} \frac{\left(W_{v_2} + W_{x_2}\right)}{«_2 \sqrt{n}} f_2 f_3 \right.$$

$$\left.\left. + \frac{\left(W_{v_2} + W_{x_2}\right)}{«_2 \sqrt{n}} \frac{\left(W_u + W_y\right)}{\sqrt{n}} f_3 f_1 \right\} \right]$$

$$= \left[\frac{\left(\pm_u^2 + \pm_y^2\right)}{n} f_1^2 + \frac{\left(\pm_{v_1}^2 + \pm_{x_1}^2\right)}{n «_1^2} f_2^2 + \frac{\left(\pm_{v_2}^2 + \pm_{x_2}^2\right)}{n «_2^2} f_3^2 \right]$$

$$+ \frac{2}{n}\left[\frac{\pm_{(y,x_1)}}{«_1} f_1 f_2 + \frac{\pm_{(x_1,x_2)}}{«_1 «_2} f_2 f_3 + \frac{\pm_{(y,x_2)}}{«_2} f_3 f_1 \right]$$

$$= \frac{1}{n}\left[\pm_y^2 f_1^2 + \frac{\pm_{x_1}^2}{«_1^2} f_2^2 + \frac{\pm_{x_2}^2}{«_2^2} f_3^2 + 2\left\{ \frac{\pm_{(y,x_1)}}{«_1} f_1 f_2 + \frac{\pm_{(x_1,x_2)}}{«_1 «_2} f_2 f_3 + \frac{\pm_{(y,x_2)}}{«_2} f_3 f_1 \right\} \right]$$

$$+ \frac{1}{n}\left[\pm_u^2 f_1^2 + \frac{\pm_{v_1}^2}{«_1^2} f_2^2 + \frac{\pm_{v_2}^2}{«_2^2} f_3^2 \right] \qquad (8.5.8)$$

Similarly the MSE of the estimators $\hat{R}_g$ and $\hat{P}_h$ is given by

$$MSE\left(\hat{R}_g\right) = \frac{R^2}{n}\left[\left(\frac{1}{«_{Y_1}^2} \pm_{y_1}^2 + \frac{1}{«_{Y_2}^2} \pm_{y_2}^2 - \frac{2}{«_{Y_1} «_{Y_2}} \pm_{(y_1,y_2)} \right) g_1^2 \right.$$

$$+ \left(\frac{1}{R^2 «_1^2} \pm_{x_1}^2 g_2^2 + \frac{1}{R^2 «_2^2} \pm_{x_2}^2 g_3^2 + \frac{2}{R^2 «_1 «_2} \pm_{(x_1,x_2)} g_2 g_3 \right)$$

$$\left. + \frac{2}{«_{Y_1}} \left(\frac{1}{«_1} \pm_{(x_1,y_1)} g_1 g_2 + \frac{1}{«_2} \pm_{(x_2,y_1)} g_1 g_3 \right) - \frac{2}{«_{Y_2}} \left(\frac{1}{«_1} \pm_{(x_1,y_2)} g_1 g_2 + \frac{1}{«_2} \pm_{(x_2,y_2)} g_1 g_3 \right) \right]$$

$$+ \frac{R^2}{n}\left[\left(\frac{1}{«_{Y_2}^2} \pm_{u_1}^2 + \frac{1}{«_{Y_2}^2} \pm_{u_2}^2 \right) g_1^2 + \left(\frac{1}{R^2 «_1^2} \pm_{v_1}^2 g_2^2 + \frac{1}{R^2 «_2^2} \pm_{v_2}^2 g_3^2 \right) \right] \qquad (8.5.9)$$

$$MSE\left(\hat{P}_h\right) = \frac{P^2}{n}\left[\left(\frac{1}{«_{Y_2}^2} \pm_{y_1}^2 + \frac{1}{«_{Y_2}^2} \pm_{y_2}^2 + \frac{1}{«_{Y_1} «_{Y_2}} \pm_{(y_1,y_2)} \right) h_1^2 \right.$$

$$+\left(\frac{1}{P^2\ll_1^2}\pm_{x_1}^2h_2^{\,2}+\frac{1}{P^2\ll_2^2}\pm_{x_2}^2h_3^{\,2}+\frac{2}{P\ll_1\ll_2}\pm_{(x_1,x_2)}h_2h_3\right)$$

$$+\frac{2}{\ll_{Y_1}}\left(\frac{1}{\ll_1}\pm_{(x_1,y_1)}h_1h_2+\frac{1}{\ll_2}\pm_{(x_2,y_1)}h_1h_3\right)+\frac{2}{\ll_{Y_2}}\left(\frac{1}{\ll_1}\pm_{(x_1,y_2)}h_1h_2+\frac{1}{\ll_2}\pm_{(x_2,y_2)}h_1h_3\right)\Bigg]$$

$$+\frac{P^2}{n}\left[\left(\frac{1}{\ll_{Y_2}^2}\pm_{u_1}^2+\frac{1}{\ll_{Y_2}^2}\pm_{u_2}^2\right)h_1^{\,2}+\left(\frac{1}{P^2\ll_1^2}\pm_{x_1}^2h_2^{\,2}+\frac{1}{P^2\ll_2^2}\pm_{x_2}^2h_3^{\,2}\right)\right] \qquad (8.5.10)$$

8.6 PROPOSED GENERALIZED CLASS OF UNBIASED JACK-KNIFE ESTIMATORS I

Now we consider a simple random sample of size $2m$ and split this sample randomly into two sub samples each of size m. Let $\left(\bar{y}_{2m,}\,\bar{x}_{12m},\bar{x}_{22m}\right)$ be the sample means of values on $\left(Y,X_1,X_2\right)$ respectively for the entire sample of size $2m$ and $\left(\bar{y}_m^{(i)},\bar{x}_{1,m}^{(i)},\bar{x}_{2,m}^{(i)}\right)$ be the sample means of values$\left(Y,X_1,X_2\right)$ respectively for $i^{th}\left(i=1,2\right)$ sub-sample of size m.

Let $\bar{y}_a^{(3)}$ is a generalized estimator for the entire sample of size $2m$; $\bar{y}_a^{(1)}$ and $\bar{y}_a^{(2)}$ be the generalized estimators for the two randomly split sub-samples of size m each given by

$$\bar{y}_a^{(3)}=F\left[\bar{y}_{2m},\frac{\bar{x}_{1,2m}}{\ll_1},\frac{\bar{x}_{2,2m}}{\ll_2}\right] \qquad (8.6.2)$$

$$\bar{y}_a^{(1)}=F\left[\bar{y}_m^{(1)},\frac{\bar{x}_{1,m}^{(1)}}{\ll_1},\frac{\bar{x}_{2,m}^{(1)}}{\ll_2}\right] \qquad (8.6.3)$$

$$\bar{y}_a^{(2)}=F\left[\bar{y}_m^{(1)},\frac{\bar{x}_{1,m}^{(2)}}{\ll_1},\frac{\bar{x}_{2,m}^{(2)}}{\ll_2}\right] \qquad (8.6.4)$$

respectively, where

$F\left[\bar{y}_{2m}, \frac{\bar{x}_{1,2m}}{«_1}, \frac{\bar{x}_{2,2m}}{«_2}\right]$, $F\left[\bar{y}_m^{(1)}, \frac{\bar{x}_{1,m}^{(1)}}{«_1}, \frac{\bar{x}_{2,m}^{(1)}}{«_2}\right]$ and $F\left[\bar{y}_m^{(1)}, \frac{\bar{x}_{1,m}^{(2)}}{«_1}, \frac{\bar{x}_{2,m}^{(2)}}{«_2}\right]$ are the bounded functions of $\left[\bar{y}_{2m}, \frac{\bar{x}_{1,2m}}{«_1}, \frac{\bar{x}_{2,2m}}{«_2}\right]$, $\left[\bar{y}_m^{(1)}, \frac{\bar{x}_{1,m}^{(1)}}{«_1}, \frac{\bar{x}_{2,m}^{(1)}}{«_2}\right]$ and $\left[\bar{y}_m^{(1)}, \frac{\bar{x}_{1,m}^{(2)}}{«_1}, \frac{\bar{x}_{2,m}^{(2)}}{«_2}\right]$ respectively, satisfying the regularity conditions of $f(.)$ in (8.2.1) for $F(.)$ involved in generalized estimator $\bar{y}_a^{(i)}$ where i=1, 2, 3.

On the lines of Sukhatme and Sukhatme (Chapter IV, page 162), for N large, we propose the generalized Jack-knife generalized $\bar{y}_{aj}$ under measurement error given by

$$\bar{y}_{aj} = 2\bar{y}_a^{(3)} - \frac{1}{2}\left(\bar{y}_a^{(1)} + \bar{y}_a^{(2)}\right) \tag{8.6.5}$$

8.7 PROPOSED GENERALIZED CLASS OF UNBIASED JACK-KNIFE ESTIMATORS II

We consider a simple random sample of size n=2m and split this sample randomly into two sub samples each of size m. Let $\left(\frac{\bar{y}_{1,2m,}}{\bar{y}_{2,2m}}\right)$ be the ratio sample means of values on $\left(\frac{Y_1}{Y_2}\right)$ and $\left(\bar{x}_{1,2m}, \bar{x}_{2,2m}\right)$ be the sample means of $\left(X_1, X_2\right)$ respectively for the entire sample of size $2m$ and $\left(\frac{\bar{y}_{1,m,}^{(i)}}{\bar{y}_{2,m}^{(i)}}\right)$ be the ratio of sample means of $\left(\frac{Y_1}{Y_2}\right)$ and $\left(\bar{x}_{1,2m}, \bar{x}_{2,2m}\right)$ be the sample means of $\left(X_1, X_2\right)$ respectively for $i^{th}\left(i=1,2\right)$ sub-sample of size m.

The generalized estimator $\hat{R}_G^{(3)}$ based on sample of size $2m$ of ratio of population means $\frac{\mu_{Y_1}}{\mu_{Y_2}}$ using means μ_1 and μ_2 of auxiliary variables X_1 and X_2 and $\hat{R}_G^{(1)}$ and $\hat{R}_G^{(2)}$ be the generalized estimators for the two randomly split sub-samples of size m each respectively is given by

$$\hat{R}_G^{(3)} = G\left(\hat{R}_{2m}, \frac{\overline{x}_{1,2m}}{\mu_1}, \frac{\overline{x}_{2,2m}}{\mu_2} \right) \tag{8.7.1}$$

$$\hat{R}_G^{(1)} = G\left(\hat{R}_{m}, \frac{\overline{x}_{1,m}^{(1)}}{\mu_1}, \frac{\overline{x}_{2,m}^{(1)}}{\mu_2} \right) \tag{8.7.2}$$

and

$$\hat{R}_G^{(2)} = G\left(\hat{R}_{m}, \frac{\overline{x}_{1,m}^{(2)}}{\mu_1}, \frac{\overline{x}_{2,m}^{(2)}}{\mu_2} \right) \tag{8.7.3}$$

respectively, where

$G\left[\hat{R}_{2m}, \frac{\overline{x}_{1,2m}}{\mu_1}, \frac{\overline{x}_{2,2m}}{\mu_2} \right]$, $G\left[\hat{R}_{m}^{(1)}, \frac{\overline{x}_{1,m}^{(1)}}{\mu_1}, \frac{\overline{x}_{2,m}^{(1)}}{\mu_2} \right]$ and $G\left[\hat{R}_{m}^{(2)}, \frac{\overline{x}_{1,m}^{(2)}}{\mu_1}, \frac{\overline{x}_{2,m}^{(2)}}{\mu_2} \right]$ are the bounded functions of $\left(\hat{R}_{2m}, \frac{\overline{x}_{1,2m}}{\mu_1}, \frac{\overline{x}_{2,2m}}{\mu_2} \right)$, $\left(\hat{R}_{m}^{(1)}, \frac{\overline{x}_{1,m}^{(1)}}{\mu_1}, \frac{\overline{x}_{2,m}^{(1)}}{\mu_2} \right)$ and $\left(\hat{R}_{m}^{(2)}, \frac{\overline{x}_{1,m}^{(2)}}{\mu_1}, \frac{\overline{x}_{2,m}^{(2)}}{\mu_2} \right)$ respectively satisfying the regularity conditions of $g(.)$ in (8.3.1) for $G(.)$ involved in generalized estimator $\hat{R}_G^{(i)}$ where i=1, 2, 3.

On the lines of Sukhatme and Sukhatme (Chapter IV, page 162), for N large, we propose the generalized Jack-knife estimator $\hat{R}_G$ under measurement error given by

$$\hat{R}_{Gj} = 2\hat{R}_G^{(3)} - \frac{1}{2}\left(\hat{R}_G^{(1)} + \hat{R}_G^{(2)} \right) \tag{8.7.4}$$

8.8 PROPOSED GENERALIZED CLASS OF UNBIASED JACK-KNIFE ESTIMATORS III

In this section, we consider a simple random sample of size n=2m and split this sample randomly into two sub samples each of size m. Let $\bar{y}_{1,2m}\bar{y}_{2,2m}$ be the product of sample means of values on Y_1Y_2 and $\left(\bar{x}_{1,2m},\bar{x}_{2,2m}\right)$ be the sample means of $\left(X_1,X_2\right)$ respectively for the entire sample of size $2m$ and $\bar{y}_{1,m}^{(i)}\bar{y}_{2,m}^{(i)}$ be the product of sample means of Y_1Y_2 and $\left(\bar{x}_{1,m}^{(i)},\bar{x}_{2,m}^{(i)}\right)$ be the sample means of $\left(X_1,X_2\right)$ respectively for $i^{th}\left(i=1,2\right)$ sub-sample of size m.

The generalized estimator $\hat{P}_H^{(3)}$ based on sample of size $2m$ of product of population means $\mu_{Y_1}\mu_{Y_2}$ using means μ_1 and μ_2 of auxiliary variables X_1, X_2 and $\hat{P}_H^{(1)}$, $\hat{P}_H^{(2)}$ be the generalized estimators for the two randomly split sub-samples of size m each, given by

$$\hat{P}_H^{(3)}=H\left(\hat{P}_{2m},\frac{\bar{x}_{1,2m}}{\mu_1},\frac{\bar{x}_{2,2m}}{\mu_2}\right) \tag{8.8.1}$$

and

$$\hat{P}_H^{(1)}=H\left(\hat{P}_m^{(1)},\frac{\bar{x}_{1,m}^{(1)}}{\mu_1},\frac{\bar{x}_{2,m}^{(1)}}{\mu_2}\right) \tag{8.8.2}$$

$$\hat{P}_H^{(2)}=H\left(\hat{P}_m^{(2)},\frac{\bar{x}_{1,m}^{(2)}}{\mu_1},\frac{\bar{x}_{2,m}^{(2)}}{\mu_2}\right) \tag{8.8.3}$$

respectively, where

$H\left[\hat{P}_{2m},\frac{\bar{x}_{1,2m}}{\mu_1},\frac{\bar{x}_{2,2m}}{\mu_2}\right]$, $H\left[\hat{P}_m^{(1)},\frac{\bar{x}_{1,m}^{(1)}}{\mu_1},\frac{\bar{x}_{2,m}^{(1)}}{\mu_2}\right]$ and $H\left[\hat{P}_m^{(2)},\frac{\bar{x}_{1,m}^{(2)}}{\mu_1},\frac{\bar{x}_{2,m}^{(2)}}{\mu_2}\right]$ are the bounded functions of $\left(\hat{P}_{2m},\frac{\bar{x}_{1,2m}}{\mu_1},\frac{\bar{x}_{2,2m}}{\mu_2}\right)$, $\left(\hat{P}_m^{(1)},\frac{\bar{x}_{1,m}^{(1)}}{\mu_1},\frac{\bar{x}_{2,m}^{(1)}}{\mu_2}\right)$ and $\left(\hat{P}_m^{(2)},\frac{\bar{x}_{1,m}^{(2)}}{\mu_1},\frac{\bar{x}_{2,m}^{(2)}}{\mu_2}\right)$ respectively satisfying the regularity conditions of $h(.)$ in (8.4.1) for $H(.)$ involved in generalized estimator $\hat{P}_G^{(i)}$ where i=1, 2, 3.

On the lines of Sukhatme and Sukhatme (Chapter IV, page 162), for N large, we propose the generalized Jack-knife estimator $\hat{P}_{Hj}$ under measurement error given by

$$\hat{P}_{Hj} = 2\hat{P}_H^{(3)} - \frac{1}{2}\left(\hat{P}_H^{(1)} + \hat{P}_H^{(2)}\right) \qquad (8.8.4)$$

8.9 MSE OF THE PROPOSED CLASS OF GENERALIZED UNBIASED ESTIMATORS

We can define the following terms similarly as mention in (8.5.1), (8.5.2) and (8.5.3).

$$\bar{y}_{2m} = \frac{1}{2m}\left[\sum_{j=1}^{2m} y_j\right]$$

$$= \frac{1}{(2m)^{1/2}}\left[\frac{1}{(2m)^{1/2}}\sum_{j=1}^{2m} u_j + (Y_j - «_Y) + «_Y\right]$$

$$= «_Y + \frac{1}{(2m)^{1/2}}\left(W_u + W_y\right) \qquad (8.9.1)$$

where $W_u = \frac{1}{(2m)^{1/2}}\sum_{j=1}^{2m} u_j$ and $W_Y = \frac{1}{(2m)^{1/2}}\sum_{j=1}^{2m}(Y_j - «_Y)$ are of order $O_p(1)$.

Similarly we also define W_{v_1}, W_{x_1}, W_{V_2}, W_{X_2}, $W_u^{(1)}$, $W_y^{(1)}$, $W_u^{(2)}$, $W_y^{(2)}$, $W_{v_1}^{(1)}$, $W_{x_1}^{(1)}$, $W_{v_1}^{(2)}$, $W_{x_1}^{(2)}$, $W_{v_2}^{(1)}$, $W_{x_2}^{(1)}$, $W_{v_2}^{(2)}$ and $W_{x_2}^{(2)}$ as we did for W_u and W_y.

$$\bar{x}_{1,2m} = \frac{1}{\sqrt{2m}}\left[W_{v_1} + W_{x_1}\right] + «_1 \qquad (8.9.2)$$

$$\bar{x}_{2,2m} = \frac{1}{\sqrt{2m}}\left[W_{V_2} + W_{X_2}\right] + «_2 \qquad (8.9.3)$$

$$\bar{y}_m^{(1)} = \frac{1}{\sqrt{m}}\left[W_u^{(1)} + W_y^{(1)}\right] + «_Y \qquad (8.9.4)$$

$$\bar{x}_{1,m}^{(1)} = \frac{1}{\sqrt{m}}\left[W_{v_1}^{(1)} + W_{x_1}^{(1)}\right] + «_1 \qquad (8.9.5)$$

$$\bar{x}_{2,m}^{(1)} = \frac{1}{\sqrt{m}}\left[W_{v_2}^{(1)} + W_{x_2}^{(1)}\right] + «_2 \qquad (8.9.6)$$

$$\bar{y}_m^{(2)} = \frac{1}{\sqrt{m}}\left[W_u^{(2)} + W_y^{(2)}\right] + \mu_Y \tag{8.9.7}$$

$$\bar{x}_{1,m}^{(2)} = \frac{1}{\sqrt{m}}\left[W_{v_1}^{(2)} + W_{x_1}^{(2)}\right] + \mu_1 \tag{8.9.8}$$

$$\bar{x}_{2,m}^{(2)} = \frac{1}{\sqrt{m}}\left[W_{v_2}^{(2)} + W_{x_2}^{(2)}\right] + \mu_2 \tag{8.9.9}$$

Expanding $F\left[\bar{y}_{2m}, \frac{\bar{x}_{1,2m}}{\mu_1}, \frac{\bar{x}_{2,2m}}{\mu_2}\right]$, $F\left[\bar{y}_m^{(1)}, \frac{\bar{x}_{1,m}^{(1)}}{\mu_1}, \frac{\bar{x}_{2,m}^{(1)}}{\mu_2}\right]$ and $F\left[\bar{y}_m^{(1)}, \frac{\bar{x}_{1,m}^{(2)}}{\mu_1}, \frac{\bar{x}_{2,m}^{(2)}}{\mu_2}\right]$ in third order Taylor's series about the point $(\mu_Y, 1, 1)$ and noting that $F(\mu_Y, 1, 1) = \mu_Y$ for i=1, 2,3, we have

$$\bar{y}_a^{(3)} = \Bigg[\mu_Y + \frac{(W_u + W_y)}{(2m)^{1/2}}F_1 + \frac{(W_{v_1} + W_{x_1})}{\mu_1 (2m)^{1/2}}F_2 + \frac{(W_{v_2} + W_{x_2})}{\mu_2 (2m)^{1/2}}F_3 + \frac{1}{2!2m}\Bigg\{(W_u + W_y)^2 F_{200}$$

$$+ \frac{(W_{v_1} + W_{x_1})^2}{\mu_1^2}F_{020} + \frac{(W_{v_2} + W_{x_2})^2}{\mu_2^2}F_{002}\Bigg\} + 2\Bigg\{\frac{(W_u + W_y)(W_{v_1} + W_{x_1})}{\mu_1}F_{110}$$

$$+ \frac{(W_{v_1} + W_{x_1})(W_{v_2} + W_{x_2})}{\mu_1 \mu_2}F_{011} + \frac{(W_{v_2} + W_{x_2})(W_u + W_y)}{\mu_2}F_{101}\Bigg\} + \ldots\Bigg] \tag{8.9.10}$$

Similarly, we have

$$\bar{y}_a^{(i)} = \Bigg[\mu_Y + \frac{\left(W_{u^{(i)}} + W_{y^{(i)}}\right)}{(m)^{1/2}}F_1 + \frac{\left(W_{v_1^{(i)}} + W_{x_1^{(i)}}\right)}{\mu_1 (m)^{1/2}}F_2 + \frac{\left(W_{v_2^{(i)}} + W_{x_2^{(i)}}\right)}{\mu_2 (m)^{1/2}}F_3 + \frac{1}{2!m}\Bigg\{\left(W_{u^{(i)}} + W_{Y^{(i)}}\right)^2 F_{200}$$

$$+ \frac{\left(W_{v_1^{(i)}} + W_{x_1^{(i)}}\right)^2}{\mu_1^2}F_{020} + \frac{\left(W_{v_2^{(i)}} + W_{x_2^{(i)}}\right)^2}{\mu_2^2}F_{002}\Bigg\} + 2\Bigg\{\frac{\left(W_{u(i)} + W_{Y^{(i)}}\right)\left(W_{v_1^{(i)}} + W_{x_1^{(i)}}\right)}{\mu_1}F_{110}$$

$$+ \frac{\left(W_{v_1^{(i)}} + W_{x_1^{(i)}}\right)\left(W_{v_2^{(i)}} + W_{x_2^{(i)}}\right)}{\mu_1 \mu_2}F_{011} + \frac{\left(W_{v_2^{(i)}} + W_{x_2^{(i)}}\right)\left(W_{u^{(i)}} + W_{y^{(i)}}\right)}{\mu_2}F_{101}\Bigg\} + \ldots\Bigg] \tag{8.9.11}$$

where, *i*=1, 2. On putting these values in $\bar{y}_{aj}$, we have

$$\bar{y}_{aj} = 2\Bigg[\mu_Y + \frac{(W_u + W_y)}{(2m)^{1/2}}F_1 + \frac{(W_{v_1} + W_{x_1})}{\mu_1 (2m)^{1/2}}F_2 + \frac{(W_{v_2} + W_{x_2})}{\mu_2 (2m)^{1/2}}F_3 + \frac{1}{2!2m}\Bigg\{(W_u + W_y)^2 F_{200}$$

$$+\frac{\left(W_{v_1}+W_{x_1}\right)^2}{«_1^2}F_{020}+\frac{\left(W_{v_2}+W_{x_2}\right)^2}{«_2^2}F_{002}\Bigg\}+2\Bigg\{\frac{\left(W_u+W_y\right)\left(W_{v_1}+W_{x_1}\right)}{«_1}F_{110}$$

$$+\frac{\left(W_{v_1}+W_{x_1}\right)\left(W_{v_2}+W_{x_2}\right)}{«_1«_2}F_{011}+\frac{\left(W_{v_2}+W_{x_2}\right)\left(W_u+W_y\right)}{«_2}F_{101}\Bigg\}+\dots\Bigg]$$

$$-\frac{1}{2}\Bigg[«_Y+\frac{\left(w_{u^{(1)}}+w_{y^{(1)}}\right)}{(m)^{1/2}}F_1+\frac{\left(W_{v_1^{(1)}}+W_{x_1^{(1)}}\right)}{«_1(m)^{1/2}}F_2+\frac{\left(W_{v_2^{(1)}}+W_{x_2^{(1)}}\right)}{«_2(m)^{1/2}}F_3+\frac{1}{2!m}\Bigg\{\left(w_{u^{(1)}}+w_{y^{(1)}}\right)^2F_{200}$$

$$+\frac{\left(W_{v_1^{(1)}}+W_{x_1^{(1)}}\right)^2}{«_1^2}F_{020}+\frac{\left(W_{v_2^{(1)}}+W_{x_2^{(1)}}\right)^2}{«_2^2}F_{002}\Bigg\}+2\Bigg\{\frac{\left(W_{u^{(1)}}+W_{y^{(1)}}\right)\left(W_{v_1^{(1)}}+W_{x_1^{(1)}}\right)}{«_1}F_{110}$$

$$+\frac{\left(W_{v_1^{(1)}}+W_{x_1^{(1)}}\right)\left(W_{v_2^{(1)}}+W_{x_2^{(1)}}\right)}{«_1«_2}F_{011}+\frac{\left(W_{v_2^{(1)}}+W_{x_2^{(1)}}\right)\left(w_{u^{(1)}}+w_{y^{(1)}}\right)}{«_2}F_{101}\Bigg\}+\dots$$

$$+«_Y+\frac{\left(w_{u^{(2)}}+w_{y^{(2)}}\right)}{(m)^{1/2}}F_1+\frac{\left(W_{v_1^{(2)}}+W_{x_1^{(2)}}\right)}{«_1(m)^{1/2}}F_2+\frac{\left(W_{v_2^{(2)}}+W_{x_2^{(2)}}\right)}{«_2(m)^{1/2}}F_3+\frac{1}{2!m}\Bigg\{\left(w_{u^{(2)}}+w_{y^{(2)}}\right)^2F_{200}$$

$$+\frac{\left(W_{v_1^{(2)}}+W_{x_1^{(2)}}\right)^2}{«_1^2}F_{020}+\frac{\left(W_{v_2^{(2)}}+W_{x_2^{(2)}}\right)^2}{«_2^2}F_{002}\Bigg\}+2\Bigg\{\frac{\left(W_{u^{(2)}}+W_{y^{(2)}}\right)\left(W_{v_1^{(2)}}+W_{x_1^{(2)}}\right)}{«_1}F_{110}$$

$$+\frac{\left(W_{v_1^{(2)}}+W_{x_1^{(2)}}\right)\left(W_{v_2^{(2)}}+W_{x_2^{(2)}}\right)}{«_1«_2}F_{011}+\frac{\left(W_{v_2^{(2)}}+W_{x_2^{(2)}}\right)\left(w_{u^{(2)}}+w_{y^{(2)}}\right)}{«_2}F_{101}\Bigg\}+\dots\Bigg]$$

so that

$$\bar{y}_{aj}-«_Y=\frac{2}{(2m)^{1/2}}\left[\left(W_u+W_y\right)F_1+\frac{1}{«_1}\left(W_{v_1}+W_{x_1}\right)F_2+\frac{1}{«_2}\left(W_{v_2}+W_{x_2}\right)F_3\right]+\frac{2}{2!2m}\Bigg[\left(W_u+W_y\right)^2F_{200}$$

$$+\frac{1}{«_1^2}\left(W_{v_1}+W_{x_1}\right)^2F_{020}+\frac{1}{«_2^2}\left(W_{v_2}+W_{x_2}\right)^2F_{002}+2\Bigg\{\frac{1}{«_1}\left(W_u+W_y\right)\left(W_{v_1}+W_{x_1}\right)F_{110}$$

$$+\frac{1}{«_1«_2}\left(W_{v_1}+W_{x_1}\right)\left(W_{v_2}+W_{x_2}\right)F_{011}+\frac{1}{«_2}\left(W_{v_2}+W_{x_2}\right)\left(W_u+W_Y\right)F_{101}\Bigg]$$

$$
-\frac{1}{2}\Bigg[\frac{1}{(m)^{1/2}}\Big\{\left(W_{u^{(1)}}+W_{y^{(1)}}\right)F_1+\frac{1}{«_1}\left(W_{v_1^{(1)}}+W_{x_1^{(1)}}\right)F_2+\frac{1}{«_2}\left(W_{v_2^{(1)}}+W_{x_2^{(1)}}\right)F_3\Big\}
$$

$$
+\frac{1}{2!m}\Big\{\left(W_{u^{(1)}}+W_{y^{(1)}}\right)^2F_{200}+\frac{1}{«_1^2}\left(W_{v_1^{(1)}}+W_{x_1^{(1)}}\right)^2F_{020}+\frac{1}{«_2^2}\left(W_{v_2^{(1)}}+W_{x_2^{(1)}}\right)^2F_{002}\Big\}
$$

$$
+2\Big\{\frac{1}{«_1}\left(W_{u^{(1)}}+W_{y^{(1)}}\right)\left(W_{v_1^{(1)}}+W_{x_1^{(1)}}\right)F_{110}+\frac{1}{«_1«_2}\left(W_{v_1^{(1)}}+W_{x_1^{(1)}}\right)\left(W_{v_2^{(1)}}+W_{x_2^{(1)}}\right)F_{011}
$$

$$
+\frac{1}{«_2}\left(W_{v_2^{(1)}}+W_{x_2^{(1)}}\right)\left(W_{u^{(1)}}+W_{y^{(1)}}\right)F_{101}\Big\}
$$

$$
+\frac{1}{(m)^{1/2}}\Big\{\left(W_{u^{(2)}}+W_{y^{(2)}}\right)F_1+\frac{1}{«_1}\left(W_{v_1^{(2)}}+W_{x_1^{(2)}}\right)F_2+\frac{1}{«_2}\left(W_{v_2^{(2)}}+W_{x_2^{(2)}}\right)F_3\Big\}
$$

$$
+\frac{1}{2!m}\Big\{\left(W_{u^{(2)}}+W_{y^{(2)}}\right)^2F_{200}+\frac{1}{«_1^2}\left(W_{v_1^{(2)}}+W_{x_1^{(2)}}\right)^2F_{020}+\frac{1}{«_2^2}\left(W_{v_2^{(2)}}+W_{x_2^{(2)}}\right)^2F_{002}\Big\}
$$

$$
+2\Big\{\frac{1}{«_1}\left(W_{u^{(2)}}+W_{y^{(2)}}\right)\left(W_{v_1^{(2)}}+W_{x_1^{(2)}}\right)F_{110}+\frac{1}{«_1«_2}\left(W_{v_1^{(2)}}+W_{x_1^{(2)}}\right)\left(W_{v_2^{(2)}}+W_{x_2^{(2)}}\right)F_{011}
$$

$$
+\frac{1}{«_2}\left(W_{v_2^{(2)}}+W_{x_2^{(2)}}\right)\left(W_{u^{(2)}}+W_{y^{(2)}}\right)F_{101}\Big\}\Bigg] \qquad (8.9.12)
$$

On taking expectation on both sides of (8.9.12) we get the bias of $\bar{y}_{aj}$ given by

$$
E\left(\bar{y}_{aj}-«_Y\right)=\Bigg[\frac{2}{2!2m}\left(\pm_u^2+\pm_y^2\right)F_{200}+\frac{1}{«_1^2}\left(\pm_{v_1}^2+\pm_{x_1}^2\right)F_{020}+\frac{1}{«_2^2}\left(\pm_{v_2}^2+\pm_{x_2}^2\right)F_{002}
$$

$$
+\frac{2}{«_1}\pm_{(y,x_1)}F_{110}+\frac{2}{«_2}\pm_{(y,x_2)}F_{101}+\frac{2}{«_1«_2}\pm_{(x_1,x_2)}F_{011}\Bigg]
$$

$$
-\frac{1}{2}\Bigg[\frac{2}{2!m}\left(\pm_u^2+\pm_y^2\right)F_{200}+\frac{1}{«_1^2}\left(\pm_{v_1}^2+\pm_{x_1}^2\right)F_{020}+\frac{1}{«_2^2}\left(\pm_{v_2}^2+\pm_{x_2}^2\right)F_{002}
$$

$$
+\frac{2}{«_1}\pm_{(y,x_1)}F_{110}+\frac{2}{«_2}\pm_{(y,x_2)}F_{101}+\frac{2}{«_1«_2}\pm_{(x_1,x_2)}F_{011}
$$

$$
+\frac{2}{2!m}\left(\pm_u^2+\pm_y^2\right)F_{200}+\frac{1}{«_1^2}\left(\pm_{v_1}^2+\pm_{x_1}^2\right)F_{020}+\frac{1}{«_2^2}\left(\pm_{v_2}^2+\pm_{x_2}^2\right)F_{002}
$$

$$
+\frac{2}{«_1}\pm_{(y,x_1)}F_{110}+\frac{2}{«_2}\pm_{(y,x_2)}F_{101}+\frac{2}{«_1«_2}\pm_{(x_1,x_2)}F_{011}\Bigg] \qquad (8.9.13)
$$

$$
Bias\left(\bar{y}_{aj}\right)=0 \qquad (8.9.14)
$$

Squaring both side of (8.9.12) and taking expectation, the mean square error of $\bar{y}_{aj}$ up terms of $O\left(\frac{1}{n}\right)$, is given by

$$MSE\left(\bar{y}_{aj}\right)=E\left[\frac{2}{(2m)^{1/2}}\left\{\left(W_u+W_y\right)F_1+\frac{1}{\text{«}_1}\left(W_{v_1}+W_{x_1}\right)F_2+\frac{1}{\text{«}_2}\left(W_{v_2}+W_{x_2}\right)F_3\right\}\right.$$

$$\text{-}\frac{1}{2(m)^{1/2}}\left\{\left(W_{u^{(1)}}+W_{y^{(1)}}\right)F_1+\frac{1}{\text{«}_1}\left(W_{v_1^{(1)}}+W_{x_1^{(1)}}\right)F_2+\frac{1}{\text{«}_2}\left(W_{v_2^{(1)}}+W_{x_2^{(1)}}\right)F_3\right\}$$

$$\left.\text{-}\frac{1}{2}\frac{1}{(m)^{1/2}}\left\{\left(W_{u^{(2)}}+W_{y^{(2)}}\right)F_1+\frac{1}{\text{«}_1}\left(W_{v_1^{(2)}}+W_{x_1^{(2)}}\right)F_2+\frac{1}{\text{«}_2}\left(W_{v_2^{(2)}}+W_{x_2^{(2)}}\right)F_3\right\}\right]^2$$

$$=\left[\frac{4}{2m}\left\{\left(\pm_u^2+\pm_y^2\right)F_1^2+\frac{1}{\text{«}_1^2}\left(\pm_{v_1}^2+\pm_{x_1}^2\right)F_2^{\,2}+\frac{1}{\text{«}_2^2}\left(\pm_{v_2}^2+\pm_{x_2}^2\right)F_3^{\,2}\right.\right.$$

$$\left.+\frac{2}{\text{«}_1}\pm_{(y,x_1)}F_1F_2+\frac{2}{\text{«}_2}\pm_{(y,x_2)}F_3F_1+\frac{2}{\text{«}_1\text{«}_2}\pm_{(x_1,x_2)}F_2F_3\right\}$$

$$+\frac{1}{4m}\left\{\left(\pm_u^2+\pm_y^2\right)F_1^2+\frac{1}{\text{«}_1^2}\left(\pm_{v_1}^2+\pm_{x_1}^2\right)F_2^{\,2}+\frac{1}{\text{«}_2^2}\left(\pm_{v_2}^2+\pm_{x_2}^2\right)F_3^{\,2}\right.$$

$$\left.+\frac{2}{\text{«}_1}\pm_{(y,x_1)}F_1F_2+\frac{2}{\text{«}_2}\pm_{(y,x_2)}+F_3F_1+\frac{2}{\text{«}_1\text{«}_2}\pm_{(x_1,x_2)}F_2F_3\right\}$$

$$+\frac{1}{4m}\left\{\left(\pm_u^2+\pm_y^2\right)F_1^2+\frac{1}{\text{«}_1^2}\left(\pm_{v_1}^2+\pm_{x_1}^2\right)F_2^{\,2}+\frac{1}{\text{«}_2^2}\left(\pm_{v_2}^2+\pm_{x_2}^2\right)F_3^{\,2}\right.$$

$$\left.+\frac{2}{\text{«}_1}\pm_{(y,x_1)}F_1F_2+\frac{2}{\text{«}_2}\pm_{(y,x_2)}F_3F_1+\frac{2}{\text{«}_1\text{«}_2}\pm_{(x_1,x_2)}F_2F_3\right\}$$

$$-\frac{4}{2\sqrt{2}\,(2m)^{\frac{1}{2}}\,m^{\frac{1}{2}}}\left\{\left(\pm_u^2+\pm_y^2\right)F_1^2+\frac{1}{\text{«}_1^2}\left(\pm_{v_1}^2+\pm_{x_1}^2\right)F_2^{\,2}+\frac{1}{\text{«}_2^2}\left(\pm_{v_2}^2+\pm_{x_2}^2\right)F_3^{\,2}\right.$$

$$\left.+\frac{2}{\text{«}_1}\pm_{(y,x_1)}F_1F_2+\frac{2}{\text{«}_2}\pm_{(y,x_2)}F_3F_1+\frac{2}{\text{«}_1\text{«}_2}\pm_{(x_1,x_2)}F_2F_3\right\}$$

$$\text{-}\frac{4}{2\sqrt{2}\,(2m)^{\frac{1}{2}}\,m^{\frac{1}{2}}}\left\{\left(\pm_u^2+\pm_y^2\right)F_1^2+\frac{1}{\text{«}_1^2}\left(\pm_{v_1}^2+\pm_{x_1}^2\right)F_2^{\,2}+\frac{1}{\text{«}_2^2}\left(\pm_{v_2}^2+\pm_{x_2}^2\right)F_3^{\,2}\right.$$

$$\left.\left.+\frac{2}{\text{«}_1}\pm_{(y,x_1)}F_1F_2+\frac{2}{\text{«}_2}\pm_{(y,x_2)}F_3F_1+\frac{2}{\text{«}_1\text{«}_2}\pm_{(x_1,x_2)}F_2F_3\right\}\right]$$

$$
= \frac{1}{2m}\left[\pm_y^2 F_1^2 + \frac{1}{«_1^2}\pm_{x_1}^2 F_2^2 + \frac{1}{«_2^2}\pm_{x_2}^2 F_3^2 + 2\left\{\frac{1}{«_1}\pm_{(y,x_1)} F_1F_2 + \frac{1}{«_1«_2}\pm_{(x_1,x_2)}F_2F_3 + \frac{1}{«_2}\pm_{(y,x_2)}F_3F_1\right\}\right] + \frac{1}{2m}\left[\pm_u^2F_1^2 + \frac{1}{«_1^2}\pm_{v_1}^2F_2^2 + \frac{1}{«_2^2}\pm_{v_2}^2F_3^2\right] \tag{8.9.15}
$$

Proceeding similarly we can find the expressions of Biases and MSE's of $\hat{R}_{Gj}$ and $\hat{P}_{Hj}$ can be easily derived on similar lines as that of MSE($\bar{y}_{aj}$) and are given by

$$
Bias\left(\hat{R}_{Gj}\right) = 0 \tag{8.9.16}
$$

$$
MSE\left(\hat{R}_{Gj}\right) = \frac{R^2}{2m}\left[\left(\frac{1}{«_{Y_2}^2}\pm_{y_1}^2 + \frac{1}{«_{Y_2}^2}\pm_{y_2}^2 - \frac{1}{«_{Y_1}«_{Y_2}}\pm_{(y_1,y_2)}\right)G_1^2 + \left(\frac{1}{R^2«_1^2}\pm_{x_1}^2G_2^2 + \frac{1}{R^2«_2^2}\pm_{x_2}^2G_3^2 + \frac{2}{R«_1«_2}\pm_{(x_1,x_2)}G_2G_3\right) + \frac{2}{«_{Y_1}}\left(\frac{1}{«_1}\pm_{(x_1,y_1)}G_1G_2 + \frac{1}{«_2}\pm_{(x_2,y_1)}G_1G_3\right) - \frac{2}{«_{Y_2}}\left(\frac{1}{«_1}\pm_{(x_1,y_2)}G_1G_2 + \frac{1}{«_2}\pm_{(x_2,y_2)}G_1G_3\right)\right] + \frac{R^2}{2m}\left[\left(\frac{1}{«_{Y_2}^2}\pm_{u_1}^2 + \frac{1}{«_{Y_2}^2}\pm_{u_2}^2\right)G_1^2 + \left(\frac{1}{R^2«_1^2}\pm_{v_1}^2G_2^2 + \frac{1}{R^2«_2^2}\pm_{v_2}^2G_3^2\right)\right] \tag{8.9.17}
$$

$$
Bias\left(\hat{P}_{Hj}\right) = 0 \tag{8.9.18}
$$

$$
MSE\left(\hat{P}_{Hj}\right) = \frac{P^2}{2m}\left[\left(\frac{1}{«_{Y_2}^2}\pm_{y_1}^2 + \frac{1}{«_{Y_2}^2}\pm_{y_2}^2 + \frac{1}{«_{Y_1}«_{Y_2}}\pm_{(y_1,y_2)}\right)H_1^2 + \left(\frac{1}{P«_1^2}\pm_{x_1}^2H_2^2 + \frac{1}{P^2«_2^2}\pm_{x_2}^2H_3^2 + \frac{2}{P«_1«_2}\pm_{(x_1,x_2)}H_2H_3\right) + \frac{2}{«_{Y_1}}\left(\frac{1}{«_1}\pm_{(x_1,y_1)}H_1H_2 + \frac{1}{«_2}\pm_{(x_2,y_1)}H_1H_3\right) + \frac{2}{«_{Y_2}}\left(\frac{1}{«_1}\pm_{(x_1,y_2)}H_1H_2 + \frac{1}{«_2}\pm_{(x_2,y_2)}H_1H_3\right)\right] + \frac{P^2}{2m}\left[\left(\frac{1}{«_{Y_2}^2}\pm_{u_1}^2 + \frac{1}{«_{Y_2}^2}\pm_{u_2}^2\right)H_1^2 + \left(\frac{1}{P^2«_1^2}\pm_{v_1}^2H_2^2 + \frac{1}{P^2«_2^2}\pm_{v_2}^2H_3^2\right)\right] \tag{8.9.19}
$$

8.10 CONCLUDING REMARKS OF PROPOSED ESTIMATORS FOR MEAN

We can easily see that bias from (8.5.5), (8.5.8), (8.9.14) and (8.9.15) the bias and the MSE of proposed class of generalized estimators and proposed class of unbiased jack-knife estimators are given by

$$Bias\left(\bar{y}_a\right)=\frac{1}{2n}\left[\frac{1}{\text{«}_1^2}\pm_{x_1}^2 f_{020}+\frac{1}{\text{«}_2^2}\pm_{x_2}^2 f_{002}+2\left\{\frac{\pm_{(y,x_1)}}{\text{«}_1}f_{100}+\frac{\pm_{(x_1,x_2)}f_{011}}{\text{«}_1\text{«}_2}+\frac{\pm_{(y,x_2)}}{\text{«}_2}f_{101}\right\}\right]$$

$$+\frac{1}{2n}\left[\frac{\pm_{v_1}^2}{\text{«}_1^2}f_{020}+\frac{\pm_{v_2}^2}{\text{«}_2^2}f_{002}\right] \quad (8.10.1)$$

$$MSE\left(\bar{y}_a\right)=\frac{1}{n}\left[\pm_y^2 f_1^2+\frac{\pm_{x_1}^2}{\text{«}_1^2}f_2^2+\frac{\pm_{x_2}^2}{\text{«}_2^2}f_3^2+2\left\{\frac{\pm_{(y,x_1)}}{\text{«}_1}f_1 f_2+\frac{\pm_{(x_1,x_2)}}{\text{«}_1\text{«}_2}f_2 f_3+\frac{\pm_{(y,x_2)}}{\text{«}_2}f_3 f_1\right\}\right]$$

$$+\frac{1}{n}\left[\pm_u^2 f_1^2+\frac{\pm_{v_1}^2}{\text{«}_1^2}f_2^2+\frac{\pm_{v_2}^2}{\text{«}_2^2}f_3^2\right] \quad (8.10.2)$$

$$Bias\left(\bar{y}_{aj}\right)=0 \quad (8.10.3)$$

$$MSE\left(\bar{y}_{aj}\right)=\frac{1}{2m}\left[\pm_y^2 F_1^2+\frac{1}{\text{«}_1^2}\pm_{x_1}^2 F_2^2+\frac{1}{\text{«}_2^2}\pm_{x_2}^2 F_3^2+2\left\{\frac{1}{\text{«}_1}\pm_{(y,x_1)}F_1F_2+\frac{1}{\text{«}_1\text{«}_2}\pm_{(x_1,x_2)}F_2F_3\right.\right.$$

$$\left.\left.+\frac{1}{\text{«}_2}\pm_{(y,x_2)}F_3F_1\right\}\right]+\frac{1}{2m}\left[\pm_u^2 F_1^2+\frac{1}{\text{«}_1^2}\pm_{v_1}^2 F_2^2+\frac{1}{\text{«}_2^2}\pm_{v_2}^2 F_3^2\right] \quad (8.10.4)$$

From (8.10.2) and (8.10.4), we see that both estimators $\bar{y}_a$ and $\bar{y}_{aj}$ have the same mean square error but from (8.10.1) bias of $\bar{y}_a$ is not zero whereas from (8.10.3) bias of the jack-knifed estimator $\bar{y}_{aj}$ is zero. whereas from (8.10.3) bias of the jack-knifed estimator $\bar{y}_{aj}$ is zero and both the estimators $\bar{y}_a$ and $\bar{y}_{aj}$ having the same mean square error, the jack-knifed estimator $\bar{y}_{aj}$ may be preferred to the estimator $\bar{y}_a$ in the presence of measurement errors also.

Therefore, the proposed unbiased jack-knifed estimator should be preferred than the conventional estimators under measurement error as they help in removing the bias while still preserving the efficiency. Many important estimators like that of proposed by Abu-dayyeh (2003) given by

$$\bar{y}_a = \bar{y}\left(\frac{\bar{x}_1}{«_1}\right)^{\sim_1}\left(\frac{\bar{x}_2}{«_2}\right)^{\sim_2}$$

are special cases of the present study when we assume the measurement errors to be absent. The conventional results can be obtained as special cases of this study by setting the measurement error variances to be zero.

8.11 CONCLUDING REMARKS OF PROPOSED ESTIMATOR FOR RATIO

For the proposed generalized estimators $\hat{R}_g$ and $\hat{R}_{Gj}$, it can be easily seen from (8.5.6), (8.9.18), (8.5.9) and (8.9.19) that bias the bias and the MSE are given as

$$Bias\left(\hat{R}_g\right) = \frac{R}{n}\left[\left(\frac{1}{2«_1^2}\pm_{x_1}^2 g_{030} + \frac{1}{2«_2^2}\pm_{x_2}^2 g_{003} + \frac{1}{R«_1«_2}\pm_{(x_1,x_2)} g_{011}\right)\right.$$
$$+\frac{1}{«_{Y_1}}\left(\frac{1}{«_1}\pm_{(x_1,y_1)} g_{110} + \frac{1}{«_2}\pm_{(x_2,y_1)} g_{101}\right)$$
$$\left.-\frac{1}{«_{Y_2}}\left(\frac{1}{«_1}\pm_{(x_1,y_2)} g_{110} + \frac{1}{«_2}\pm_{(x_2,y_2)} g_{101}\right)\right] + \frac{1}{n}\left[\frac{1}{2«_1^2}\pm_{v_1}^2 g_{030} + \frac{1}{2«_2^2}\pm_{v_2}^2 g_{003}\right] \quad (8.11.1)$$

$$MSE\left(\hat{R}_g\right) = \frac{R^2}{n}\left[\left(\frac{1}{«_{Y_1}^2}\pm_{y_1}^2 + \frac{1}{«_{Y_2}^2}\pm_{y_2}^2 - \frac{2}{«_{Y_1}«_{Y_2}}\pm_{(y_1,y_2)}\right) g_1^{\ 2}\right.$$
$$+\left(\frac{1}{R^2«_1^2}\pm_{x_1}^2 g_2^{\ 2} + \frac{1}{R^2«_2^2}\pm_{x_2}^2 g_3^{\ 2} + \frac{2}{R^2«_1«_2}\pm_{(x_1,x_2)} g_2 g_3\right)$$
$$\left.+\frac{2}{«_{Y_1}}\left(\frac{1}{«_1}\pm_{(x_1,y_1)} g_1 g_2 + \frac{1}{«_2}\pm_{(x_2,y_1)} g_1 g_3\right) - \frac{2}{«_{Y_2}}\left(\frac{1}{«_1}\pm_{(x_1,y_2)} g_1 g_2 + \frac{1}{«_2}\pm_{(x_2,y_2)} g_1 g_3\right)\right]$$

$$+\frac{R^2}{n}\left[\left(\frac{1}{«_{Y_2}^2}\pm_{u_1}^2 + \frac{1}{«_{Y_2}^2}\pm_{u_2}^2\right) g_1^{\ 2} + \left(\frac{1}{R^2«_1^2}\pm_{v_1}^2 g_2^{\ 2} + \frac{1}{R^2«_2^2}\pm_{v_2}^2 g_3^{\ 2}\right)\right] \quad (8.11.2)$$

$$Bias\left(\bar{y}_{Rj}\right) = 0 \quad (8.11.3)$$

$$MSE\left(\hat{R}_{Gj}\right) = \frac{R^2}{2m}\left[\left(\frac{1}{«_{Y_2}^2}\pm_{y_1}^2 + \frac{1}{«_{Y_2}^2}\pm_{y_2}^2 - \frac{1}{«_{Y_1}«_{Y_2}}\pm_{(y_1,y_2)}\right) G_1^{\ 2}\right.$$

$$+\left(\frac{1}{R^2\mu_1^2}\sigma_{x_1}^2 G_2^{\,2}+\frac{1}{R^2\mu_2^2}\sigma_{x_2}^2 G_3^{\,2}+\frac{2}{R\mu_1\mu_2}\sigma_{(x_1,x_2)}G_2G_3\right)$$

$$+\frac{2}{\mu_{Y_1}}\left(\frac{1}{\mu_1}\sigma_{(x_1,y_1)}\,G_1G_2+\frac{1}{\mu_2}\sigma_{(x_2,y_1)}G_1G_3\right)-\frac{2}{\mu_{Y_2}}\left(\frac{1}{\mu_1}\sigma_{(x_1,y_2)}G_1G_2+\frac{1}{\mu_2}\sigma_{(x_2,y_2)}G_1G_3\right)\Bigg]$$

$$+\frac{R^2}{2m}\left[\left(\frac{1}{\mu_{Y_2}^2}\sigma_{u_1}^2+\frac{1}{\mu_{Y_2}^2}\sigma_{u_2}^2\right)(G_1)^2+\left(\frac{1}{R^2\mu_1^2}\sigma_{v_1}^2G_2^{\,2}+\frac{1}{R^2\mu_2^2}\sigma_{v_2}^2G_3^{\,2}\right)\right] \qquad (8.11.4)$$

From (8.11.2) and (8.11.4), we see that both estimators $\hat{R}_g$ and $\hat{R}_{Gj}$ have the same mean square error but from (8.11.1) bias of $\hat{R}_g$ is not zero whereas from (8.11.3) bias of the jack-knifed estimator $\hat{R}_{Gj}$ is zero. Hence, both the estimators $\hat{R}_g$ and $\hat{R}_{Gj}$ having the same mean square error, but the jack-knifed estimator $\hat{R}_{Gj}$ may be preferred to the estimator $\hat{R}_g$ in the sense of unbiasedness under the presence of measurement errors.

Therefore, the proposed unbiased jack-knifed estimator should be preferred than the conventional estimators under measurement error as they help in removing the bias while still preserving the efficiency. Many important estimators like adapted estimators of Srivastava (1967) type for estimation of ratio of population mean given by

$$\hat{R}_{\alpha}=\hat{R}\left(\frac{\bar{x}_1}{\mu_1}\right)^{\alpha_1}\left(\frac{\bar{x}_2}{\mu_2}\right)^{\alpha_2}$$

are special cases of the present study when we assume the measurement errors to be absent. Such conventional results can be obtained as special cases of this study by setting the measurement error variances to be zero or by choosing the derivatives judiciously under measurement errors.

8.12 CONCLUDING REMARKS OF PROPOSED ESTIMATOR FOR PRODUCT

Also, it can be seen from (8.5.7), (8.9.20), (8.5.10) and (8.9.21) for proposed generalized estimators $\hat{P}_h$ and $\hat{P}_{Hj}$ that bias the bias and the MSE are given as

$$\begin{aligned}
Bias\left(\hat{P}_h\right) = \frac{P}{n}\Bigg[\Bigg(& \frac{1}{2«_1^2}\pm_{x_1}^2 h_{030} + \frac{1}{2«_2^2}\pm_{x_2}^2 h_{003} + \frac{1}{P«_1«_2}\pm_{(x_1,x_2)} h_{011} \\
& + \frac{1}{«_{Y_1}}\left(\frac{1}{«_1}\pm_{(x_1,y_1)} h_{110} + \frac{1}{«_2}\pm_{(x_2,y_1)} h_{101}\right) + \frac{1}{«_{Y_2}}\left(\frac{1}{«_1}\pm_{(x_1,y_2)} h_{110} + \frac{1}{«_2}\pm_{(x_2,y_2)} h_{101}\right)\Bigg)\Bigg] \\
& + \frac{P}{n}\left[\frac{1}{2«_1^2}\pm_{v_1}^2 h_{030} + \frac{1}{2«_2^2}\pm_{v_2}^2 h_{003}\right]
\end{aligned} \tag{8.12.1}$$

$$\begin{aligned}
MSE\left(\hat{P}_h\right) = \frac{P^2}{n}\Bigg[& \left(\frac{1}{«_{Y_2}^2}\pm_{y_1}^2 + \frac{1}{«_{Y_2}^2}\pm_{y_2}^2 + \frac{1}{«_{Y_1}«_{Y_2}}\pm_{(y_1,y_2)}\right) h_1^2 \\
& + \left(\frac{1}{P^2«_1^2}\pm_{x_1}^2 h_2^2 + \frac{1}{P^2«_2^2}\pm_{x_2}^2 h_3^2 + \frac{2}{P«_1«_2}\pm_{(x_1,x_2)} h_2 h_3\right) \\
& + \frac{2}{«_{Y_1}}\left(\frac{1}{«_1}\pm_{(x_1,y_1)} h_1 h_2 + \frac{1}{«_2}\pm_{(x_2,y_1)} h_1 h_3\right) + \frac{2}{«_{Y_2}}\left(\frac{1}{«_1}\pm_{(x_1,y_2)} h_1 h_2 + \frac{1}{«_2}\pm_{(x_2,y_2)} h_1 h_3\right)\Bigg] \\
& + \frac{P^2}{n}\left[\left(\frac{1}{«_{Y_2}^2}\pm_{u_1}^2 + \frac{1}{«_{Y_2}^2}\pm_{u_2}^2\right) h_1^2 + \left(\frac{1}{P^2«_1^2}\pm_{x_1}^2 h_2^2 + \frac{1}{P^2«_2^2}\pm_{x_2}^2 h_3^2\right)\right]
\end{aligned} \tag{8.12.2}$$

$$Bias\left(\hat{P}_{Hj}\right) = 0 \tag{8.12.3}$$

$$\begin{aligned}
MSE\left(\hat{P}_{Hj}\right) = \frac{P^2}{2m}\Bigg[& \left(\frac{1}{«_{Y_2}^2}\pm_{y_1}^2 + \frac{1}{«_{Y_2}^2}\pm_{y_2}^2 + \frac{1}{«_{Y_1}«_{Y_2}}\pm_{(y_1,y_2)}\right) H_1^2 \\
& + \left(\frac{1}{P^2«_1^2}\pm_{x_1}^2 H_2^2 + \frac{1}{P^2«_2^2}\pm_{x_2}^2 H_3^2 + \frac{2}{P«_1«_2}\pm_{(x_1,x_2)} H_2 H_3\right) \\
& + \frac{2}{«_{Y_1}}\left(\frac{1}{«_1}\pm_{(x_1,y_1)} H_1 H_2 + \frac{1}{«_2}\pm_{(x_2,y_1)} H_1 H_3\right) + \frac{2}{«_{Y_2}}\left(\frac{1}{«_1}\pm_{(x_1,y_2)} H_1 H_2 + \frac{1}{«_2}\pm_{(x_2,y_2)} H_1 H_3\right)\Bigg]
\end{aligned}$$

$$+\frac{P^2}{2m}\left[\left(\frac{1}{\mu_{Y_2}^2}\sigma_{u_1}^2+\frac{1}{\mu_{Y_2}^2}\sigma_{u_2}^2\right)H_1{}^2+\left(\frac{1}{P^2\mu_1^2}\sigma_{v_1}^2H_2{}^2+\frac{1}{P^2\mu_2^2}\sigma_{v_2}^2H_3{}^2\right)\right] \qquad (8.12.4)$$

From (8.12.2) and (8.12.4), we see that both estimators $\hat{P}_h$ and $\hat{P}_{Hj}$ have the same mean square error but from (8.12.1) bias of $\hat{P}_h$ is not zero. Whereas from (8.12.3) bias of the jack-knifed estimator $\hat{P}_{Hj}$ is zero, hence, both the estimators $\hat{P}_h$ and $\hat{P}_{Hj}$ having the same mean square error, but the jack-knifed estimator $\hat{P}_{Hj}$ may be preferred to the estimator $\hat{P}_h$ in the sense of unbiasedness under the presence of measurement errors.

Therefore, the proposed unbiased jack-knifed estimator for product of population means should be preferred than the conventional estimators under measurement error as they help in removing the bias while still preserving the efficiency. Many important estimators like adapted estimators of Srivastava (1967) type for estimation of product of population mean given by

$$\hat{P}_{\alpha}=\hat{P}\left(\frac{\overline{x}_1}{\mu_1}\right)^{\alpha_1}\left(\frac{\overline{x}_2}{\mu_2}\right)^{\alpha_2}$$

are special cases of the present study when we assume the measurement errors to be absent. Such conventional results can be obtained as special cases of this study by setting the measurement error variances to be zero or by choosing the derivatives judiciously under measurement errors.

The unbiased of generalized jack-knife estimator helps in finding unbiased or almost unbiased jack-knife estimators for various special cases in similar fashion without increasing the mean square error. The expression of bias and MSE of such special cases can easily be found by replacing the suitable values of the derivatives in the respective equations of the bias and MSE of the proposed generalized estimators.

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