Heat and Mass Transfer in Fluid and Porous Medium

THESIS

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2019

Dedicated

to

My Son

and

My Family Members

DECLARATION

I declare that the work embodied in this Ph.D. thesis entitled as **"Heat and Mass Transfer in Fluid and Porous Medium"** is carried out by me under the supervision of Prof. B.S. Bhadauria, Department of Applied Mathematics, Babasaheb Bhimrao Ambedkar University (A Central University) Lucknow, India. The information presented in this Ph.D. thesis has not been submitted for the award of any degree or diploma. I declare that I have faithfully acknowledged, given credit to and referred to the research workers whenever their works have been cited in the text and the body of the thesis. I further declare that I have not willful lifted up some others work, para, text, data, results, etc. reported in the journals, books, magazines, reports, dissertations, thesis, etc., or available at website and included them in this thesis and cited my own work.

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Date: 15/05/2019

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CERTIFICATE

This is to certify that the thesis titled **"Heat and Mass Transfer in Fluid and Porous Medium"** submitted by **Mrs. Kanchan Shakya** is an original research work and has not been previously submitted in part or full for the award of any other degree or diploma to this or any other university.

The thesis submitted to the Babasaheb Bhimrao Ambedkar University Lucknow satisfies all the requirements as stipulated in the *Doctor of Philosophy (Ph.D.) regulations -1999 as amended in 2008/2010/2013* and it is fit for submission and evaluation for the award of the degree of Doctor of Philosophy of the University.

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PREFACE

The thesis entitled **"Heat and Mass Transfer in Fluid and Porous Medium"** comprising of analytical/numerical solutions of some problems related with the topic, is an outcome of the research work carried out by me during the course of investigations under the supervision of Prof. B.S. Bhadauria, Professor, Department of Applied Mathematics, School for Physical Sciences, Babasaheb Bhimrao Ambedkar University, Lucknow.

Flow through porous media occurs in many science and engineering systems. The important topics investigated in this thesis are that of flow, heat and mass transfer in porous media. It is a subject of engineering interest, and an important field of study in itself. Several contributions have been made in modelling fluid flow, heat, and mass transfer through a porous medium. These contributions include the introduction of non-Darcy effects on momentum, energy, and mass transport in porous media for various geometrical configurations and boundary conditions (Nield and Bejan, 1999). A medium which is a solid body containing pores is called a porous medium. Flow through porous media is also of interest in chemical engineering (adsorption, filtration, and flow in packed columns), petroleum engineering, hydrology, soil physics, biophysics and geophysics. Two macroscopic properties of porous media which may be used to describe fluid flow are porosity and permeability. Therefore, the relevant studies on regulation of heat and mass transfer in the following chapters is performed. For more detailed analysis on thermal instability, refer some excellent books: Ingham and Pop(2005), Nield and Bejan(2012) and Vafai(2000).

The **first problem** deals with linear and nonlinear analyses has been done and the combine effect of internal heating and Soret effect on Darcy - Brinkman convection in a binary viscoelastic fluid saturated porous layer, heated and salted from below, has been studied, analytically. Linear stability analysis has been performed by using normal mode technique and nonlinear analysis is done using truncated Fourier series. The modified Darcy-Brinkman-Oldroyd model, including the time derivative term, is employed for the momentum equation. The effects of Darcy number, Soret parameter, relaxation and retardation parameters, solute Rayleigh number, internal heat source, Lewis number and Darcy-

Prandtl number on stationary and oscillatory convection are shown graphically. Also heat and mass transports are calculated in terms of the Nusselt number and Sherwood number and presented graphically.

In the **second problem**, the effect of internal heat source and Soret effect has been investigated on double diffusive convection in a rotating anisotropic porous medium saturated with a couple stress fluids, heated and salted from below. Linear stability analysis has been performed by using Normal mode technique and for nonlinear analysis, minimal representation of Fourier series up to two terms has been considered. The modified Darcy model, which includes the time derivative term and Coriolis term, has been employed in the momentum equation. The effect of Taylor number, couple stress parameter, solute Rayleigh number, internal heat source parameter, Lewis number, Darcy-Prandtl number, thermal and mechanical anisotropy parameter on the stationary and oscillatory modes of convection has been obtained and shown graphically, Also the heat and mass transports are obtained in terms of the Nusselt number and Sherwood number respectively, and shown graphically.

The **third study** deals with the effect of internal heating on double diffusive convection in a rotating anisotropic porous medium saturated with a viscoelastic fluid, which is heated and salted from below. Linear stability analysis has been performed by using normal mode technique and nonlinear theory is based on minimal representation of Fourier series up to two terms. The modified Darcy model, which includes the time derivative and Coriolis terms has been employed in the momentum equation. The effects of Taylor number, solute Rayleigh number, internal heat source parameter, diffusivity ratio, relaxation and retardation parameters, thermal and mechanical anisotropy parameters on the stationary and oscillatory convection are obtained and shown graphically. Also, heat and mass transports have been obtained in terms of the Nusselt number and Sherwood number respectively and presented through Figs.

The **fourth study** deals with the investigation of the effect of internal heating and Soret effect on linear and nonlinear double diffusive convection in a couple stress fluid saturated anisotropic porous layer, heated and salted from below, analytically. Linear stability analysis has been performed by using normal mode technique and nonlinear analysis is done using truncated Fourier series. The modified Darcy model, which includes the time derivative term, has been employed in the momentum equation. The effects of Vadasz number, anisotropic parameter, Soret parameter, couple stress parameter, solute Rayleigh number, internal heat source parameter, Lewis number, Darcy-Prandtl number and normalized porosity on the stationary and oscillatory are shown graphically. Also heat and mass transports are calculated in terms of the Nusselt number and Sherwood number and shown graphically.

In the **last problem**, the double diffusive convection in a Maxwell fluid saturated rotating anisotropic porous layer in the presence of the Soret and Dufour effects has been investigated. Linear stability analysis has been performed by using normal mode technique, while nonlinear analysis is done using truncated Fourier series. The flow is also affected by temperature and concentration gradients in their medium. The modified Darcy model has been employed in the momentum equation. Effects of mechanical anisotropy parameter, relaxation parameter, retardation parameter, Darcy-Prandtl number, Dufour parameter, Soret parameter, solute Rayleigh number and Lewis number on the stationary and oscillatory modes of convection have been obtained and are shown graphically. Further, heat and mass transports across the porous medium are also presented in the figures.

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Nomenclature

Latin Symbols

a	wave number
a_c	critical wave number
d	depth of fluid layer
ΔT	temperature difference across the porous layer
$\vec{g}$	acceleration due to gravity
P_e	peclet number
R	scaled Rayleigh number
C	Couple stress parameter $C = \frac{\mu_c}{\mu d^2}$
L_e	Lewis number $Le = \frac{\kappa_{Tz}}{\kappa_s}$
D_a	Darcy number $Da = \frac{K_z}{d^2}$
D	cross diffusion due to T component
K_{ST}	cross diffusion due to S component
K_{TS}	cross diffusion due to T component
Pr_d	Prandtl number $Pr_d = \frac{\phi\gamma\nu d^2}{\kappa_T K}$
V_a	Vadasz number $V_a = (\frac{P_r}{D_a})$
T_a	Taylor number $T_a = (\frac{2\Omega\kappa_T}{\nu\varepsilon})^2$
Ra_T	thermal Rayleigh number $Ra_T = \frac{\beta_T g \Delta T K_z d}{\nu\kappa_T}$
Ra_S	solutal Rayleigh number $Ra_S = \frac{\beta_S g \Delta S K_z d}{\nu\kappa_T}$
K	permeability of porous media $K_x(\hat{i}\hat{i} + \hat{j}\hat{j}) + K_z(\hat{k}\hat{k})$
N_u	Nusselt number
S_h	Sharewood number

p	reduced pressure
T	temperature
S	solute concentration
ΔT	temperature difference across the fluid layer
ΔS	solute difference across the porous layer
t	time
τ	diffusivity ratio
$\mathbf{q}$	fluid velocity(u, v, w)
Q	internal heat source
R_i	internal Rayleigh number
T_0	reference temperature
S_0	reference concentration
(x, y, z)	horizontal and vertical co-ordinates

Greek symbols

K_{11}	effective thermal diffusivity
K_{22}	effective solutal diffusivity
K_{21}	soret coefficient
α_T	coefficient of thermal expansion
β_T	coefficient of thermal expansion
β_S	coefficient of solute expansion
α_S	coefficient of solute expansion
κ_T	thermal diffusivity $\kappa_{Tx}(\hat{i}\hat{i} + \hat{j}\hat{j}) + \kappa_{Tz}(\hat{k}\hat{k})$
K_x	permeability in x direction
K_z	permeability in z direction
η	thermal anisotropic parameter κ_{Tx}/κ_{Tz}
ξ	mechanical anisotropic parameter K_x/K_z
$\overline{\lambda_1}$	stress relaxation time
$\overline{\lambda_2}$	strain retardation time

λ_1	relaxation parameter $(\frac{\kappa_{Tz}}{\gamma d^2})\overline{\lambda_1}$
λ_2	retardation parameter $(\frac{\kappa_{Tz}}{\gamma d^2})\overline{\lambda_2}$
Ω	angular velocity of rotation
σ	dimensionless oscillatory frequency
μ	dynamic viscosity of the fluid
μ_c	couple stress viscosity of the fluid
κ_s	effective concentration diffusivity
κ_{Tx}	effective thermal diffusivity in x-direction
κ_{Tz}	effective thermal diffusivity in z-direction
ϕ	porosity
ν	kinematic viscosity
ρ	fluid density
ρ_0	reference density
ψ	stream function
γ	heat capacity ratio $\frac{(\rho c_p)_m}{(\rho c_p)_f}$
ϵ_n	normalised porosity $\frac{\epsilon}{\gamma}$

Other symbols

∇_1^2	$\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$
∇^2	$\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$

subscripts

b	basic state
c	critical
0	reference value

superscripts

'	perturbed quantity
*	dimensionless quantity

osc	oscillatory
st	stationary

Chapter 1

Introduction

Firstly, we introduce some definitions, hydrodynamic equations, methods and laws, along with literature review and applications to comprehend the upcoming chapters.

1.1 Fluid Mechanics

Fluid mechanics is the study of flow and forces within the fluids. It is a branch of science which deals with behaviour of liquids or gases. It has a wide range of applications in many engineering and technological areas involving aerospace, chemical, civil, environmental, mechanical, climatological, geological, meteorological and ocean engineering. The matter is divided into two categories, namely, fluids and solids. If under given thermodynamic conditions and in the absence of any external force, a matter does not change its shape, it is called a solid. On the contrary, if it changes its shape, it is called a fluid. Fluids are further classified into three categories, namely, liquids, gases and plasma. The subject of fluid mechanics is concerned with the properties and behavior of fluids, whether at rest or in motion. The subject deals with pressures, velocities, and accelerations in the fluid, including fluid deformation and compression or expansion. More applications of fluid mechanics can be seen in the study of flows in natural rivers and artificial channels and the flow of ground water, the dispersion of pollutants in the atmosphere, lakes, rivers, and oceans and the flows in the pipelines of crude oil and natural gas from the petroleum fields.

1.1.1 Newtonian and non-Newtonian fluids

In the **Newtonian** fluid, viscosity directly depends upon shear stress. Some examples of **Newtonian** fluid are mineral oil, glycerin, air, water and alcohol etc. While **Non-Newtonian** fluids show completely opposite behaviour i.e. there is no direct dependency of viscosity upon shear stress. Some examples of **Non-Newtonian** fluids are paint, tooth paste etc.

1.1.2 Viscoelastic fluid

Viscoelastic fluid is a type of non-Newtonian fluid which is in the core of research and industry. Viscoelastic fluids are those that show partial elastic recovery upon the removal of a deforming stress. Such materials possess properties of both viscous fluids and elastic solids. Polymeric fluids often show strong viscoelastic effects or in other word. Fluids that have a stress recovery when shear is relaxed are called viscoelastic. Viscoelasticity is divided into two major fields: linear and nonlinear. Linear viscoelasticity is the field of rheology devoted to the study of viscoelastic materials under very small strain or deformation where the displacement gradients are very small and the flow regime can be described by a linear relationship between stress and rate of strain. Nonlinear viscoelasticity is the field of rheology devoted to the study of viscoelastic materials under large deformation, and hence it is the subject of primary interest to the study of flow of viscoelastic fluids. Nonlinear viscoelastic constitutive equations are sufficiently complex that very few flow problems can be solved analytically. The mathematical models have been developed for determining the stress or strain interactions in viscoelastic fluids and their temporal dependencies. Some viscoelastic fluids are given by:

- Maxwell fluid.
- Rivlin-Ericksen fluid.
- Jeffrey (Oldroyd) fluid.
- Walters B' Fluid

Maxwell fluid

Maxwell (1867) proposed a model of viscoelastic fluids, now known as Maxwell fluids. Such fluids have the properties both of viscosity and elasticity. For steady flow, fluid behaves like a Newtonian fluid.

Oldroyd fluid

This model was proposed by Jeffreys (1929). Oldroyd (1953) generalized Jefferys model and provided a different equation of state for viscoelastic fluids. Such fluids, in the absence of retardation, behave like Maxwell fluids while in the absence of stress relaxation and retardation both, yield Newtonian fluids.

1.2 Overview Heat and Mass transfer

Heat transfer and mass transfer are kinetic processes that may occur and be studied separately or jointly. The study of convective heat and mass transfer is based on terms or concepts of mass, momentum and energy. The fluid flow obeys certain principles of mass, momentum and energy. Engineering applications of convective heat and mass transfer are extremely varied. The heat and mass transfer problems deal with the transfer of heat and mass by moving fluids within these thermal and concentration boundary layers.

1.2.1 Heat transfer

Heat transfer takes place in a system when heat flows from high temperature to low temperature. Because of this heat flow thermal instability occurs in the system. Heat transfer process is mainly of three types:

- **Radiation**

  In the process of Radiation, energy transmits in the form of electromagnetic waves. Radiation may take place in the absence of medium. One of the best examples of radiation is Sunlight.

- **Conduction**

  In the process of Conduction, heat transfer occurs between two adjacent parts of the system because of their temperature difference in stationary medium i.e. the medium, is at rest, which may be a solid or fluid. For example, when a metallic rod is heated at one end, the heat gets transferred to the other end by conduction.

- **Convection**

  In the process of Convection, heat transfer occurs by actual motion of matter. Convection can be mainly of two types: Natural Convection and Forced Convection. Convection can take place because of irregular motion of fluid particles in the fluid and also due to temperature gradients. Boiling of water is the best example of convection. Figure 1.1, is the best example to understand all types of heat transfer.

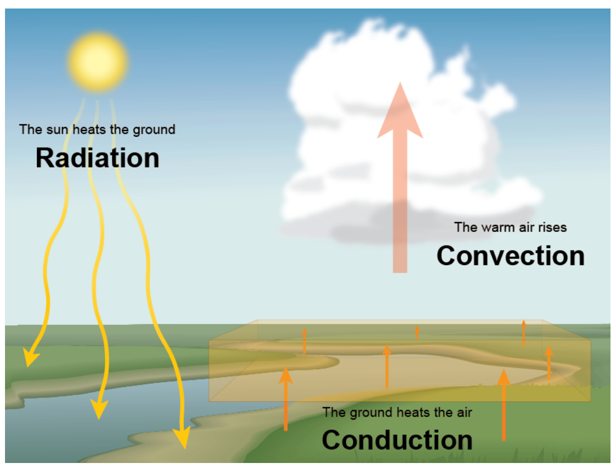

Figure 1.1: kinds of heat transfer

We can divide the convective mechanism as follows:

- **Natural convection** is a type of convection in which heat transfers in fluid because of the difference of density which occurs due to temperature gradient without any external disturbances. For Example boiling of water, mantle convection, and oceanic motions etc.

- **Forced convection** is another type of convection in which heat transport is not only because of temperature gradients but it is also driven by some external force such as fan, pump etc.

- **Mixed convection**, is a type of convection when both types of convection mentioned above take place simultaneously. For Example solar receivers exposed to wind currents, electronic devices cooled by fans etc.

1.2.2 Mass transfer

The driving forces for mass transfer are relatively easy to define. These forces give the expressions for the flux, which can be used in the equations for conservation of mass. When these equations are discretized using a numerical method and when the resulting numerical model equations are solved, the results give the concentration distribution and fluxes in a system as functions of the modeled space coordinates and time. Mass transfer refers to mass in transit due to species concentration gradient in a mixture, must have a mixture of two or more species to occur. Mass transfer occurs by two Mechanisms:

- **Diffusion mass transfer**

  In diffusion mass transfer the transfer of matter occurs by the movement of molecules or species or particles of one component to other. Diffusional mass transfer may occur either due to concentration gradient (Molecular Diffusion) or temperature gradient (Thermal Diffusion) or pressure gradient (Pressure Diffusion).

- **Convective mass transfer**

  Convective mass transfer is a mechanism in which mass is transferred between the fluid and the solid surface as a result of movement of matter from the fluid to the solid surface or fluid. Convective mass transfer is again classified as Natural or Free Convection mass transfer and Forced Convection Mass Transfer.

1.3 Rayleigh Bénard convection

The mathematical theory first developed by Rayleigh is now known as Rayleigh-Benard convection. When the temperature variations within the fluid layer remain small, the Boussinesq approximation is adopted, which neglects compressibility except for the presence of a buoyancy term in the momentum equation. Furthermore, viscous heating is often negligible compared to thermal driving from the boundary conditions. Rayleigh Bénard convection is essentially a type of natural convection. It takes place when a horizontal fluid layer is heated from below, because of this fluid acquires regular patterns of convection cells known as Bénard cells. The main reason of generation of these convective cells is the buoyant force acting between the fluid layers because of temperature differences, see Figure 1.2. Rayleigh Bénard convection depends on the buoyancy force, viscous force and depends on a non-dimensional number known as Rayleigh number defined as $Ra = \frac{\alpha_T g \Delta T d^3}{\nu \kappa_T}$, where d is the depth of fluid layers, ΔT is the temperature difference between the fluid layers, ν is the kinematic viscosity of fluid, κ_T is the thermal diffusivity of the fluid, α_T is the coefficient of thermal expansion and g is the acceleration due to gravity. Chandrasekhar (1961) introduced the classical Rayleigh-Bénard convection as an interesting phenomenon, due to bottom heating of a fluid layer. The convection takes place in the system when the Rayleigh number exceeds a certain value, known as critical Rayleigh number.

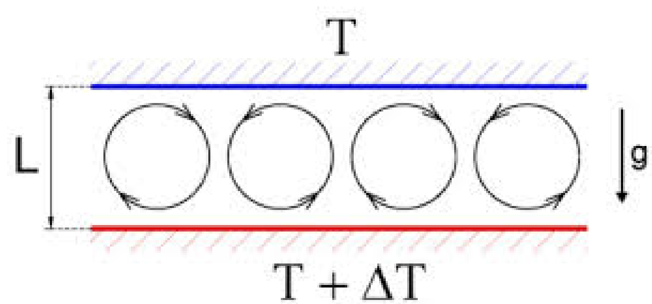

Figure 1.2: Rayleigh Bénard convection

1.3.1 Thermal instability

Consider an infinitely extended horizontal fluid layer of depth d which is confined between two parallel plates, the bottom plane at $z = 0$ while top one is at $z = d$. A Cartesian coordinate system is adopted in such a way that the origin lies on the lower plate and z axis is considered to be vertically upward. The fluid layer is heated from below and cooled from above. The physical configuration of the system is explained in Figure 1.3. After getting enough heat fluid molecules expand and become lighter and so they are forced by heavy molecules to move up and void space is created due to these molecules, which is filled by nearby molecules. The repetition of this mechanism starts the movement in the fluid. When the temperature gradient is so enough between the fluid layers, a small pocket of fluids starts moving up into the colder region having higher density. During this period onset of convection takes place in the system. Bénard(1900), gave the first quantitative experiments of thermal instability in fluid layers. Lord Rayleigh (1916) gave the first theoretical treatment to Bénard experiments.

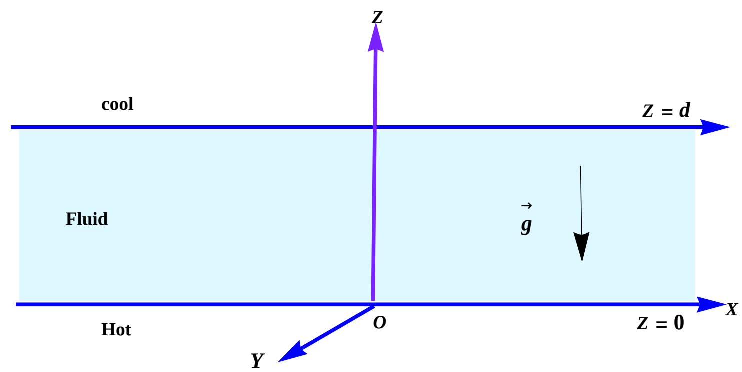

Figure 1.3: Thermal instability Diagram

1.3.2 Porosity

It is ratio of the volume of the voids to the total volume of the porous medium. The porosity of any porous medium ranges between $(0, 1)$. The porosity of porous medium is depicted in Figure 1.4. Let, V_f denotes the volume of the fluid (voids), and V_m denotes the volume of the material (total volume), then the porosity ϵ is given by,

$$\epsilon = \frac{V_f}{V_m}. \tag{1.3.1}$$

Generally, it is assumed that all the void space are connected. If in a medium some pore space is disconnected, then one has to introduce an **effective porosity** defined as the ratio of connected void space to total volume.

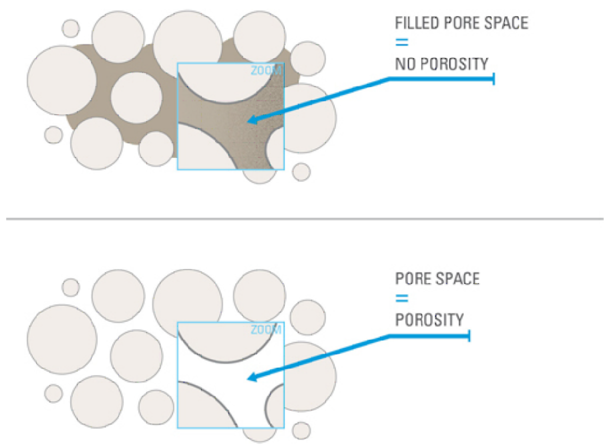

Figure 1.4: Figure of Porosity

1.3.3 Permeability

Permeability is a measure of the ease with which a fluid flows through the porous medium. The degree to which porous within the material is inter-connected is known as effective porosity. The permeability is independent of the nature of the fluid although it depends on the geometry of the medium. This permeability constant was first introduced by Darcy (1856) with the help of his famous experiment known as Darcys law which introduced permeability as a measurable quantity. The permeability of porous medium is depicted in Figure 1.5. Here it is denoted by parameter K.

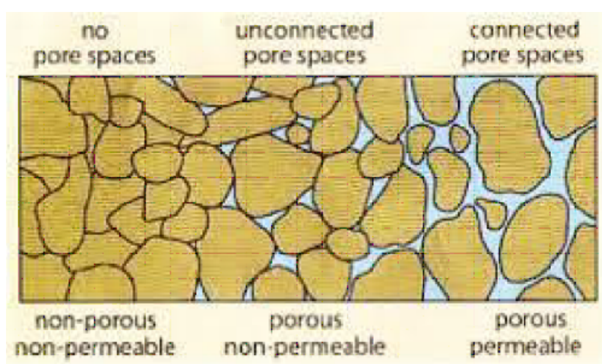

Figure 1.5: Figure of Permeable

1.4 Mechanism of Dimensionless parameters

The magnitude of individual quantities encountered in a physical problem can assemble into dimensionless group using dimensional analysis and the differential equations governing the fluid flow can be recast into dimensionless form. Such equations involve one or more dimensionless parameters and these are the key factors in determining the qualitative and quantitative nature of the flow phenomenon. The following dimensionless parameters appear in this thesis:

1.4.1 Thermal Rayleigh number

The thermal Rayleigh number plays a significant role in liquid layers where the buoyancy forces are important. Physically, it represents the balance of energy released by the buoyancy force and the energy dissipation by viscous and thermal effects. Mathematically, this number denotes the eigen value in the study of stability of thermal convective instability problems. The thermal Rayleigh number is defined as

$$R_T = \frac{\beta_T \rho_0 g \Delta T d^3}{\mu_0 \kappa_T} \tag{1.4.1}$$

where, ΔT is the temperature difference between the boundaries, β_T thermal expansion coefficient, g is the acceleration due to gravity, d is the thickness of the fluid layer, κ thermal diffusivity, μ is dynamic viscosity of the fluid, ρ is fluid density.

1.4.2 Solute Rayleigh number

The Solute Rayleigh number is defined by taking the solute concentration gradient into account. The solute Rayleigh number is defined as

$$R_S = \frac{\beta_S g \Delta S d^3}{\nu \kappa_T} \tag{1.4.2}$$

where, ΔS is the solute concentration difference between the boundaries, β_S salute expansion coefficient, g is the acceleration due to gravity, d is the thickness of the fluid layer, κ

thermal diffusivity, ν is kinematic viscosity of the fluid.

1.4.3 Taylor number

When a layer of liquid is rotating with a uniform angular velocity, a dimensional parameter called Taylor number arises and is defined as

$$Ta = \left(\frac{4\Omega^2 d^4}{\nu^2}\right) = \left(\frac{2\Omega d^2}{\nu}\right)^2 \tag{1.4.3}$$

It is a measure of the square of the ratio of coriolis force to viscous force, where Ω is the characteristic angular velocity, d is depth of fluid layer, and ν is the kinematic viscosity. Generally, the studies show that onset of convection gets delayed by the effect of rotation.

1.4.4 Internal Rayleigh number

The internal Rayleigh number represents the relative importance of the temperature dependent heat source(sink) over thermal diffusion. The internal Rayleigh number is defined as

$$R_i = \frac{Qd^2}{\kappa_T} \tag{1.4.4}$$

where Q is the quantity of internal heat generation, d is the thickness of the fluid layer.

1.4.5 Prandtl number

It represents the ratio of the kinematic viscosity to the thermal diffusivity. It is defined as

$$P_r = \frac{\nu}{\kappa} \tag{1.4.5}$$

This parameter is a measure of the relative importance of viscosity and thermal diffusivity in the flow field.

1.4.6 Lewis number

It is defined as the ratio of thermal diffusivity to mass diffusivity. It is used to characterize fluid flows where there is simultaneous heat and mass transfer by convection. It is named after Warren K. Lewis (1882-1975).

$$Le = \frac{\kappa_T}{\kappa_S} \tag{1.4.6}$$

where κ_T is the thermal diffusivity, κ_S is the mass diffusivity.

1.4.7 Nusselt number

This parameter is an important one associated with heat transfer problems where the efficiency of convection is measured by a normalized heat transport called a Nusselt number.

$$N_u = \frac{\text{Heat transport by(conduction + convection)}}{\text{Heat transport by(conduction)}} \tag{1.4.7}$$

1.4.8 Sherwood number

It represents the ratio of the convective mass transfer to the rate of diffusive mass transport, and is named in honor of Thomas Kilgore Sherwood. The Sherwood number (also called the mass transfer Nusselt number) is a dimensionless number used in mass-transfer operation.
.

$$S_h = \frac{\text{Convective mass transfer rate}}{\text{Diffusion rate}} \tag{1.4.8}$$

1.5 Couple-stress fluid

Stokes (1966) was the first to propose the model for couple-stress fluid. Couple-stress fluid is a type of non-Newtonian fluid having polar effects. Couple stress fluid theory is a simple generalization of the classical theory of viscous Newtonian fluids that allow the sustenance of couple stresses and body couples in the fluid medium. The utilization of couple-stress fluid is in the study of mechanisms of lubrication of synovial joints. The synovial fluid has been modelled as a couple-stress fluid in human joints by Walicki and Walicka (1999).

Some more examples are in the fields of extrusion of polymer fluids, solidification of liquid crystals, animal bloods etc. The couple-stress parameter affects the momentum equation of the system. Some examples of couple-stress fluid are; Paints, Polymer, blood. Let C is the couple-stress parameter, then

$$C = \frac{\mu_c}{\mu d^2}, \tag{1.5.1}$$

where μ, μ_c are fluid viscosity and couple stress viscosity respectively.

1.6 Internal heating

Internal heat is the main cause of energy for celestial bodies caused by nuclear fusion and decaying of radioactive materials, which keeps the celestial objects warm and active. It is due to the internal heating of the earth that there exists a thermal gradient between the interior and exterior of the earth's crust, saturated by multi-components fluids which helps convective flow. Therefore, there are huge number of applications of internal heat generation like in geophysics, reactor safety analyses, metal waste form development for spent nuclear fuel. This term affects the energy equation of convective system. In this work, it is considered as a convective parameter R_i, which shows a destabilize effect and given by

$$R_i = \frac{Qd^2}{\kappa_T}, \tag{1.6.1}$$

where Q is the internal heat coefficient.

1.7 Porous medium

A porous medium is a material containing a large number of voids (also called pores). In other words, it is a rigid body with a large number of openings (pores). All rocks in the upper parts of the earths crust, irrespective of their type, age or origin, contain openings. However, it is difficult to provide an exact geometrical definition of the notion of a pore. Pores may be either linked (connected) or isolated (non-connected), distributed in the material more or less frequently either in a regular manner or in an indiscriminate manner.

Interconnected pores are the effective pores, and ineffective pores are those pores through which fluid can not pass. This may be either due to surface tension caused by fine holes or the holes may not be interconnected, so that they do not influence the flow behavior but may influence the compressibility of a medium. In a porous medium, the voids are classified

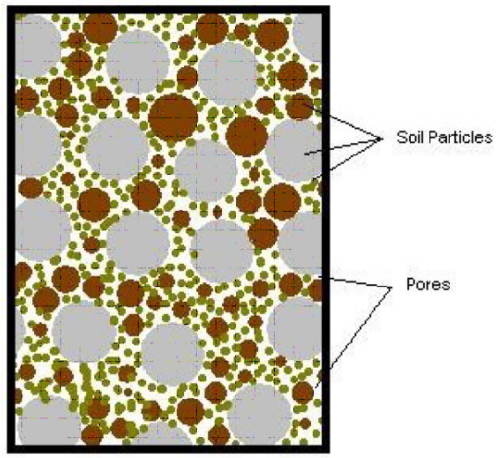

Figure 1.6: Pore

according to the behavior of the fluid within these spaces. The interstices or capillaries are the smallest void spaces in which molecular forces between the molecules of the solid and those of the fluid are significant. Caverns are the largest void spaces in which the fluids are partially influenced by the walls of the voids. Intermediary space in size between capillaries and caverns are referred to as pores. In all situations, the void spaces partially or completely influence the motion of the flowing fluids in these spaces. The interconnectedness of the pores allows the fluids to flow through the material. The nature of the porous medium depends on the interconnected pores and their location, size and shape. A porous medium is homogeneous if its average properties are independent of location, and heterogeneous if they depend on location. A natural porous medium has an irregular distribution of pores with respect to shape and size. A porous medium is not necessarily restricted to have the pores belonging to only one class and may be embedded with the pores of different sizes and shapes. According to this description, the term porous medium covers a very wide range of substances. Beach sand, sandstones, limestone, rye bread, wood, rocks, soil and human lungs are examples of the natural porous medium whereas concrete, cement, ceramics, bricks, paper cloth and filter paper are some examples of the man-made porous medium,biological tissues (e.g. bones). All kinds of porous media both natural and artificial have a random void structure. It is quite difficult to characterize a porous medium in terms of sizes, shapes, orientations and interconnections of voids in the medium, but it is rather characterized by its two properties namely porosity and permeability, which take care of all the complexity of a porous medium. In a porous medium, the distribution of pores with respect to shape and size is irregular. The shape of porous medium is depicted in Figure 1.7.

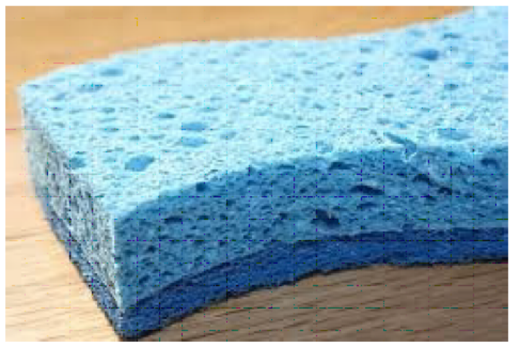

Figure 1.7: Porous Medium Diagram

1.8 Heterogeneity of Porous media

1.8.1 Homogenous porous medium

A medium is homogenous with respect to a property if the property is independent of position within the medium.

1.8.2 Isotropic porous medium

A medium is isotropic with respect to a property if the property is independent of the direction within the medium.

1.8.3 Anisotropic porous medium

Epherre(1977) was the first to study the anisotropic porous medium. Anisotropic porous medium is defined as a porous medium having different permeability in different direction. It has its applications in chemical engineering process and insulating purpose. Castinel and Combarnous (1974) conducted the first study on the convective flow in anisotropic porous media. Their work was supplemented by Epherre(1977) and Tyvand (1980) which took into account respectively the anisotropy of the thermal diffusivity and the effect of dispersion in the case of an uniform flow. The effects of anisotropy on the convective stability of the porous layer are reported by McKibbin (1992).

1.8.4 Saturated porous medium

A porous medium is said to be saturated porous medium if its pore volume is connected and inhabited by a specific fluid. As all pores filled with water.

1.8.5 Unsaturated porous medium

A porous medium is said to be unsaturated porous medium if its pore volume is not fully inhabited by a fluid. e.g. Dry sponge.

1.9 Hoton-Rogers-Lapwood convection

Horton and Rogers (1945) and Lapwood (1948) studied the Rayleigh Bénard convection in a horizontal porous medium, therefore named as Hoton-Rogers-Lapwood convection or Darcy Bénard convection. In this case the non-dimensionlised Darcy Rayleigh number is defined as $Ra_D = \frac{\alpha_T g \Delta T K d}{\nu \kappa_T}$, here K is permeability. Thermal convection in porous media has attracted the researchers very much because of its huge applications in various fields like petroleum industry, chemical engineering and geophysics etc. Interested authors can read the unique books (Problem related to thermal instability in a fluid saturated porous medium) are given by Ingham and Pop(2005), Nield and Bejan(2012) and Vafai(2000).

1.10 Double diffusive convection

Double-diffusive convection arises due to combined diffusion of heat and mass, driven by buoyancy effect. The density gradients that provide the driving buoyancy force are induced by the combined effects of temperature and concentration in the fluid non-uniformly. If heat and salt are involved in fluid saturated medium, the heat diffuses more rapidly than a dissolved substance and convection sets in. A two component fluid layer is displaced vertically and loses excess heat more rapidly than excess solute. To compare with mono diffusion, a single component fluid layer is stable if the density decreases in the vertically upward direction whereas a fluid layer consisting of two components can diffuse relative to each other and may be dynamically unstable. The resulting buoyancy may act either to increase the displacement of particle, cause of a monotonic instability, or reverse the direction of displacement of particles, cause of an oscillatory instability, depending on whether the solute gradient is destabilizing and the temperature gradient is stabilizing or vice-versa. In terms of temperature T and concentration S, the density of mixture is expressed as

$$\rho = \rho_0[1 - \alpha_T(T - T_0) + \beta_T(S - S_0)], \tag{1.10.1}$$

where α_T and β_T are the thermal and solute expansion coefficients, T and S are the temperature and the solute content, respectively.

1.11 Cross diffusive convection

The convection driven by buoyancy that is contributed by two different diffusion components, as temperature and solutal concentration with differing rates of diffusion is widely known as double diffusive convection or two component convection. Furthermore,when two transport processes take place simultaneously they interfere with each other, producing cross diffusion effects. This has two types of effects; one as Soret effect and other is Dufour effect. Soret effect was introduced by Soret (1879) in fluids, it refers to mass flux produced by a temperature gradient. The energy flux caused by a composition gradient was discovered in 1873 by Dufour and was correspondingly referred to the Dufour effect. It can also be known as diffusion thermo effect to indicate that it is the inverse of thermal diffusion or Dufour effect refers to heat flux produced by a concentration gradient. For porous media, the phenomena of cross diffusion are complicated because of the interaction between the fluid and porous matrix, and accurate values of cross-diffusion coefficients are not available. The quantitative experimental data suitable for model validation are quite rare.

1.12 Governing equations

In fluid dynamics, every fluid follows certain basic properties, where the hydrodynamical flow of a viscous fluid is governed by varying density and temperature, which are given below:

(a) Equation of continuity

The equation of continuity basically represents the conservation of mass. It states that, the rate of generation of mass within a given volume must be balanced by an equal net outward flow of mass through the volume. The differential form of equation of continuity is:

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \vec{q}) = 0. \tag{1.12.1}$$

For incompressible fluid, the above expression is reduces to

$$\nabla.\vec{q} = 0, \tag{1.12.2}$$

where $\vec{q}$ is fluid velocity and ρ is fluid density.

(b) Momentum equation

$$\frac{\partial \vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \vec{q} + F, \tag{1.12.3}$$

where ν is kinetic viscosity, p is pressure, F is the body force term, represents an external forces that act on the fluid, for example: gravity, wind, etc.

(c) Energy or temperature equation

$$\frac{\partial T}{\partial t} + (\vec{q}.\nabla)T = \kappa_T \nabla^2 T + Q(T - T_0). \tag{1.12.4}$$

It is based on the law of conservation of energy. κ_T is the thermal conductivity which is proportional constant in Fourier's law of heat conduction. Q is coefficient of internal heat.

(d) Boussinesq approximation

Boussinesq approximation introduced by Boussinesq (1903) consists essentially of neglecting variation of density in the inertia term but retaining them in the buoyancy term. The Boussinesq approximation is a common method to deal with the convective problems. This approximation states that, the density differences are sufficiently small that it can be neglected everywhere except where they appear in terms multiplied by $\vec{g}$, the acceleration due to gravity. Mathematically, it is defined as

$$\rho = \rho_0[1 - \alpha_T(T - T_0)]. \tag{1.12.5}$$

Here α_T is the thermal expansion coefficient, and subscript 0 is the reference value.

1.13 Hydrodynamic equations for porous medium

(a) Equation of continuity

$$\nabla.\vec{q} = 0. \tag{1.13.1}$$

(b) Momentum equation

$$\frac{\rho}{\epsilon}\frac{\partial \vec{q}}{\partial t} = -\nabla p - \frac{\mu}{K}\vec{q} + \rho_f g. \tag{1.13.2}$$

(c) Energy or temperature equation

$$\frac{\partial T}{\partial t} + (\vec{q}.\nabla)T = \kappa_T \nabla^2 T. \tag{1.13.3}$$

(d) Oberbeck-Boussinesq approximation

$$\rho_f = \rho_0[1 - \alpha_T(T - T_0)], \tag{1.13.4}$$

where ϵ is porosity of the porous medium, μ is the dynamic viscosity, K is permeability and κ_T is overall thermal conductivity.

1.13.1 Darcy law

The fluid flow in a porous medium is governed by this law, which is given by Henry Darcy in (1856). He found that there is a proportionality relation between flow rate and the applied pressure difference. Darcys law is applicable when the flow is streamlined or laminar in which the movement of individual fluid particles in a pore flow along paths nearly parallel to the walls of the pores. The laminar flow region is at low flow rates, where inertial effects are very small that they can be neglected and it breaks down at adequately high flow rates. Another restriction on the use of Darcys law applies to the flow of gases at low pressures. If the velocities in the pores become significantly high, the inertial forces may become commensurable with the frictional forces. Mathematically defined as

$$\vec{q} = -\frac{K}{\mu}\nabla p, \tag{1.13.5}$$

where all the parameters already discussed in above sections.

1.13.2 Brinkman-extended Darcy model

The above Darcy law is valid for only sufficiently small and linear seepage velocity ($\vec{q}$). Thus for high velocity i.e. for high Reynold numbers, the extension of Darcy law is given by

$$\nabla p = -\frac{\mu}{K}\vec{q} + \overline{\mu}\nabla^2\vec{q}, \tag{1.13.6}$$

where $\overline{\mu}$ is a quantity having the dimension of viscosity and it is known as the effective viscosity. The effective viscosity is not expected to be the same as the viscosity of the fluid.

1.14 Boundary conditions

To study any dynamical system, the boundary conditions are used for the calculation of dependent variables. On the basis of lower and upper boundaries of a convective system, given below are some different types of boundary conditions such as rigid-rigid, free-free, rigid-free and free-rigid.

1. Zero normal velocity: $q = 0$ for both rigid and free boundaries.

2. **Rigid boundary**

   (a) Zero tangential velocity (no slip): For a viscous fluid, the boundary condition on a surface assumes no relative velocity between the surface and the liquid immediately at a the surface. If the boundary surface is stationary, with the flow moving past it, then

   $$u = v = w = 0$$

   (b) For an inviscid fluid, the flow slips over the surface, hence, at the surface the flow must be tangent to the surface.

   $$\vec{q}.\vec{n} = 0$$

   where $\vec{n}$ is a unit vector perpendicular to the surface. The boundary conditions elsewhere in the flow depend on the type of problem being considered, and usually pertain to inflow and outflow boundaries at a finite distance from the surfaces, or aninfinity boundary condition infinitely far from the surfaces.

   Boundary condition for the z-component of velocity are

   $$w = \frac{\partial w}{\partial z} = 0, \text{ for rigid boundaries.}$$

3. **Free boundary**

   Zero tangential stress: In the case of a free surface the boundary conditions for velocity depends on the whether. If there is no surface tension at the boundary,i.e. the free surface does not deform in the direction normal to itself, then

   $$w = 0$$

   Consider the z-axis perpendicular to the x y plane,therefore w does not vary with respect to x and y, i.e.

   $$\frac{\partial w}{\partial x} = 0 \qquad \text{and} \qquad \frac{\partial w}{\partial y} = 0$$

   Boundary condition for the z-component of velocity are

   $$w = \frac{\partial^2 w}{\partial z^2} = 0, \text{ for free boundaries.}$$

   This is the stress-free condition.

4. **Conducting (isothermal)**: For isothermal boundary wall, the temperature disturbances must be zero at the boundary.

5. **Insulated (adiabatic)**: For adiabatic boundary wall, the temperature of the wall change, but there should be no through-flow of temperature.

1.15 Methods of solution

1.15.1 Numerical and Analytical methods

Stability Theory: Consider a basic state (steady state) of a physical problem whose stability is to be examined. Consider a state near this basic state and examine whether the system will tend to the considered basic state as time passes. To examine the stability or instability of a given physical system, examine as to how the equilibrium configuration reacts to small fluctuations (disturbances) which are either inherent in the system or man made. The system is said to be stable with respect to a particular disturbance (also called perturbation) if the perturbation dies down gradually and the system returns to its original position. On the other hand, if even a single perturbation grows with time and the system never returns to its original position, the system is said to be unstable. A system is said to be stable if it is stable with respect to all perturbations to which it could be subjected. If the system is unstable even for one special mode of perturbation, it is said to be unstable. The stability of any flow depends upon the nature of acting infinitesimal disturbances. If the initial disturbances are small, the governing equations may be linear. For analysing the stability there are two methods, given below:

- **Energy method**

  We calculate the kinetic energy of the perturbations in this method, and if this kinetic energy increases along with time increases, then the flow is unstable, and if it tends to zero at time tends to infinity then the flow is stable. This method fails to provide the information of unstable system, so it is not generally used for convective problems. Energy method is global in nature as the kinetic energy of the whole system is calculated and because of this its applications are restricted. While this provides a surest limit for the stability of the flow, it is crude in giving the unstable limit. Moreover, it gives very little information about the local behavior of the perturbations. For investigating the stability of a flow, once the fluid is confined within rigid boundaries, sometimes the vorticity of the perturbations is considered in place of kinetic energy.

- **Normal mode method**

  In this method, for determining the stability of a system, the linearized perturbation equations are formed, within the linear framework, from the equations of conservation of mass, momentum and energy, retaining only the linear terms in perturbed quantities. These equations are then solved either analytically or with the help of variational method or through an integral equation with a set of appropriate boundary conditions. The stability of the system leads to the dispersion relation in the parameters. Sometimes this dispersion relation is quite complex and an analytical interpretation may not be possible. Therefore, to determine the effect of a particular physical parameter on the growth rate of the perturbations, the change by varying that parameter is analyzed while keeping the other parameters fixed. An increase in the growth rate implies the destabilizing influence of that particular parameter and a decrease in the growth rate shows the stabilizing influence of the parameter. For providing a complete stability analysis of a given system, an arbitrary disturbance is expressed as a superposition of certain possible modes, and then the stability of the system with respect to each of these modes is investigated, based on oscillations theory. Thus if $f'(x, y, z, t)$ is a typical wave component describing a disturbance, it is expressed mathematically, as $f'(x, y, z, t) = f(z)exp[i(k_x x + k_y y) + st]$ where $k^2 = k_x^2 + k_y^2$ is the wave number and s is a constant to be determined which, in general, is complex so that $s = s_r + is_i$. Infinitesimal perturbations of the rest state may either damp or grow depending on the value of the parameter s. The stability analysis is performed by using frequency of perturbation (s). If $Re(s > 0)$, then the disturbance will grow exponentially with time, and it will represent an unstable system. If $Re(s = 0)$, then the system will be neutrally stable. If $Re(s < 0)$, then it shows a stable system as the disturbance reduces exponentially with time. When $s_r = 0$ $s_i = 0$, the system is marginally stable under the principle of exchange of stabilities while $s_r = 0$ but $s_r \neq 0$ represents overstability of periodic oscillatory motion.

In particular, the stability of the system will depend on its stability to the disturbances of all wave numbers, and its instability will emanate from the instability with respect to even one wave number.

Perturbation method: Sometimes, it is not easy to solve an engineering problems, mathematical non-linear governing equation in real life, directly. In this method, the solution is represented by the initial few terms of an asymptotic expansion. The expansions may be carried out in terms of introduced parameter which appears in the considered equation, known as perturbation parameter. Here, χ is assumed as a perturbation parameter such that the solution is available and reasonably simple for $\chi = 0$, this process is known as regular perturbation method. There is one more kind of perturbation method, known as singular perturbation method, where the solution cannot be approximated by setting the parameter value to zero. Mathematically, an expression can be used for approximation to the solution of U

$$U = U_0 + \chi U_1 + \chi^2 U_2 + ..., \tag{1.15.1}$$

where U_0 would be the known solution to the exactly solvable initial problem and $U_1, U_2, ..$ are represent the higher-order terms which may be found by successive iterations.

A truncated representation of Fourier series method: The linear stability analysis is sufficient to find the stability condition of the motionless solution and the corresponding eigenfunctions describing qualitatively the convective flow, however it cannot give information related to the values of the convection amplitudes, nor regarding the rate of heat and mass transfer. In order to get this additional information, non-linear analysis is performed which will help in understanding the physical mechanism with minimum amount of mathematical analysis and is a step forward toward understanding full non-linear problem. The Fourier expressions for the physical variables such as stream function, temperature and solute concentration, are given by

$$\psi = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} A_{mn}(\tau)\sin(max)\sin(n\pi z), \tag{1.15.2}$$

$$T = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} B_{mn}(\tau)\cos(max)\sin(n\pi z), \tag{1.15.3}$$

$$\phi = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} C_{mn}(\tau)\cos(max)\sin(n\pi z). \tag{1.15.4}$$

1.16 Heat and Mass transfer in fluid layer and porous medium

Heat and mass transfer phenomena are found everywhere in nature and are important in all branches of science and technology. The application of heat mass transfer processes go to greater lengths in numerous fields of science, engineering and technology. Heat and mass transfer operations quite often occur in the fields of electric engineering, civil engineering, aeronautics, metallurgy, environmental engineering, refrigeration, air conditioning, biological and industrial processes. The study of geophysics, astronomy, meteorology, agriculture, oceanography and food processing demand the knowledge of heat and mass transfer. Heat and mass transfer flows are highly significant for their varied practical importance. Many examples of heat and mass transfer applications can be cited from the environment. Beavers and Joseph (1967) proposed boundary conditions at a naturally permeable wall. Natural convection in a volumetrically heated fluid layer at high Rayleigh numbers is investigated by Fan-bill B. Cheung (1977). Chen and Chen (1988) studied onset of finger convection in a horizontal porous layer underlying a fluid layer. Poulikakos (1986) proposed Buoyancy-driven convection in a horizontal fluid layer extending over a porous substrate. Beavers et al. (1974) analysed boundary conditions at a porous surface which bounds a fluid flow. Kim and Kim (2001) studied the onset of natural convection and heat transfer correlation in horizontal fluid layer heated uniformly from below. Abdullah AbbasKendoush (2009) shows theoretical analysis of heat and mass transfer to fluids flowing across a flat plate.

The porous media analogue of Rayleigh-Bénard convection is known as Horton Rogers Lapwood convection, and was first studied by Horton and Rogers (1945) and by Lapwood (1948), independently. Nield (1968) was the first to investigate double diffusive generalization of the Horton Rogers Lapwood problem. Lapwood examined the breakdown of a stability of a fluid subject to a vertical temperature gradient in a porous medium and also discussed the possibility of convective flow. The linear stability analysis of the thermosolutal convection in a sparsely packed porous layer was made by Poulikakos (1986) using DarcyBrinkman model. The double- diffusive convection in porous media in the presence of Soret and Dufour coefficients has been analyzed by Rudraiah and Malashetty (1986). Malashetty and Gaikwad (2002) studied the effect of cross diffusion for Soret and Dufour coefficients on the double-diffusive convection in an unbounded vertically stratified two component system with compensating horizontal thermal and solute gradients.

The theory of polar fluids has received wider attention in recent years because the traditional newtonian fluids cannot precisely describe the characteristics of the fluid flow involved therein. These fluids deform and produce a spin field due to the microrotation of suspended particles forming micropolar fluid developed by Eringen (1966). The micropolar fluids take care of local effects arising from microstructure and as well as the intrinsic motions of microfluidics. The spin field due to microrotation of freely suspended particles set up an anti-symmetric stress, known as couple stress, and thus forming couple-stress fluid. Thus, couple-stress fluid, according to Eringen (1966), is a particular case of micropolar fluid when microrotation balances with the natural vorticity of the fluid. Malashetty et al. (2006) investigated the double-diffusive convection in a two-component couple stress liquid layer with Soret effect using both linear and nonlinear stability analyses. Stability analysis of a Maxwell fluid in a porous medium heated from below was investigated by Wenchang Tan and Takashi Masuoka (2007). Gaikwad and Kouser (2013) performed onset of Darcy-Brinkman convection in a binary viscoelastic fluid-saturated porous layer with internal heat source. The work was extended by Gaikwad and Dhanraj (2014) to study Soret effect on DarcyBrinkman convection in a binary Viscoelastic fluid-saturated porous layer. Alok Srivastava et. al. (2014) investigated the effect of internal heating on double diffusive convection in a couple stress fluid saturated anisotropic porous medium. Moli Zhao (2014) analysed double diffusive convection in a Maxwell fluid saturated porous layer with internal heat source.

More recently Gaikwad and Javaji (2016) examined the onset of Darcy-Brinkman Convection in a Maxwell Fluid saturated anisotropic porous layer. Gaikwad et. al. (2016) investigated the effect of Soret parameter on the onset of double diffusive convection in a binary viscoelastic fluid saturated porous layer with internal heat source using linear stability theory. Sumathi and Aiswarya (2017) studied a stability analysis of thermal convection of a saturated rotating, viscoelastic porous layer. Altawallbeh et. al.(2017) examined the linear stability analysis of double diffusive convection in a viscoelastic fluid saturated porous layer with cross diffusion effects and internal heat source. Srivastava and Singh (2018) performed linear and weak nonlinear double diffusive convection in a viscoelastic fluid saturated anisotropic porous medium with internal heat source.

1.17 Applications

Convection in porous media has so many practical applications in modern science and engineering. Scientists and Engineers study flow aspects like momentum, heat and mass transfer etc in porous media to apply them to a huge area of modern industrial fields that include:

(a) Petroleum industries need complete knowledge of flow mechanism in order to increase their recovery.

(b) Since the earth has huge amount of energy resources trapped inside it, especially near the core. This heat needs to be brought upto the surface from deep interiors travelling its way through different porous rocks. Here seems to be a need for effective thermal conductive fluids.

(c) Chemical engineers require the knowledge for mass transfer and chemical reaction processes in porous reactors.

(d) Nuclear scientists also require porous media knowledge for effective coolants and mass transfer fluids for proper functioning of nuclear rectors.

(e) Medical implants are a very crucial applications of porous materials. Porous materials like stainless steels and titanium are mainly used for this purpose.

(f) Porous materials can also be used as cells, grinding wheels, thermal insulators, capacitors, catalysts etc.

Chapter 2

Internal heating and Soret effect on Darcy - Brinkman convection in a binary viscoelastic fluid saturated porous layer

2.1 Introduction

There is a large number of practical situations in which convection is driven by internal heat source in a porous medium. The wide applications of such convection occur in nuclear reactions, nuclear heat cores, nuclear energy, nuclear waste disposals, oil extractions and crystal growth. The study concerning internal heat source in porous media is provided by Tveitereid (1977), who obtained the steady solution in the form of hexagons and two dimensional rolls for convection in a horizontal porous layer with internal heat source. Horton and Rogers (1945) and Lapwood (1948) were the first to obtained analytically the expression for critical Rayleigh number for the onset of convection in a fluid-saturated porous layer

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heated from below. Bejan (1978) studied analytically the buoyancy induced convection with internal heat source, Parthiban and Patil (1995) studied the effect of non-uniform boundaries temperature on thermal instability in a porous medium with internal heat source and predicted that internal heat source parameter advances the onset of convection. Hill(2005) performed linear and nonlinear analyses on double-diffusive convection in a porous layer with a concentration based internal heat source. Bhadauria et al. (2014)-(2012) studied effect of internal heating on double diffusive convection in a couple stress fluid saturated anisotropic porous medium and also natural convection in a rotating anisotropic porous layer with internal heat source. Khan and Aziz(2012) studied transient heat transfer in a heat-generating fin with radiation and convection with temperature-dependent heat transfer coefficient. Further, there are many studies available on the effect of cross-diffusion on onset of double-diffusive convection in a porous medium. Thermal convection in a binary fluid driven by the Soret and Dufour effects has been investigated by Knobloch (1980). Hurle and Jakeman (1971) performed a theoretical study of Soret driven thermosolutal convection in a binary fluid mixture. Linear and nonlinear analyses of double diffusive convection in a fluid saturated porous layer with cross-diffusion effects has been carried out by Malashetty and Biradar (2012). Rudraiah and Malashetty (1986) carried out a study on double diffusive convection in a porous medium in the presence of Soret and Dufour effects, while Gaikwad et. al.(2009 a,b, 2012 a,b) performed a linear and nonlinear double diffusive convection in a fluid-saturated anisotropic porous layer with cross-diffusion and obtained the effect of cross diffusion coefficients. Bhadauria et al. (2013) investigated the double diffusive convection in a fluid saturated anisotropic porous layer with Soret effect and internal heat source. Rudraiah and Siddheshwar (1998) did a weak nonlinear stability analysis of double diffusive convection with cross diffusion in a fluid saturated porous medium and obtained some very interesting results. Convection in binary fluids is a complex process. The presence of concentration currents as well as thermal currents leads to linear and nonlinear behavior. In a binary fluid, the density depends on both temperature and solute concentration. This leads to a competition between heat diffusion and solute diffusion, and consequently oscillatory motions may occur. The oscillatory convective instability in binary fluid mixtures is well understood by Platten and Legros (1984).Taslim and

Narusawa (1986) investigated binary fluid composition and double diffusive convection in a porous medium. Further, the studies of double diffusive convection in porous media plays very significant roles in many areas such as in petroleum industry, solidification of binary mixture, migration of solutes in water saturated soils. Other examples include; geophysics system, crystal growth, electrochemistry, the migration of moisture through air contained in fibrous insulation, Earth's oceans, magma chambers etc. The studies on double diffusive convection in a porous media has been presented in details by Ingham and Pop (2005), Nield and Bejan (2013), Vafai (2000),(2005) and Vadasz (2008) in their books. Further, it was performed by many other researchers, namely; Poulikakos (1986), Travison and Bejan (1986), Momou (2002) etc. The very first study on double diffusive convection in porous media was mainly concerned with linear stability analysis, and was performed by Nield (1968). It is well known that the Darcy's law is not valid for non-Newtonian fluid flows in porous media. Swamy et. al.(2012) studied the onset of Darcy-Brinkman convection in a binary viscoelastic fluid saturated porous layer, where the modified Darcy-Brinkman-Oldroyd model has been developed. However, published works on thermal convection of viscoelastic fluids in porous media are fairly limited. Rudraiah et al.(1990) have studied the thermal stability of a viscoelastic fluid saturated sparsely packed porous layer. Kim et al.(2003) studied the thermal instability of viscoelastic fluids in a porous medium by performing linear and nonlinear analyses. Yoon et al.(2004) analyzed a linear theory the onset of thermal convection in a horizontal porous layer saturated with a viscoelastic liquid. Zhang et. al.(2008) carried out thermal convection for Oldroyd-B fluids in porous media with linear and nonlinear, heated from below. Gaikwad and Kouser (2013) investigated the onset of convection in a binary viscoelastic fluid saturated porous layer with internal heat source use of Darcy-Brinkman. Gaikwad and Dhanraj (2014) studied Soret effect on Darcy-Brinkman convection in a binary viscoelastic fluid-saturated porous layer and studied the cross diffusion effects on convective instability. Stability analysis of Soret-driven double diffusive convection of a Maxwell fluid in a porous medium has been investigated by Wang and Tan (2011). Narayana et al. (2012) performed linear and nonlinear stability analysis of binary Maxwell fluid convection in a porous medium with Soret and Dufour effects. Malashetty et al. (2009) have investigated the onset of convection in a binary viscoelastic fluid saturated porous layer. Kumar and Bhadauria (2011) performed stability analysis to study thermal instability in a rotating anisotropic porous layer saturated by a viscoelastic fluid. Malashetty et al. (2006) did an analytical study of linear and nonlinear double diffusive convection with Soret effect in couple stress liquids. More recently, Gaikwad and Kamble (2015) have studied theoretically, the cross diffusion effects on convective instability in porous media and Gaikwad et al. (2016) have performed a study on double diffusive convection in a binary viscoelastic fluid saturated porous layer with Soret effect and internal heat source. Therefore in the present chapter, linear and nonlinear stability analyses have been performed to study the effect of internal heat and Soret parameter on Darcy-Brinkman convection in a binary viscoelastic fluid saturated porous layer.

2.2 Mathematical Formulation

Consider a viscoelastic fluid saturated porous layer, confined between two infinitely extended horizontal planes at $z = 0$ and $z = d$, heated from below and cooled from above. An internal heat source term has been included in the energy equation. A cartesian frame of reference is chosen so that the origin lies on the lower plate and the z-axis as vertical upward. An adverse temperature gradient is applied across the porous layer. The lower planes is kept at temperature $T_0 + \Delta T$, while upper planes is kept at temperature T_0 with concentration $S_0 + \Delta S$, and S_0 respectively. The governing equations are as given

$$\begin{cases} \nabla.\vec{q} = 0, \\ (1 + \overline{\lambda_1}\frac{\partial}{\partial t})(\frac{\rho_0}{\varepsilon}\frac{\partial q}{\partial t} + \nabla p - \rho g) = (1 + \overline{\lambda_2}\frac{\partial}{\partial t})(\mu_c \nabla^2 q - \frac{\mu}{k} q), \\ \gamma\frac{\partial T}{\partial t} + (\vec{q}.\nabla)T = K_{11}\nabla^2 T + Q(T - T_0), \\ \varepsilon\frac{\partial S}{\partial t} + (\vec{q}.\nabla)S = K_{22}\nabla^2 S + K_{21}.\nabla^2 T, \\ \rho = \rho_0[1 - \beta_T(T - T_0) + \beta_S(S - S_0)] \end{cases} \tag{2.2.1}$$

where the physical variables have their usual meanings as given in the nomenclature. The externally imposed thermal and solutal boundary conditions are given by

$$\begin{cases} T = T_0 + \Delta T, & \text{at} \quad z = 0 \quad \text{and} \quad T = T_0, \quad \text{at} \quad z = d, \\ S = S_0 + \Delta S, & \text{at} \quad z = 0 \quad \text{and} \quad S = S_0, \quad \text{at} \quad z = d, \end{cases} \tag{2.2.2}$$

2.3 Basic state

At the state, velocity, pressure, temperature, concentration and density profiles are given by

$$\vec{q_b} = 0, p = p_b(z), T = T_b(z), S = S_b(z), \rho = \rho_b(z). \tag{2.3.1}$$

Substituting Eq. (2.3.1) in Eq. (2.2.1), the following relations are obtained:

$$\frac{dp_b}{dz} = -\rho_b g, \tag{2.3.2}$$

$$K_{11}\frac{d^2T_b}{dz^2} + Q(T_b - T_0) = 0, \tag{2.3.3}$$

$$\frac{d^2S_b}{dz^2} = 0, \tag{2.3.4}$$

$$\rho_b = \rho_0[1 - \beta_T(T_b - T_0) + \beta_S(S_b - S_0)]. \tag{2.3.5}$$

Use of the boundary conditions (2.2.2),in the solution of Eq. (2.3.3), is given by

$$T_b = T_0 + \Delta T\frac{\sin\sqrt{R_i}(1 - \frac{z}{d})}{\sin\sqrt{R_i}}. \tag{2.3.6}$$

Use of the boundary conditions (2.2.2),in the solution of Eq. (2.3.4)

$$S_b = S_0 + \Delta S(1 - \frac{z}{d}) \tag{2.3.7}$$

Now, superimposing infinite amplitude disturbances on the basic state in the form:

$$\vec{q} = q_b + q', T = T_b + T', p = p_b + p', S = S_b + S', \rho = \rho_b + \rho', \tag{2.3.8}$$

the following set of equations is obtained:

$$\begin{cases} \nabla.\vec{q'} = 0, \\ (1+\lambda_1\frac{\partial}{\partial t})(\frac{\rho_0}{\varepsilon}\frac{\partial q'}{\partial t} + \nabla p' - \rho(\beta_T T' - \beta_S S')g) = (1+\lambda_2\frac{\partial}{\partial t})(\mu_c\nabla^2 q' - \frac{\mu}{k}q'), \\ \gamma\frac{\partial T'}{\partial t} + (\vec{q'}.\nabla)T' + w'\frac{\partial T_b}{\partial z} = K_{11}\nabla^2 T' + QT', \\ \varepsilon\frac{\partial S'}{\partial t} + (\vec{q'}.\nabla)S' - w'\frac{\Delta S}{d} = K_{22}\nabla^2 S' + K_{21}.\nabla^2 T', \\ \rho' = -\rho_0(\beta_T T' - \beta_S S') \end{cases} \tag{2.3.9}$$

Infinitesimal perturbation was applied to the basic state of the system and then the pressure term was eliminated by taking curl twice of Eq. (2.2.1)(b). The above resulting equations are non-dimensionalized using the following transformations,

$$(x', y', z') = (x^*, y^*, z^*)d, \quad t' = t^*(\frac{\gamma d^2}{K_{11}}), q; = \frac{K_{11}}{d}q, \tag{2.3.10}$$

$$(u, v, w) = (u^*, v^*, w^*)(\frac{K_{11}}{d}), \quad T' = (\Delta T)T^*, S' = (\Delta S)S^*$$

T_b , S_b in dimensionless form are given as

$$T_b = (1 - z), \tag{2.3.11}$$

$$S_b = (1 - z).$$

The non dimensionalized equations (on dropping the asterisks for simplicity) are

$$(1+\lambda_1\frac{\partial}{\partial t})(\frac{1}{Pr_D}\frac{\partial}{\partial t}\nabla^2 w - Ra_T\nabla_1^2 T + Ra_S\nabla_1^2 S) - (1+\lambda_2\frac{\partial}{\partial t})(D_a\nabla^4 w - \nabla^2 w) = 0 \tag{2.3.12}$$

$$\Big[\frac{\partial}{\partial t} - \nabla^2 - R_i + q.\nabla\Big]T - w = 0 \tag{2.3.13}$$

$$\left[\epsilon_n \frac{\partial}{\partial t} - \nabla^2 \frac{1}{L_e}\right] S + (q.\nabla)S - S_r \nabla^2 T - w = 0 \quad (2.3.14)$$

where $Pr_D = \frac{\epsilon\gamma\nu d^2}{K_{11}K}$ is Darcy-Prandtl number, $Ra_T = \frac{\beta_T g \Delta T K d}{\nu K_{11}}$ is the thermal Rayleigh number,$Ra_S = \frac{\beta_S g \Delta S K d}{\nu K_{11}}$ is the solute Rayleigh number, $R_i = \frac{Qd^2}{K_{11}}$ is the internal Rayleigh parameter, $\lambda_1 = (\frac{K_{11}}{\gamma d^2})\overline{\lambda_1}$ is relaxation parameter , $\lambda_2 = (\frac{K_{11}}{\gamma d^2})\overline{\lambda_2}$ is retardation parameter, $L_e = \frac{K_{11}}{K_{22}}$ is Lewis number, $S_r = \frac{K_{21}\Delta T}{K_{11}\Delta S}$ the Soret parameter, $\epsilon_n = \frac{\epsilon}{\gamma}$ normalised porosity. The above system will be solved by considering stress free and isothermal boundary conditions as given below:

$$w = \frac{\partial^2 w}{\partial z^2} = T = S = 0 \;\; on \;\; z = 0, z = 1. \quad (2.3.15)$$

2.4 Linear stability Analysis

In order to do the linear stability analysis of the system, Eq. (2.3.12)-(2.3.14) subject to the boundary condition given in Eq.(2.3.15), time dependent periodic disturbance in horizontal plane are used as

$$(w, T, S) = (W, \Theta, \phi) exp(i(lx + my) + \sigma t) \quad (2.4.1)$$

where a are horizontal wave number and $\sigma = \sigma_r + i\sigma_j$, growth rate. Substituting eq.(2.4.1) into the linearized eq.(2.3.12)-(2.3.14), it is obtained

$$(1+\lambda_1\sigma)(\frac{\sigma}{Pr_D}(D^2-a^2)W + a^2 Ra_T \Theta - a^2 Ra_S \phi) - (1+\lambda_2\sigma)(D^2-a^2)[(D_a(D^2-a^2)^2-1)]W = 0 \quad (2.4.2)$$

$$[\sigma - (D^2 - a^2) - R_i]\Theta - W = 0 \quad (2.4.3)$$

$$[\epsilon_n \sigma - \frac{D^2 - a^2}{L_e}]\phi - W - (D^2 - a^2) S_r \Theta = 0. \quad (2.4.4)$$

where $a^2 = l^2 + m^2$, The boundary conditions (2.3.15) are now

$$W = \frac{d^2 W}{dz^2} = \Theta = \phi = 0 \;\; on \;\; z = 0, z = 1. \quad (2.4.5)$$

The solution w, Θ, ϕ are assumed as

$$(W(z), \Theta(z), \phi(z)) = (W_0, \Theta_0, \phi_0) sin n\pi z \quad (n = 1, 2, 3, ...) \tag{2.4.6}$$

The most unstable mode corresponds to n = 1 (fundamental mode). Therefore, substituting Eq. (2.4.6) with n = 1 into Eq. (2.4.2)-(2.4.4), the matrix form A x=0 is obtained as

$$\begin{bmatrix} (\frac{\sigma}{Pr_D} + \wedge(D_a\delta^2 + 1))\delta^2 & -a^2 Ra_T & a^2 Ra_S \\ -1 & \sigma + \delta^2 - R_i & 0 \\ -1 & S_r\delta^2 & \epsilon_n\sigma + \frac{\delta^2}{L_e} \end{bmatrix} \begin{bmatrix} W_0 \\ \Theta_0 \\ \phi_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

The thermal Rayleigh number can be expressed as

$$Ra_T = \frac{\delta^2}{a^2}\Big(\frac{\sigma}{Pr_D} + \wedge(D_a\delta^2 + 1)\Big)(\sigma + \delta^2 - R_i) + \frac{\sigma - R_i + \delta^2(1 - S_r)}{\epsilon_n\sigma + \frac{\delta^2}{L_e}} Ra_S \tag{2.4.7}$$

where $\delta^2 = \pi^2 + a^2$, $\wedge = \frac{1+\lambda_2\sigma}{1+\lambda_1\sigma}$. The growth rate σ is in general a complex quantity such that $\sigma = \sigma_r + i\sigma_i$. The system with σ_r <0 is always stable, while for σ_r >0 it will become unstable. For neutral stability state $\sigma_r = 0$.

2.4.1 Stationary State

Setting $\sigma = 0$ at the margin of stability, the expression for thermal Rayleigh number for stationary mode of convection is as given below:

$$Ra_T^{st} = \frac{\pi^2 + a^2}{a^2}(1 + D_a\delta^2)(\delta^2 - R_i) + \frac{(\delta^2(1 - S_r) - R_i)L_e}{\delta^2} Ra_S, \tag{2.4.8}$$

It is important to note that the critical wave number $a = a_c^{St}$, where $a_c^{St} = \sqrt{S}$ satisfies the following equation

$$2D_aS^3 + (3D_a\pi^2 + 1)S^2 - \pi^4(D_a\pi^2 + 1) = 0 \tag{2.4.9}$$

In the absence of Soret effect,i.e. $S_r = 0$, Eq.(2.4.8) becomes

$$Ra_T^{st} = \frac{\pi^2 + a^2}{a^2}(1 + D_a\delta^2)(\delta^2 - R_i) + \frac{(\delta^2 - R_i)L_e}{\delta^2} Ra_S. \tag{2.4.10}$$

For the system without internal-heating, i.e., $R_i = 0$

$$Ra_T^{st} = \frac{(\pi^2 + a^2)^2}{a^2}(1 + D_a\delta^2) + L_e Ra_S \tag{2.4.11}$$

This is exactly the same as obtained by Swamy et. al.(2012) When $D_a \to 0$

$$Ra_T^{st} = \frac{(\pi^2 + a^2)^2}{a^2} + L_e Ra_S \tag{2.4.12}$$

In case of single component fluid, the solutal Rayleigh number is zero i.e. $Ra_S = 0$,

$$Ra_T^{st} = \frac{(\pi^2 + a^2)^2}{a^2}, \tag{2.4.13}$$

which is the classical result obtained by Horton and Rogers (1945) and Lapwood (1948) for single component fluid in porous layer.

2.4.2 Oscillatory State

Now set $\sigma = i\sigma_i$ in Eq.(2.4.7) and removing the complex quantities from the denominator, to obtain $Ra_T^{osc} = \Delta_1 + i\sigma_i\Delta_2.$

where

$$\Delta_1 = \frac{\delta^2}{a^2}\Big[(\delta^2 - R_i)(D_a\delta^2 + 1)\Big(\frac{1 + \lambda_1\lambda_2\sigma^2}{1 + \lambda_1^2\sigma^2}\Big) - \sigma^2\Big(\frac{1}{Pr_D} + \frac{(D_a\delta^2 + 1)(\lambda_2 - \lambda_1)}{1 + \lambda_1^2\sigma^2}\Big)\Big] \tag{2.4.14}$$

$$+\frac{\epsilon_n\sigma^2 + \delta^4 L_e^{-1}(1 - S_r) - \delta^2 L_e^{-1} R_i}{(\delta^2 L_e^{-1})^2 + \epsilon_n^2\sigma^2} Ra_S$$

$$\Delta_2 = \frac{\delta^2}{a^2}\Big[(\delta^2 - R_i)\Big(\frac{1}{Pr_D} + \frac{(D_a\delta^2 + 1)(\lambda_2 - \lambda_1)}{1 + \lambda_1^2\sigma^2}\Big) + (D_a\delta^2 + 1)\Big(\frac{1 + \lambda_1\lambda_2\sigma^2}{1 + \lambda_1^2\sigma^2}\Big)\Big] \tag{2.4.15}$$

$$+\frac{\delta^2 L_e^{-1} - \epsilon_n(\delta^2(1 - S_r) - R_i)}{(\delta^2 L_e^{-1})^2 + \epsilon_n^2\sigma^2} Ra_S$$

for oscillatory mode $\Delta_2 = 0$ and ($\sigma_i \neq 0$).which is not given for brevity. The thermal Rayleigh number for oscillatory mode is given as:

$$Ra_T^{osc} = \Delta_1. \tag{2.4.16}$$

2.5 Nonlinear stability Analysis

In this section, nonlinear stability analysis is done using minimal truncated Fourier series. For simplicity, one can confine oneself only to two dimensional rolls, so that all the physical quantities do not dependent of y. Introducing the stream function ψ as $u = \frac{\partial\psi}{\partial z}, w = -\frac{\partial\psi}{\partial x}$ and taking curl of Eq. (2.2.1)(b) to eliminate pressure term,

$$(1+\lambda_1\frac{\partial}{\partial t})\Big(\frac{1}{Pr_D}\frac{\partial}{\partial t}(\nabla^2\psi) + Ra_T\frac{\partial T}{\partial x} - Ra_S\frac{\partial S}{\partial x}\Big) = (1+\lambda_2\frac{\partial}{\partial t})(D_a\nabla^4\psi - \nabla^2\psi) \quad (2.5.1)$$

$$\Big(\frac{\partial}{\partial t} - \nabla^2 - R_i\Big)T - \frac{\partial(\psi, T)}{\partial(x, Z)} + \frac{\partial\psi}{\partial x} = 0 \quad (2.5.2)$$

$$\Big(\epsilon_n\frac{\partial}{\partial t} - L_e^{-1}\nabla^2\Big)S - \frac{\partial(\psi, S)}{\partial(x, Z)} + \frac{\partial\psi}{\partial x} - S_r\nabla^2 T = 0 \quad (2.5.3)$$

It is to be noted that the effect of nonlinearity is to distort the temperature and concentration fields through the interaction of ψ and T , ψ and S. As a result a component of the form $sin(2\pi z)$ will be generated. A minimal Fourier series which describes the finite amplitude convection is given by

$$\psi = A_1(t)sin(ax)sin(\pi z), \quad (2.5.4)$$

$$T = A_2(t)cos(ax)sin(\pi z) + A_3(t)sin(2\pi z), \quad (2.5.5)$$

$$S = A_4(t)cos(ax)sin(\pi z) + A_5(t)sin(2\pi z), \quad (2.5.6)$$

where the amplitudes $A_1(t)$, $A_2(t)$, $A_3(t)$,$A_4(t)$, $A_5(t)$ are functions of time and are to be determined. Substituting above expressions in Eqs. (2.5.1)-(2.5.3) and equating the like terms, the following set of nonlinear autonomous differential equations were obtained

$$\frac{dA_1}{dt} = B \quad (2.5.7)$$

$$\frac{dB}{dt} = -\frac{Pr_D}{\delta^2\lambda_1}\Big[(\frac{\delta^2}{Pr_D} + \lambda_2 D_a\delta^4 + \delta^2\lambda_2)B + \delta^2(1 + D_a\delta^2)A_1 \quad (2.5.8)$$

$$+aRa_T A_2 - aRa_S A_4 + a\lambda_1 Ra_T\frac{dA_2}{dt} - a\lambda_1 Ra_S\frac{dA_4}{dt}\Big]$$

$$\frac{dA_2}{dt} = -\Big[aA_1 + (\delta^2 - R_i)A_2 + \pi a A_1 A_3\Big] \tag{2.5.9}$$

$$\frac{dA_3}{dt} = (R_i - 4\pi^2)A_3 + \frac{\pi a}{2}A_1 A_2 \tag{2.5.10}$$

$$\frac{dA_4}{dt} = -\frac{1}{\epsilon_n}(aA_1 + L_e^{-1}\delta^2 A_4 + \pi a A_1 A_5 + S_r \delta^2 A_2) \tag{2.5.11}$$

$$\frac{dA_5}{dt} = -\frac{1}{\epsilon_n}(4\pi^2 L_e^{-1} A_5 - \frac{\pi a}{2}A_1 A_4 + 4\pi^2 S_r A_3) \tag{2.5.12}$$

2.5.1 Steady finite amplitude motions

Set $\frac{\partial}{\partial t}$ =0, the above system becomes

$$B = 0 \tag{2.5.13}$$

$$\delta^2(1 + D_a\delta^2)A_1 + aRa_T A_2 - aRa_S A_4 = 0 \tag{2.5.14}$$

$$aA_1 + (\delta^2 - R_i)A_2 + \pi a A_1 A_3 = 0 \tag{2.5.15}$$

$$(R_i - 4\pi^2)A_3 + \frac{\pi a}{2}A_1 A_2 = 0 \tag{2.5.16}$$

$$aA_1 + L_e^{-1}\delta^2 A_4 + \pi a A_1 A_5 + S_r\delta^2 A_2 = 0 \tag{2.5.17}$$

$$4\pi^2 L_e^{-1} A_5 - \frac{\pi a}{2}A_1 A_4 + 4\pi^2 S_r A_3 = 0 \tag{2.5.18}$$

Numerical method was used to solve the above nonlinear differential equation to find the amplitudes. On solving for the amplitudes in terms of A_1, A_2, A_3, A_4, A_5 are obtained.

2.5.2 Steady Heat and Mass Transports

In the study of this type problem, quantification of heat and mass transport is very important. If H and J are the rate of heat and mass transport per unit area,then

$$H = -K_{11}\Big\langle \frac{\partial T_{total}}{\partial z} \Big\rangle_{z=0} \tag{2.5.19}$$

$$J = -K_{21}\Big\langle \frac{\partial T_{total}}{\partial z} \Big\rangle_{z=0} - K_{22}\Big\langle \frac{\partial S_{total}}{\partial z} \Big\rangle_{z=0} \tag{2.5.20}$$

where the angular bracket corresponds to a horizontal average and

$$T_{total} = T_0 - \Delta T \frac{z}{d} + T(x,z,t) \tag{2.5.21}$$

$$S_{total} = S_0 - \Delta S \frac{z}{d} + S(x,z,t).$$

Subsitituting eq. (2.5.5)-(2.5.6)into eq.(2.5.21) and using the resultant eq.(2.5.19),(2.5.20),

$$H = \frac{K_{11}\Delta T}{d}(1 - 2\pi A_3) \tag{2.5.22}$$

$$J = \frac{K_{22}\Delta S}{d}\Big[(1 - 2\pi A_5) + S_r L_e (1 - 2\pi A_3)\Big] \tag{2.5.23}$$

The Nusselt number and Sherwood number, which denote the rate of heat and mass transport respectively, are given by

$$Nu = \frac{H}{\frac{K_{11}\Delta T}{d}} = (1 - 2\pi A_3) \tag{2.5.24}$$

$$Sh = \frac{J}{\frac{K_{22}\Delta S}{d}} = (1 - 2\pi A_5) + S_r L_e (1 - 2\pi A_3)$$

Using the expressions (2.5.22)-(2.5.23) and substituting A_3, A_5 of into eqs.(2.5.24),finally the expressions for N_u , S_h are obtained.

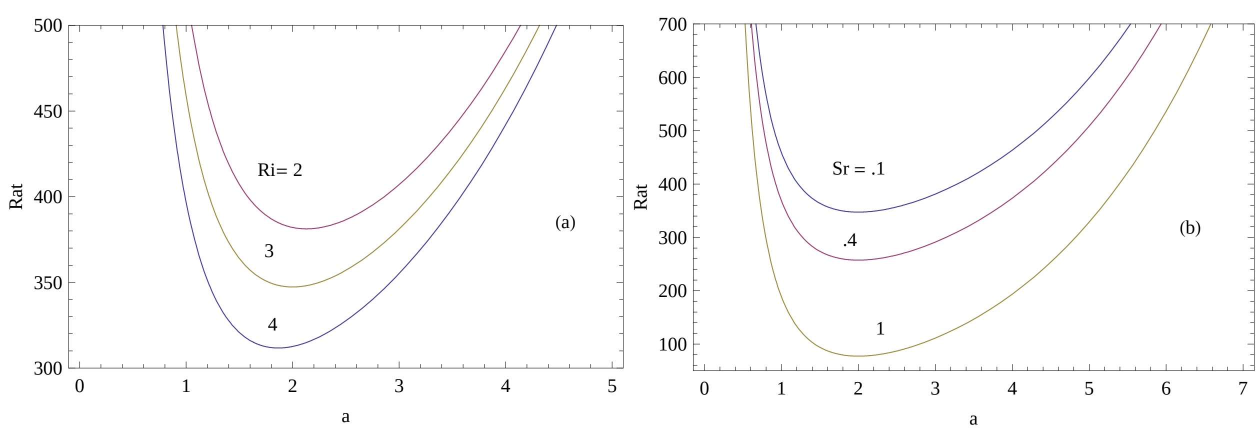

Figure 2.1: Variation of Stationary Rayleigh number with wave number for the different values of R_i and S_r

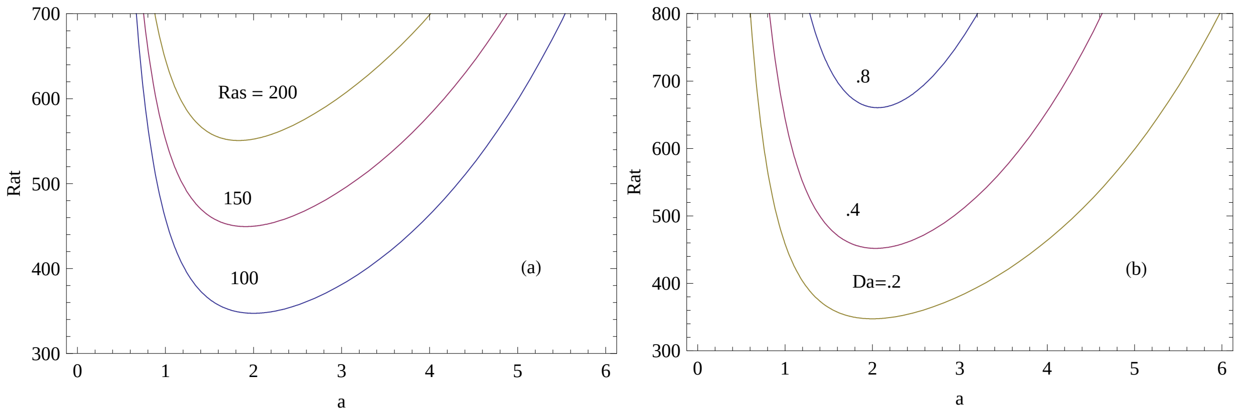

Figure 2.2: Variation of Stationary Rayleigh number with wave number for the different values of Ra_S and D_a

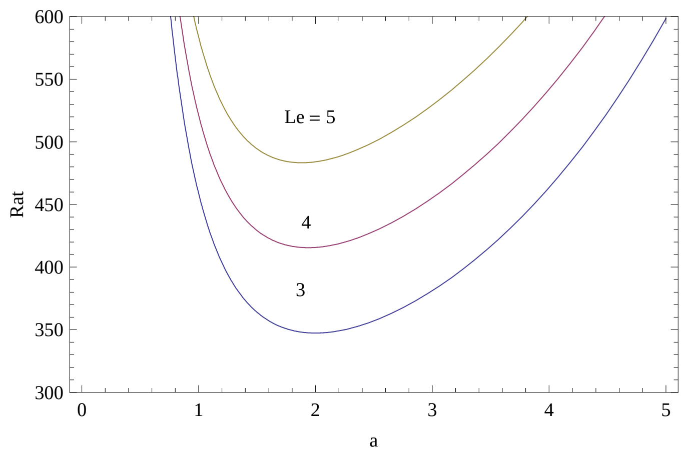

Figure 2.3: Variation of Stationary Rayleigh number with wave number for the different values of L_e

2.6 Results and Discussion

This paper investigates the combined effect of internal heating and Soret parameter on stationary and oscillatory convection in a porous medium saturated with a binary viscoelastic fluid and discusses the effects of various parameters on the onset of double diffusive convection. The expressions for the stationary and oscillatory modes of convection for different values of the parameters such as Prantdl number, relaxation parameter, retardation parameter, solute Rayleigh number, Lewis number, Soret parameter and Darcy number are computed, and the results are depicted in figures. The neutral stability curves in the (Ra_t,a) plane for various parameter values are as shown in Figs. 2.1 - 2.6. The values of the parameters are fixed as $Pr_D = 10$, $\lambda_1 = .8$, $\lambda_2 = .1$, $Ra_S = 100$, $L_e = 2$, S_r =.05, D_a=.1 and $R_i = 3$,

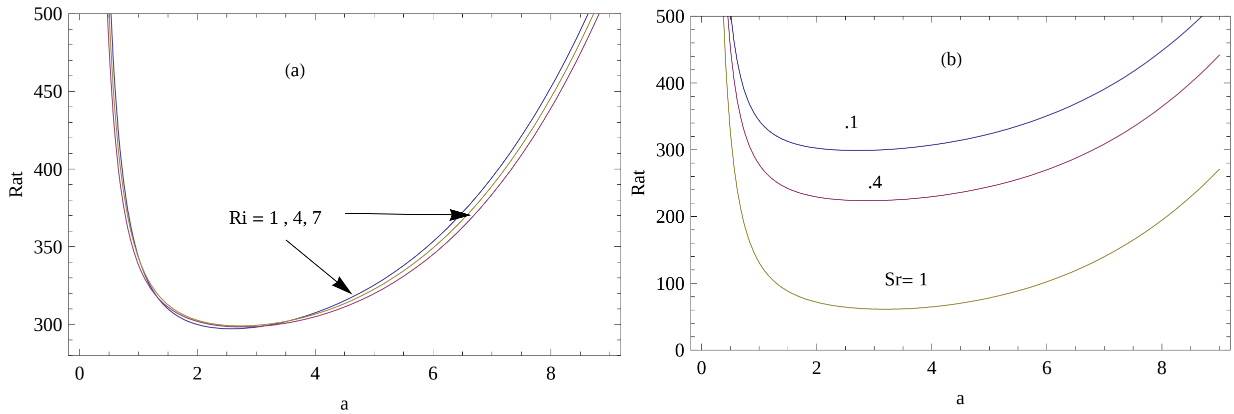

Figure 2.4: Variation of Oscillatory Rayleigh number with wave number for the different values of R_i and S_r

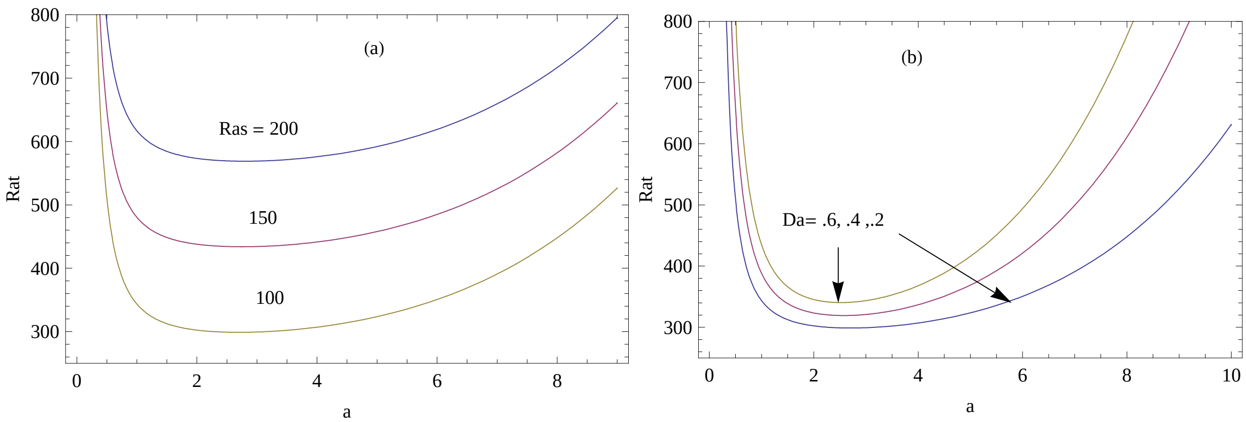

Figure 2.5: Variation of Oscillatory Rayleigh number with wave number for the different values of Ra_S, D_a

and except the varying parameter. From Figs. 2.1, 2.4(a) , it is observed that an increment in the value of internal heat source R_i , decreases the values of stationary and oscillatory Rayleigh number, which means that the effect of increasing the internal heat source R_i is to destabilize the system. In Figs. 2.1, 2.4(b), the effect of Soret parameter (S_r) is depicted, respectively for both stationary and oscillatory convection. It is found that an increment in the value of Soret parameter, decreases the value of Rayleigh numbers for both stationary and oscillatory mode of convection, thus onset of convection takes place at an early point. Figs. 2.2, 2.5(a) depict the effect of solute Rayleigh number Ra_S on the onset of convection. It is found that the effect of increasing the value of Ra_S is to increase the value of Rayleigh number Ra_T thus stabilizing the system in both stationary and oscillatory modes. Further, Figs. 2.2, 2.5(b) show that the effect of increasing the Darcy number D_a is to increase the

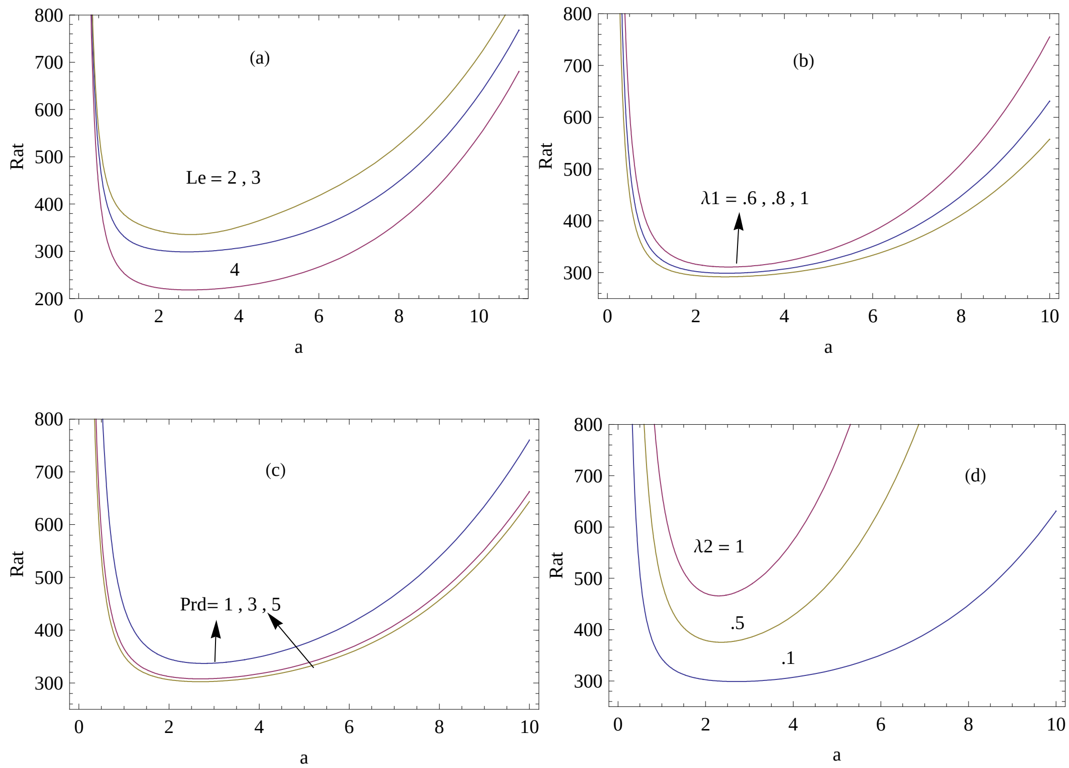

Figure 2.6: Variation of Oscillatory Rayleigh number with wave number for the different values of L_e, λ_1,Pr_D, λ_2

value of Rayleigh number Ra_T thus stabilizing the system that is the onset of convection will take place at a later point. However, the effect of increasing the Lewis number L_e is found to increase the value of Rayleigh number for stationary mode and decrease the value for oscillatory modes, thus to stabilize the stationary mode of convection and destabilize the oscillatory convection [Figs.2.3, 2.6(a)]. Also, from Figs.2.6(b, c),It is found that the oscillatory Rayleigh number decreases on increasing the value of the relaxation parameter and Prandtl number , indicating that the effect of relaxation parameter λ_1 and the Prandtl number Pr_D is to destabilize the system. Thus, the oscillatory convection takes place at an early point. However, from Fig. 2.6(d), the effect of retardation parameter λ_2 is found to stabilize the system, thus opposite to that due to λ_1.

Now, fix the values of the parameters as $Ra_S = 100$, $L_e = 2$,D_a =.1,$S_r = .05$ and $R_i = 3$ to compute the heat and mass transports across the porous medium. The results have been obtained for steady state motion, in terms of the Nussult and Sherwood numbers and depicted in the Figs. 2.7, 2.8 respectively. It is found that the steady state values of N_u and S_h approach 3 as Ra_T increases. Further, it is found from Figs. 2.7, 2.8(a) that the

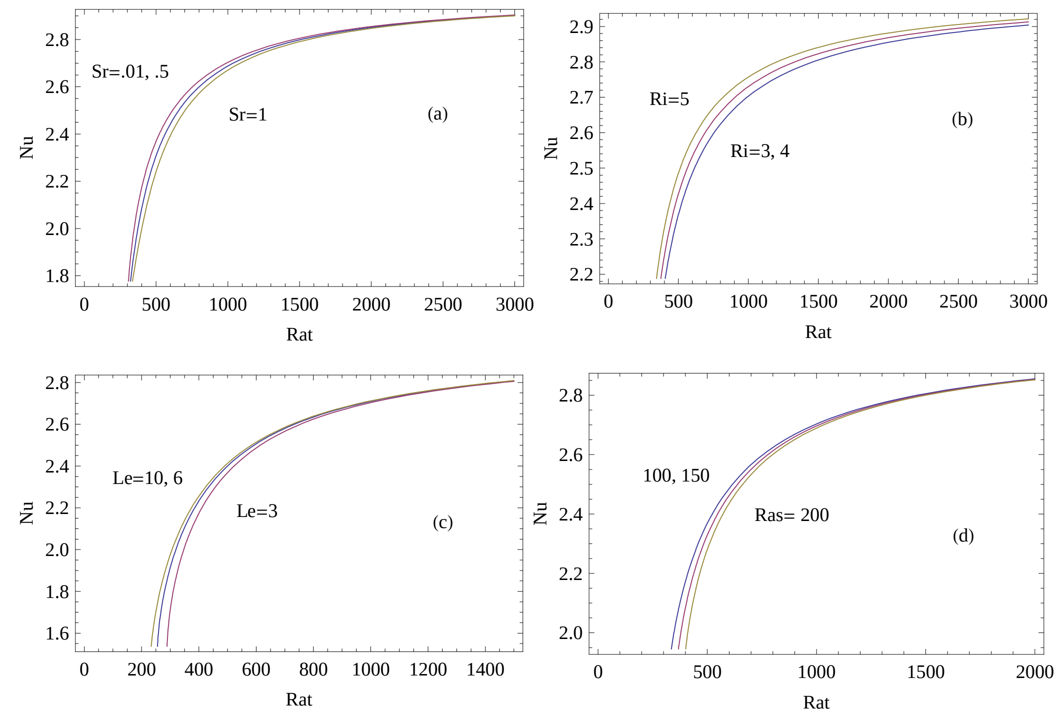

Figure 2.7: Graph between Nusselt number and Rayleigh number for the different values of parameter (a), (b), (c), (d)

value of N_u decreases, while that of S_h increases on increasing the values of Soret parameter S_r. This shows that the effect of Soret parameter is to decrease the heat transport, thus stabilizing the system and increase the mass transport in the system. In Figs. 2.7(b, c) and Figs.2.8(b, c), it is found that heat and mass transports increase on increasing R_i and L_e , thus destabilizing the system. However,Ra_S has a stabilizing effect on the system as heat and mass transport decrease on increasing its value [Fig. 2.7, 2.8(d)].

2.7 Conclusions

Effects of Soret parameter and internal heat source on double diffusive convection in a binary viscoelastic fluid saturated porous layer, heated and salted from below, is investigated analytically using linear and nonlinear stability analysis. Following conclusions are drawn:

1) The Internal heat source R_i and Soret parameter S_r have destabilizing effect on the system in both stationary and oscillatory modes of convection.

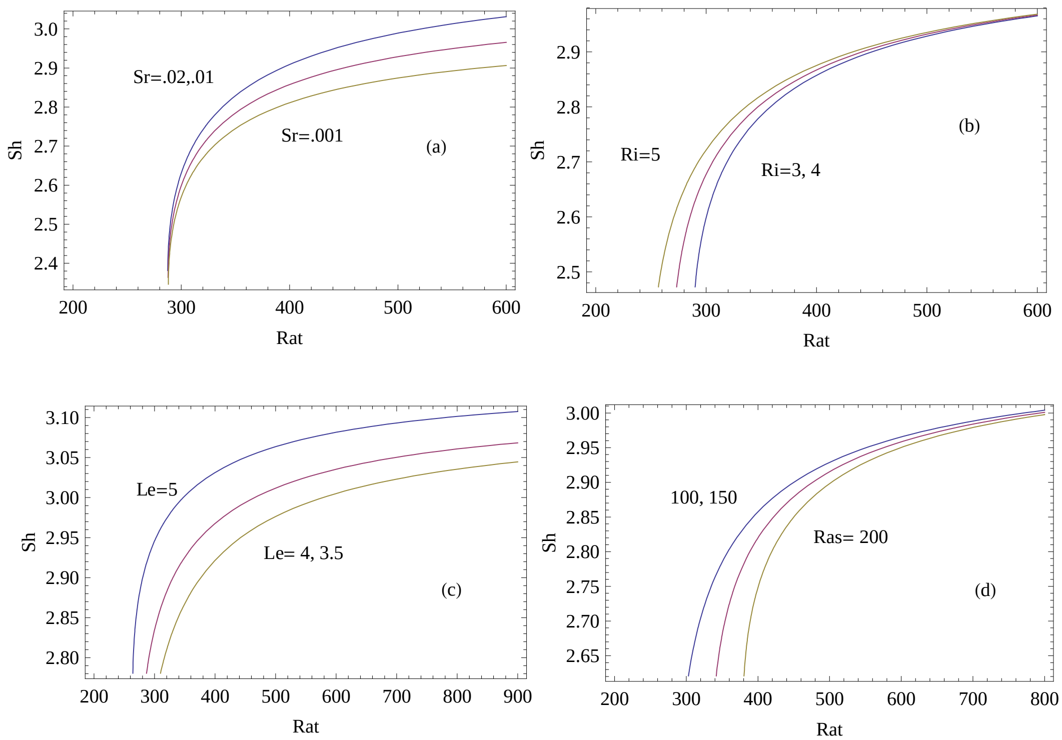

Figure 2.8: Graph between of Sherwood number and Rayleigh number for the different values of parameter (a), (b), (c), (d)

2) The Darcy number D_a and Solute Rayleigh number Ra_S have stabilizing effect on both stationary and oscillatory convection.

3) The Lewis number L_e has stabilizing effect on stationary mode of convection while destabilizing effect on oscillatory mode of convection.

4) Relaxation parameter λ_1 and Prandtl number Pr_D have destabilizing effect, while retardation parameter λ_2 has stabilizing effect on the oscillatory convection.

5) Increments in Lewis number L_e and internal Rayleigh number R_i increase, while in decrease heat and mass transports in the system.

6) Effect of Soret parameter S_r is to decrease the heat transfer and increase the mass transfer in the system.

Chapter 3

Double diffusive convection in a couple stress fluid saturated rotating anisotropic porous layer with internal heating and Soret effect

3.1 Introduction

A well-known phenomenon that involves coupled heat and mass transfer is the thermal energy flux that is generated by concentration gradients is called the Dufour (diffusion-thermal) effect. However, the effect of the temperature gradient on mass flux is known as the Soret (thermo-diffusion) effect. Swiss scientist J. Soret was the first to study the thermo diffusion in 1879. Hurle and Jakeman (1971) performed theoretical study of Soret driven thermosolutal convection in a binary fluid mixture. Gaikwad et. al.(2009) studied linear and non-linear double diffusive convection in a fluid saturated anisotropic porous layer with cross-diffusion effects. Malashetty et.al.(2012) studied linear and nonlinear double diffusive

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convection in a fluid saturated porous layer with cross diffusion effect. Recently, Altawallbeh et al. (2013) have done Soret effect and internal heat source with double diffusive convection in a fluid saturated anisotropic porous layer. Internal heat generation arises in many important situations, including reactor safety analyses, metal waste that is produced by spent nuclear fuel, fire and combustion studies, and the storage of radioactive materials. The study concerning internal heat source in porous media was provided by Tveitereid (1977), who studied thermal convection in a horizontal porous layer with internal heat sources. Hill (2005) performed linear and nonlinear analyses on the double-diffusive convection in a porous layer with a concentration based internal heat source. Bhadauria et al. (2011),(2012),(2014) studied the effect of internal heating on double diffusive convection in a couple stress fluid saturated anisotropic porous medium.

The study of double diffusive convection in a rotating porous media is importance both theoretically and due to its practical applications in engineering. Some of the important areas of application in engineering include the food and chemical process, solidification and centrifugal casting of metals, rotating machinery, petroleum industry and biomechanics problems. Chakrabarti and Gupta (1981) have analyzed the nonlinear thermohaline convection in a rotating porous medium. The effect of rotation on linear and nonlinear double diffusive convection in a sparsely packed porous medium was studied by Rudraiah et.al.(1986). Malashetty et al.(2010),(2013) performed double diffusive convection in a Darcy porous medium saturated with couple stress and also carried out effect of rotation on the onset of double diffusive convection in a Darcy porous medium saturated with couple stress fluid. Malashetty and Heera(2008) studied the onset of double diffusive convection in a horizontal anisotropic porous layer with rotation. Gaikwad (2012) have studied linear stability analysis of double diffusive convection in a horizontal sparsely packed rotating anisotropic porous layer in presence of Soret effect. Sulochana et.al (2012) performed the onset of double diffusive convection in a couple stress fluid saturated rotating anisotropic porous layer. Bhadauria et al.(2013) studied cross diffusion convection in a Newtonian fluid saturated rotating porous medium. Very first study on double diffusive convection in porous media mainly concerns with linear stability analysis, and was performed by Nield (1968).

With the growing importance of non-Newtonian fluids with suspended particles in modern technology and industries, the investigation of such fluids is desirable. Applications of such fluids are found in extrusion of polymer fluids in industry, exotic suspension, fluid film lubrication, solidification of liquid crystals, cooling of metallic plate in bath, colloidal and suspension solutions. In the category of non-Newtonian, couple stress fluids have specific features, such as polar effect. The theory of polar fluids and related theories are models for fluids whose microstructure is mechanically significant. Theory for couple stress fluid was proposed by Stokes (1966), which is a simpler polar fluid theory and shows all the important features and effect of such fluids that occur inside a deforming continuum. Stabilizing effect of couple stress parameter is reported in the works of Sharma and Thakur (2000), who investigated thermal instability in an electrically conducting couple stress fluid with magnetic field. Sunil et al. (2004) studied the effect of suspended particles on double diffusive convection in a couple stress fluid saturated porous medium, Sharma and Sharma (2004) investigated the effect of suspended particles on couple stress fluid, heated from below, in the presence of rotation and magnetic field. Malashetty et al. (2006) have done an analytical study of linear and nonlinear double diffusive convection in couple stress liquids with Soret effect. Gaikwad et al. (2014) performed linear stability analysis of double diffusive convection in a horizontal sparsely packed rotating anisotropic porous layer in the presence of Soret effect. Malashetty and Kollur (2011) investigated the onset of double diffusive convection in a couple stress fluids saturated anisotropic porous layer. Shivkumara (2011) carried out linear and non linear stability analysis of double diffusive convection in a couple stress fluid saturated porous layer. No work is available in the present literature related to the rotation with an internal heat source and Soret parameter. Therefore, in the present chapter stability analysis of Soret effect and internal heating effect on double diffusive convection in a rotating anisotropic porous medium saturated with a couple stress fluids has been investigated.

3.2 Mathematical Formulation

We consider an infinitely extended horizontal planes at z=0 and z=d fluid saturated porous medium, which is heated from below and cooled from above. Darcy model has been employed in the momentum equation. Further, an internal heat source term has been included in the energy equation. A cartesian frame of reference is chosen in such a way that the origin lies on the lower plane and the z-axis as vertical upward. The system is rotating about z-axis with a constant angular velocity Ω. An adverse temperature gradient is applied across the porous layer. The lower planes is kept at temperature $T_0 + \Delta T$, while upper planes is kept at temperature T_0, with concentration $S_0 + \Delta S$, and S_0 respectively. The governing equations are as given below

$$\nabla.\vec{q} = 0, \tag{3.2.1}$$

$$\frac{1}{\phi}(\frac{\partial q}{\partial t}) = -\nabla p + \rho_0 g(\beta_T T - \beta_S S) - \frac{2\rho_0}{\phi}\Omega \times \vec{q} - (\mu - \mu_c \nabla^2)\vec{q_a}, \tag{3.2.2}$$

$$\gamma \frac{\partial T}{\partial t} + (\vec{q}.\nabla)T = \nabla(\kappa_T.\nabla T) + QT - w\frac{\partial T_b}{\partial z}, \tag{3.2.3}$$

$$\phi \frac{\partial S}{\partial t} + (\vec{q}.\nabla)S = w\frac{\Delta S}{d} + \kappa_s \nabla^2 S + K_{21}\nabla^2 T, \tag{3.2.4}$$

$$\rho = -\rho_0[\beta_T T - \beta_S S] \tag{3.2.5}$$

where the physical variables have their usual meanings as given in the nomenclature. The externally imposed the thermal and solutal boundary conditions are given by

$$\begin{cases} T = T_0 + \Delta T, & \text{at} \quad z = 0 \quad \text{and} \quad T = T_0, & \text{at} \quad z = d, \\ S = S_0 + \Delta S, & \text{at} \quad z = 0 \quad \text{and} \quad S = S_0, & \text{at} \quad z = d, \end{cases} \tag{3.2.6}$$

3.3 Basic state

At this state the velocity, pressure, temperature, concentration and density profiles are given by

$$\vec{q_b} = 0, p = p_b(z), T = T_b(z), S = S_b(z), \rho = \rho_b(z). \tag{3.3.1}$$

Putting Eq.(3.3.1) in Eq.(3.2.1)-(3.2.4), the following equations are obtained:

$$\frac{dp_b}{dz} = -\rho_b g, \tag{3.3.2}$$

$$\kappa_T \frac{d^2T}{dz^2} + QT = 0, \tag{3.3.3}$$

$$\frac{d^2S_b}{dz^2} = 0, \tag{3.3.4}$$

$$T_b = (1-z), S_b = (1-z) \tag{3.3.5}$$

Use of the boundary conditions (3.2.6), in the solution of Eq. (3.3.3), is given by

$$T_b = T_0 + \Delta T \frac{\sin(\left(\sqrt{\frac{Qd^2}{\kappa_T}}\right)(1-\frac{z}{d}))}{\sin\left(\sqrt{\frac{Qd^2}{\kappa_T}}\right)}. \tag{3.3.6}$$

Use of the boundary conditions (3.2.6), in the solution of Eq. (3.3.4),

$$S_b = S_0 + \Delta S(1-\frac{z}{d}) \tag{3.3.7}$$

The following non dimensional variable has been introduced

$$(x,y,z) = (x^*,y^*,z^*)d, \quad t = t^*(\frac{\gamma d^2}{\kappa_{Tz}}), \tag{3.3.8}$$

$$(u,v,w) = (u^*,v^*,w^*)(\frac{\kappa_{Tz}}{d}), \quad T = (\Delta T)T^*, S = (\Delta S)S^*$$

To obtain non dimensional equation (After dropping the asterisks for simplicity and testing)$\gamma = 1$, the non dimensional equation aref

$$\Big[\Big(\frac{1}{P_{rd}}\frac{\partial}{\partial t}\nabla^2 + (\nabla_h^2 + \frac{1}{\xi}\frac{\partial^2}{\partial z^2})(1-c\nabla^2)\Big)(\frac{1}{P_{rd}}\frac{\partial}{\partial t} + \frac{1}{\xi} - c\nabla^2) + T_a\frac{\partial^2}{\partial z^2}\Big]w \tag{3.3.9}$$

$$-(\frac{1}{P_{rd}}\frac{\partial}{\partial t} + \frac{1}{\xi} - c\nabla^2)(Ra_T\nabla_h^2 T - Ra_S\nabla_h^2 S) = 0$$

$$\Big[\frac{\partial}{\partial t} - (\eta\nabla_h^2 + \frac{\partial^2}{\partial z^2}) - R_i\Big]T + w\frac{\partial T_b}{\partial z} = 0, \tag{3.3.10}$$

$$\Big[\frac{\partial}{\partial t} - \frac{1}{L_e}\nabla^2\Big]S - w - S_r\nabla^2 T = 0 \tag{3.3.11}$$

where $T_a = (\frac{2\Omega K_z}{\mu\phi})^2$ is Taylor number, $P_{rd} = \frac{\mu\phi\gamma d^2}{\kappa_T k}$ is Darcy-Prandtl number, $Ra_T = \frac{\beta_T g \Delta T K_z d}{\nu\kappa_{Tz}}$ is the thermal Rayleigh number, $Ra_S = \frac{\beta_S g \Delta S K_z d}{\nu\kappa_{Tz}}$ is the solute Rayleigh number, $R_i = \frac{Qd^2}{\kappa_{Tz}}$ is the internal heat source parameter, $C = \frac{\mu_C}{\mu d^2}$ is the couple stress fluid, $L_e = \frac{\kappa_{Tz}}{\kappa_S}$ is Lewis number, $\eta = \frac{\kappa_{Tx}}{\kappa_{Tz}}$ is thermal anisotropy parameter, $\xi = \frac{K_x}{K_z}$ is mechanical anisotropy parameter. consider the stress free and isothermal boundary conditions as given below will be solved the above system:

$$w = \frac{\partial^2 w}{\partial z^2} = T = S = 0 \;\; on \;\; z = 0, z = 1 \tag{3.3.12}$$

3.4 Linear stability Analysis

In order to do linear stability analysis one needs to solve the eigenvalue problem defined by Eq.(3.3.9)-(3.3.11) subject to the boundary conditions Eq.(3.3.3),(3.3.4), Using time dependent periodic disturbance in horizontal plane, one can write

$$(w, T, S) = (W, \Theta, \phi)exp[i(lx + my) + \sigma t] \tag{3.4.1}$$

where l,m are horizontal wave number and $\sigma = \sigma_r + i\sigma_j$, growth rate. Substituting Eq. (3.4.1) into the linearized Eq.(3.3.9)-(3.3.11). We obtain

$$\Big[\Big(\frac{\sigma(D^2-a^2)}{P_{rd}} + (\frac{D^2}{\xi} - a^2)(1 - c(D^2-a^2))\Big)(\frac{\sigma}{P_{rd}} + \frac{1}{\xi} - C(D^2-a^2)) + T_a D^2\Big]W = \tag{3.4.2}$$

$$\Big(\frac{\sigma}{P_{rd}} + \frac{1}{\xi} - C(D^2-a^2)\Big)\Big(-a^2 Ra_T\Theta + a^2 Ra_S\phi\Big)$$

$$[\sigma - (D^2 - \eta a^2) - R_i]\Theta - W = 0 \tag{3.4.3}$$

$$[\sigma - \frac{1}{L_e}(D^2 - a^2)]\phi - W - (D^2 - a^2)S_r\Theta = 0. \tag{3.4.4}$$

where $D = d/dz$ and $a^2 = l^2 + m^2$. The boundary conditions (3.4.1), now reads
W= $D^2W = \Theta = \phi = 0$ at $z = 0, 1$:

Solutions of Eqs. (3.3.9)-(3.3.11) satisfying the boundary conditions (3.4.1), are assumed as

$(W(z), \Theta(z), \phi(z)) = (W_0, \Theta_0, \phi_0) sin n\pi z \quad (n = 1, 2, 3, ...)$

$$\Big[(\frac{\sigma\delta^2}{P_{rd}} + \delta_1^2(1 + C\delta^2))(\frac{\sigma}{P_{rd}} + \frac{1}{\xi} + C\delta^2) + T_a\pi^2\Big]W_0 + \tag{3.4.5}$$

$$(\frac{\sigma}{P_{rd}} + \frac{1}{\xi} + C\delta^2)\Big[a^2 Ra_T\Theta_0 + a^2 Ra_S\phi_0\Big] = 0$$

$$\Big[\sigma + \delta_2^2 - R_i\Big]\Theta_0 - W_0 = 0 \tag{3.4.6}$$

$$\Big[\sigma + \frac{\delta^2}{L_e}\Big]\phi_0 - W_0 + \delta^2 S_r\Theta_0 = 0 \tag{3.4.7}$$

Solve the above Eqs. to get the thermal Rayleigh number in the form

$$Ra_T = \frac{(\sigma + \delta_2^2 - R_i)}{a^2}\Big[(\frac{\sigma\delta^2}{P_{rd}} + \delta_1^2(1 + C\delta^2)) + \frac{T_a\pi^2}{(\frac{\sigma}{P_{rd}} + \frac{1}{\xi} + C\delta^2)}\Big] + \tag{3.4.8}$$

$$Ra_S\Big[\frac{(\sigma + \delta_2^2 - R_i) - \delta^2 S_r}{(\sigma + \frac{1}{L_e})(\frac{\sigma}{P_{rd}} + \frac{1}{\xi} + C\delta^2)}\Big]$$

where $a^2 = l^2 + m^2$, $\delta_1^2 = \frac{\pi^2}{\xi} + a^2$, $\delta_2^2 = \pi^2 + \eta a^2$. The growth rate σ is in general a complex quantity such that $\sigma = \sigma_r + i\sigma_j$. The system with $\sigma_r < 0$ is always stable, while for $\sigma_r > 0$ it will become unstable. For neutral stability state $\sigma_r = 0$.

A. Stationary State

The steady onset corresponds to σ_r=0 (i.e.$\sigma_r = \sigma_j$=0). The expression for the thermal Rayleigh number and the corresponding wave number of the system for a stationary mode of convection are as given below:

$$Ra_T^{st} = \frac{\delta^2 - R_i}{a^2}\Big[\delta_1^2(1 + C\delta^2) + \frac{T_a\pi^2}{\frac{1}{\xi} + C\delta^2}\Big] + Ra_S\Big[\frac{(\delta_2^2 - R_i) - \delta^2 S_r}{\frac{\delta^2}{L_e}(\frac{1}{\xi} + C\delta^2)}\Big], \tag{3.4.9}$$

It is important to note that the critical wave number $a = a_c^{St}$ depends on the couple stress parameter and Taylor number. In the absence of Taylor number, i.e. when Ta = 0, Eq.(3.4.9) gives

$$Ra_T^{st} = \frac{\delta^2 - R_i}{a^2}\Big(\delta_1^2(1+C\delta^2)\Big) + Ra_S\Big[\frac{(\delta_2^2 - R_i) - \delta^2 S_r}{\frac{\delta^2}{L_e}(\frac{1}{\xi}+C\delta^2)}\Big] \tag{3.4.10}$$

For isotropic porous media when $\xi = 1$ and without internal-heating, i.e., $R_i = 0$ we get

$$Ra_T^{st} = \frac{(\pi^2+a^2)^2}{a^2}(1+C\delta^2) + Ra_S\Big[\frac{(\pi^2+a^2) - (\pi^2+a^2)S_r}{\frac{\delta^2}{L_e}(1+C\delta^2)}\Big] \tag{3.4.11}$$

which is the result given by Malashetty et. al. (2011) For single component fluid, $Ra_S = 0$

$$Ra_T^{st} = \frac{(\pi^2+a^2)^2}{a^2}(1+C\delta^2) \tag{3.4.12}$$

which is the one obtained by Shivakumara et. al. (2011). When C = 0 (i.e. Newtonian fluid case),

$$Ra_T^{st} = \frac{(\pi^2+a^2)^2}{a^2} \tag{3.4.13}$$

which has the critical value $Ra_c^{St} = 4\pi^2$ for $a_c^S t = \pi^2$ obtained and which are the classical results by Horton and Rogers (1945) and Lapwood (1948).

B. Oscillatory State

For oscillatory mode of convection set $\sigma = i\sigma_j$ in Eq.(3.4.8) and removing the complex quantities from the denominator, to obtain

$$Ra_T^{osc} = \Delta_1 + i\sigma_i\Delta_2.$$

$$\Delta_1 = \frac{d_1 d_3 + \sigma^2 d_2 d_4}{d_3^2 + \sigma^2 d_4^2} \tag{3.4.14}$$

$$\Delta_2 = \frac{d_2 d_3 - d_1 d_4}{d_3^2 + \sigma^2 d_4^2}, \tag{3.4.15}$$

For oscillatory onset $\Delta_2 = 0$ and ($\sigma_i \neq 0$. where σ is the oscillatory frequency which is not given for brevity.

where

$A_1 = T_a\pi^2 P_{rd}(\delta_2^2 - R_i) - \sigma^2\delta^2(\frac{(\delta_2^2-R_i)}{P_{rd}} + \frac{1}{\xi} + C\delta^2) + \delta_1^2 P_{rd}(1+C\delta^2)((\delta_2^2 - R_i)(\frac{1}{\xi}+C\delta^2) - \frac{\sigma^2}{P_{rd}})$

$A_2 = (\delta^2(\delta_2^2 - R_i)(\frac{1}{\xi}+C\delta^2) - \frac{\sigma^2\delta^2}{P_{rd}} + \delta_1^2 P_{rd}(1+C\delta^2)(\frac{(\delta_2^2-R_i)}{P_{rd}} + \frac{1}{\xi} + C\delta^2) + T_a\pi^2 P_{rd})$

$B_1 = Ra_S(\delta_2^2 - R_i - \delta^2 S_r)$

$B_2 = Ra_S$

$C_1 = a^2 P_{rd}(\frac{1}{\xi} + C\delta^2)$

$C_2 = a^2$

$C_3 = (\frac{\delta^2}{L_e}(\frac{1}{\xi} + C\delta^2) - \frac{\sigma^2}{P_{rd}})$

$C_4 = (\frac{\delta^2}{L_e P_{rd}} + \frac{1}{\xi} + C\delta^2)$

$d_1 = A_1C_3 + B_1C_1 - \sigma^2(A_2C_4 + B_2C_2)$

$d_2 = A_2C_3 + A_1C_4 + B_2C_1 + B_1C_2$

$d_3 = C_1C_3 - \sigma^2 C_2C_4$

$d_4 = C_2C_3 + C_1C_4$

we have the necessary expression for oscillatory Rayleigh number as:

$$Ra_T^{osc} = \Delta_1 = \frac{d_1d_3 + \sigma^2 d_2d_4}{d_3^2 + \sigma^2 d_4^2}. \tag{3.4.16}$$

3.5 Nonlinear stability Analysis

In this section, nonlinear stability analysis using minimal truncated Fourier series has been considered. For simplicity, only two dimensional rolls have been considered, so that all physical quantities do not dependent of y. Introducing stream function ψ such that $u = \frac{\partial\psi}{\partial z}$, $w = \frac{-\partial\psi}{\partial x}$ then taking curl to eliminate pressure term from Eq.(3.2.2) and then non-dimensionalizing the resulting equations by using transformation given by Eq.(3.3.8) the following equations are ontained

$$\Big(\frac{1}{P_{rd}}\frac{\partial}{\partial t}\nabla^2 + (\frac{\partial^2}{\partial x^2} + \frac{1}{\xi}\frac{\partial^2}{\partial z^2})(1 - c\nabla^2)\Big)\psi - (T_a)^{\frac{1}{2}}\frac{\partial V}{\partial z} + Ra_T\frac{\partial T}{\partial x} - Ra_S\frac{\partial S}{\partial x} = 0, \tag{3.5.1}$$

$$\Big(\frac{1}{P_{rd}}\frac{\partial}{\partial t} + (\frac{1}{\xi} - c\nabla^2)\Big)V + (T_a)^{\frac{1}{2}}\frac{\partial\psi}{\partial z} = 0, \tag{3.5.2}$$

$$\Big(\frac{\partial T}{\partial t} - (\eta\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} - R_i)\Big)T + \frac{\partial\psi}{\partial x} - \frac{\partial(\psi, T)}{\partial(x, z)} = 0, \tag{3.5.3}$$

$$\Big(\frac{\partial}{\partial t} - \frac{1}{L_e}\nabla^2\Big)S + \frac{\partial\psi}{\partial x} - \frac{\partial(\psi, S)}{\partial(x, z)} - S_r\nabla^2 T = 0 \tag{3.5.4}$$

It is to be noted that the effect of nonlinearity is to distort the temperature and concentration fields through the interaction of ψ and T, ψ and S. As a result, a component of the form $sin(2\pi z)$ will be generated. Here V is zonal velocity induced due to rotation. A minimal Fourier series which describes the finite amplitude convection is given by

$$\psi = M_0(t)sin(ax)sin(\pi z), \tag{3.5.5}$$

$$T = M_1(t)cos(ax)sin(\pi z) + M_2(t)sin(2\pi z), \tag{3.5.6}$$

$$S = M_3(t)cos(ax)sin(\pi z) + M_4(t)sin(2\pi z), \tag{3.5.7}$$

$$V = F(t)sin(ax)cos(\pi z) + G(t)sin(2\pi z), \tag{3.5.8}$$

where the amplitudes $M_0(t)$, $M_1(t)$, $M_2(t)$,$M_3(t)$, $M_4(t)$, $F(t)$ and $G(t)$ are functions of time and are to be determined. Substituting above expressions in Eqs. (3.5.1)-(3.5.4) and equating the like terms, the following set of nonlinear autonomous differential equations are obtained

$$\frac{dX}{dt} = D \tag{3.5.9}$$

where

$$X = (M_0, M_1, M_2, M_3, M_4, F, G)^T$$

$$D = (D_0, D_1, D_2, D_3, D_4, D_5, D_6)^T$$

$$D_0 = \frac{-P_{rd}}{\delta^2}[(1 + C\delta^2)\delta_1^2 M_0 - \pi(T_a)^{\frac{1}{2}} F + aRa_T M_1 - aRa_S M_3] \tag{3.5.10}$$

$$D_1 = -[aM_0 + \pi a M_0 M_2 + (\delta_2^2 - R_i)M_1] \tag{3.5.11}$$

$$D_2 = -[(4\pi^2 - R_i)M_2 - \frac{\pi a}{2} M_0 M_1] \tag{3.5.12}$$

$$D_3 = -[aM_0 + \delta^2 \frac{1}{L_e} M_3 + \pi a M_0 M_2 + S_r \delta^2 M_1] \tag{3.5.13}$$

$$D_4 = -[4\pi^2 \frac{1}{L_e} M_2 - \frac{\pi a}{2} M_0 M_3 + 4\pi^2 S_r M_2] \tag{3.5.14}$$

$$D_5 = -P_{rd}[(\frac{1}{\xi} + C\delta^2)F + \pi(T_a)^{\frac{1}{2}} M_0] \tag{3.5.15}$$

$$D_6 = -P_{rd}(\frac{1}{\xi} + C\pi^2)G \tag{3.5.16}$$

Numerical method was used to solve the above nonlinear differential equation to find the amplitudes.

A. Steady finite amplitude convection

For steady state finite amplitude convection, left hand sides of the eq.(3.5.10)-(3.5.16) are put to zero.

$$(1 + C\delta^2)\delta_1^2 M_0 - \pi(T_a)^{\frac{1}{2}}F + aRa_T M_1 - aRa_S M_3 = 0 \tag{3.5.17}$$

$$aM_0 + \pi a M_0 M_2 + (\delta_2^2 - R_i)M_1 = 0 \tag{3.5.18}$$

$$(4\pi^2 - R_i)M_2 - \frac{\pi a}{2} M_0 M_1 = 0 \tag{3.5.19}$$

$$aM_0 + \delta^2 \frac{1}{L_e} M_3 + \pi a M_0 M_2 + S_r \delta^2 M_1 = 0 \tag{3.5.20}$$

$$4\pi^2 \frac{1}{L_e} M_2 - \frac{\pi a}{2} M_0 M_3 + 4\pi^2 S_r M_2 = 0 \tag{3.5.21}$$

$$(\frac{1}{\xi} + C\delta^2)F + \pi(T_a)^{\frac{1}{2}} M_0 = 0 \tag{3.5.22}$$

$$(\frac{1}{\xi} + C\pi^2)G = 0 \tag{3.5.23}$$

On solving for the amplitudes in terms of M_0, it is obtained

$$\begin{cases} M_1 = -\frac{2a(4\pi^2 - R_i)M_0}{a^2 A_1^2 \pi^2 + 2(R_i - 4\pi^2)(R_i - \delta_2^2)}, \\ M_2 = -\frac{a^2 \pi M_0^2}{a^2 A_1^2 \pi^2 + 2(R_i - 4\pi^2)(R_i - \delta_2^2)}, \\ M_3 = -\frac{(8aM_0 L_e)[a^2 M_0^2 \pi^2 (1 + L_e S_r) + 2(R_i - 4\pi^2)(\delta^2 S_r + R_i - \delta_2^2)]}{(a^2 M_0^2 L_e^2 + 8\delta^2)(a^2 M_0^2 \pi^2 + 2(R_i - 4\pi^2)(R_i - \delta_2^2))}, \\ M_4 = -\frac{(a^2 M_0^2 L_e)[2L_e(R_i - 4\pi^2)(R_i - \delta_2^2) + \pi^2(a^2 M_0^2 L_e - 8S_r\delta^2)]}{\pi(a^2 M_0^2 L_e^2 + 8\delta^2)(a^2 M_0^2 \pi^2 + 2(R_i - 4\pi^2)(R_i - \delta_2^2))} \end{cases} \tag{3.5.24}$$

B. Steady Heat and Mass Transports

In the study of this type problem, quantification of heat and mass transport is very important in porous media. Let H and J denote the rate of heat and mass transport per unit area for the fluid phase, defined by

$$H = -K_T \left\langle \frac{\partial T_{total}}{\partial z} \right\rangle_{z=0} \tag{3.5.25}$$

$$J = -K_S \left\langle \frac{\partial T_{total}}{\partial z} \right\rangle_{z=0} - K_{21} \left\langle \frac{\partial S_{total}}{\partial z} \right\rangle_{z=0} \tag{3.5.26}$$

where the angular bracket corresponds to a horizontal average and

$$T_{total} = T_0 - \Delta T \frac{z}{d} + T(x, z, t) \tag{3.5.27}$$

$$S_{total} = S_0 - \Delta S \frac{z}{d} + S(x, z, t)$$

Substituting Eq. (3.5.6)-(3.5.7)into Eq.(3.5.27) and using the resultant eq.(3.5.25),(3.5.26)

$$H = \frac{K_T \Delta T}{d}(1 - 2\pi M_2) \tag{3.5.28}$$

$$J = \frac{K_S \Delta S}{d}\Big[(1 - 2\pi M_4) + S_r L_e (1 - 2\pi M_2)\Big] \tag{3.5.29}$$

substituting the value of eqs.(3.5.28)-(3.5.29), The Nussalt number and Sherwood number are defined by

$$Nu = \frac{H}{\frac{K_T \Delta T}{d}} = (1 - 2\pi M_2) \tag{3.5.30}$$

$$Sh = \frac{J}{\frac{K_S \Delta S}{d}} = (1 - 2\pi M_4) + S_r L_e (1 - 2\pi M_2)$$

Using the expression of M_2, M_4 in Eqs.(3.5.30) the expression of Nu and Sh can be obtained,

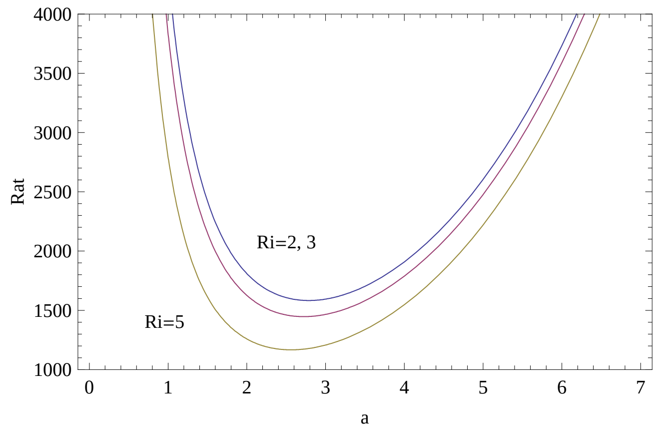


Figure 3.1: Stationary neutral stability curves for different values of R_i

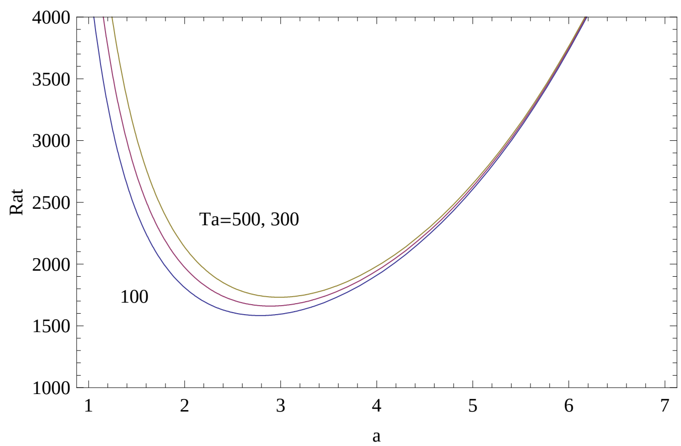


Figure 3.2: Stationary neutral stability curves for different values of T_a

3.6 Result and discussion

Effects of Soret parameter and internal heat source have been studied on double diffusive convection in a couple stress fluid saturated rotating anisotropic porous layer using linear and nonlinear stability analyses. In this section, the effects of the existing parameters on onset of double diffusive convection have been obtained numerically and presented graphically. The expressions of the thermal Rayleigh number for the stationary and oscillatory modes of convection have been computed for different values of the parameters; Taylor number, couple stress parameter, solute Rayleigh number, Darcy-Prandtl number, and Soret parameter and depicted in figures.

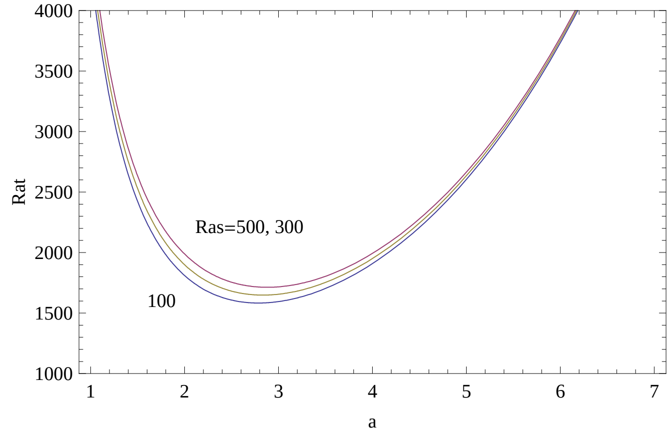


Figure 3.3: Stationary neutral stability curves for different values of Ra_S

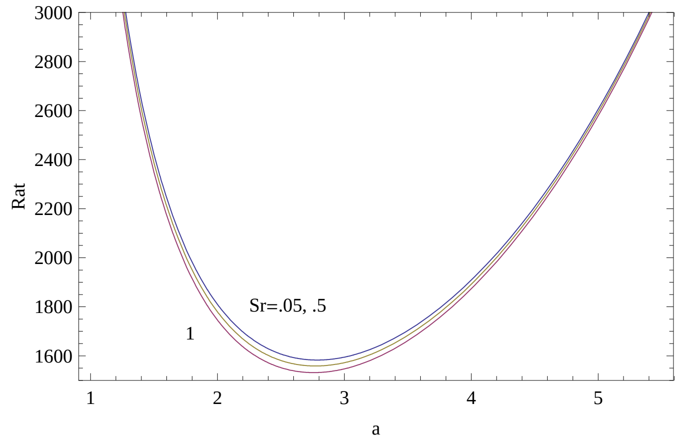


Figure 3.4: Stationary neutral stability curves for different values of S_r

3.6.1 Linear analysis

The marginal stability curves in the (Ra_T, a) plane for the stationary and oscillatory modes are presented for different values of various governing parameters. The values of the parameters are fixed as $T_a = 100$, $C = 2$, $Ra_S = 100$, $L_e = 2$, =.5, $Pr_D = 10$, S_r=.05, =.5 and $R_i = 2$, except the varying parameter.

In Fig. 3.1 and Fig. 3.9, it is observed that the stationary and oscillatory Rayleigh numbers decrease with the increase in internal Rayleigh number R_i, which indicates that the internal Rayleigh number destabilizes the system. From Fig. 3.2 and Fig. 3.10, it is observed that on increasing the value of Taylor number T_a, values of stationary Rayleigh number and oscillatory Rayleigh number increase, thus stabilizing the system and delaying the onset of convection. Besides, the critical wave number increases with increasing T_a. Fig. 3.3 and Fig. 3.11 depict that the stationary and oscillatory Rayleigh number increase

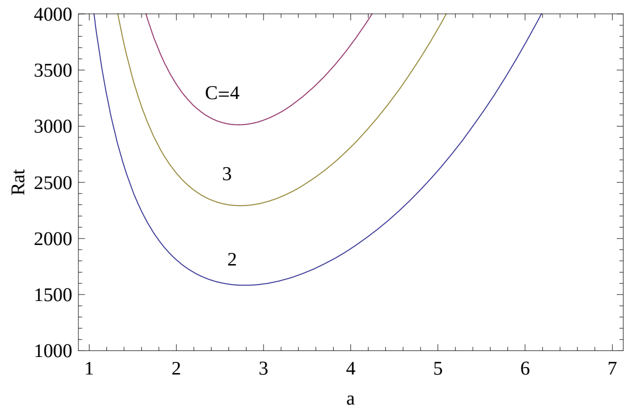


Figure 3.5: Stationary neutral stability curves for different values of C

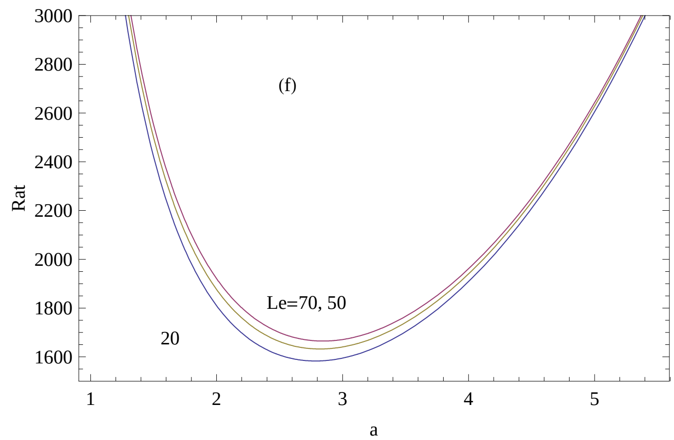


Figure 3.6: Stationary neutral stability curves for different values of L_e

with an increment in solutal Rayleigh number Ra_S, which indicates that the effect of solutal Rayleigh number is to enhance the stability of the system. Therefore, the solutal Rayleigh number Ra_S has a stabilizing effect on the system. Fig. 3.4 and Fig. 3.12 show that increasing the value of Soret parameter S_r is to decrease the stationary Rayleigh number, so it has a destabilizing effect on the stationary convection. On the other hand, the oscillatory Rayleigh number increases with increasing Soret parameter, which means that the Soret parameter S_r has a stabilizing effect on oscillatory mode of convection. From Fig. 3.5, and Figs. 3.13, 3.15 it can be found that an increment in the value of couple stress parameter C and thermal anisotropic parameter are to increase the stationary and oscillatory Rayleigh number. Thus the system has stabilizing effect for both the modes. It is found from Fig. 3.6 and Fig. 3.14 that the effect of increasing the value of Lewis number L_e is to increase stationary Rayleigh number, which indicates that the Lewis number stabilizes the stationary mode of convection, while it has opposite effect on oscillatory Rayleigh number. In Fig.

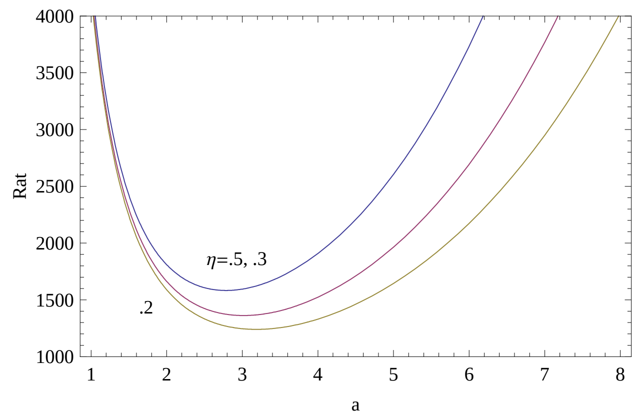


Figure 3.7: Stationary neutral stability curves for different values of η

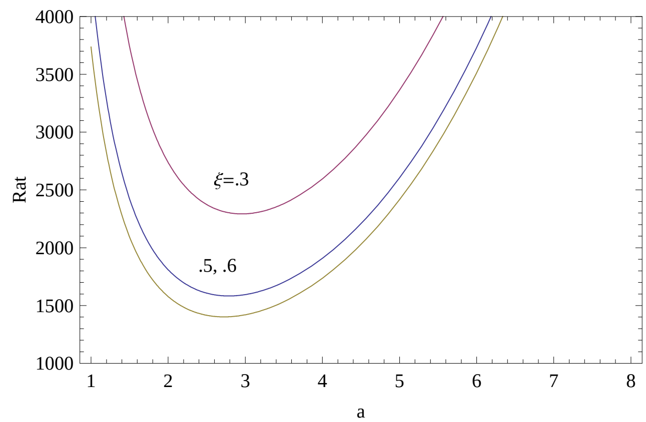


Figure 3.8: Stationary neutral stability curves for different values of ξ

3.7 and Fig. 3.15, it is found that increasing the value of thermal anisotropic parameter, stationary Rayleigh number and oscillatory Rayleigh number increase thus is to stabilize the system. However in Figs. 3.8 and Figs. 3.16, the oppose effect is found for mechanical anisotropic parameter. Fig. 3.17(i), shows that increasing the value of the Darcy-Prandtl number Pr_D decreases the oscillatory Rayleigh number, indicating that the Darcy-Prandtl number is to destabilizing the onset of oscillatory convection.

3.6.2 Nonlinear Analysis

The effect of various parameters on the rate of heat and mass transfer is shown in Figs. 3.18 and 3.19 respectively. Figs. 3.18 (a) and 3.19 (a) shows that an increment in the value of the internal Rayleigh number R_i increases both the rate of heat and mass transfer. Figs. 3.18 (b), (c) - Figs. 3.19 (b) and (c) depict that the Nusselt number N_u and Sherwood number S_h increase with increasing Soret parameter S_r and mechanical anisotropic parameter, which

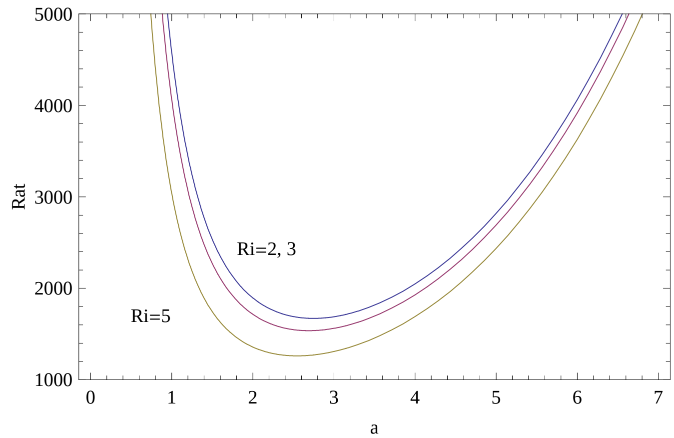


Figure 3.9: Oscillatory neutral stability curves for different values of R_i

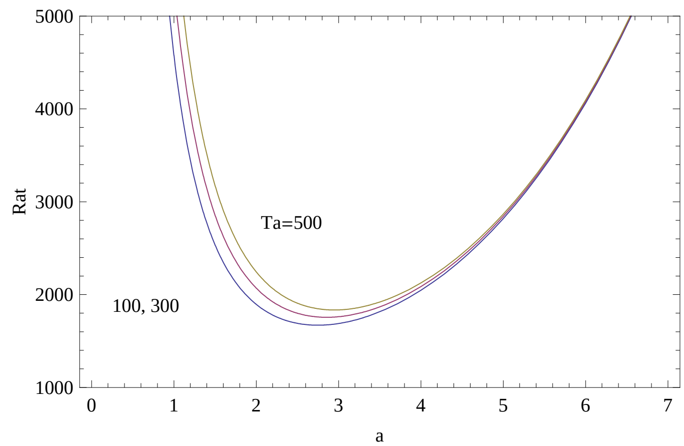


Figure 3.10: Oscillatory neutral stability curves for different values of T_a

indicates that the effect of mechanical anisotropic parameter and Soret parameter S_r is to increase the rate of heat and mass transfer. In Figs. 3.18 (d), (f) - Figs. 3.19 (d), (g), it can be seen that both the rates of heat and mass transfer decrease on increasing solute Rayleigh number Ra_S and thermal anisotropic parameter. From Figs. 3.18 (e), (g) - Figs. 3.19 (e), (h), it is noted that increasing the value of couple stress parameter C and Taylor number T_a is to decrease the Nusselt number N_u and Sherwood number S_h, thus reducing the heat and mass transfer. In Fig. 3.18 (h) and Fig. 3.19 (f), it is shown that an increment in the value of Lewis number L_e decreases the value of Nusselt number N_u and increases the value of Sherwood number S_h, thus the effect of Lewis number L_e has a stabilizing effect on heat transfer.

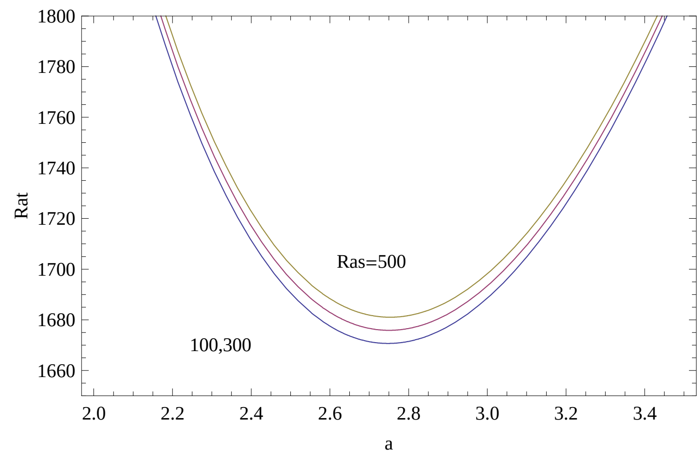


Figure 3.11: Oscillatory neutral stability curves for different values of Ra_S

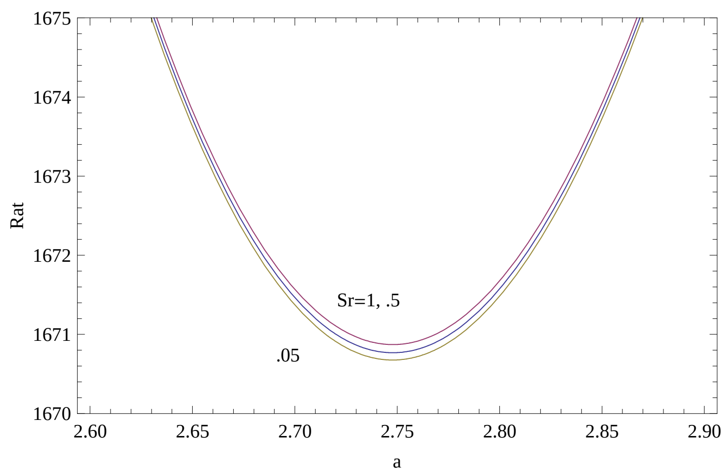


Figure 3.12: Oscillatory neutral stability curves for different values of S_r

3.7 Conclusion

In the present chapter, Soret and internal heating effect on double diffusive convection in a couple stress fluids saturated rotating anisotropic porous layer, heated and salted from below, is investigated. The problem has been solved analytically, performing linear and nonlinear analyses. The linear analysis is done using normal mode technique. The following findings are made:

1. The Taylor number T_a, couple stress fluid C, solute Rayleigh number Ra_S and thermal anisotropic parameter has a stabilizing effect on both stationary and oscillatory modes of convection.

2. The internal heat parameter R_i , mechanical anisotropic parameter destabilize the stationary and oscillatory system.

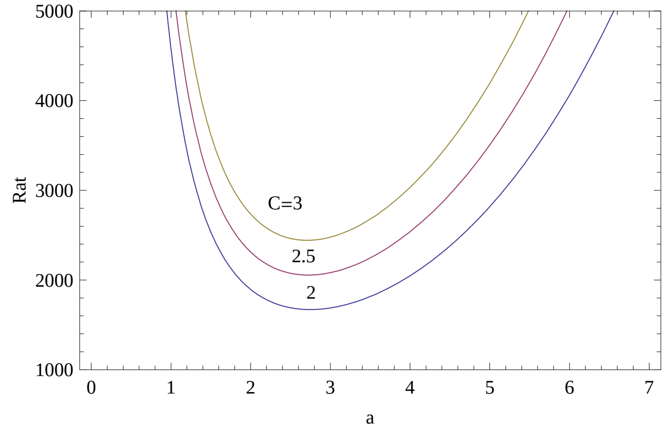


Figure 3.13: Oscillatory neutral stability curves for different values of C

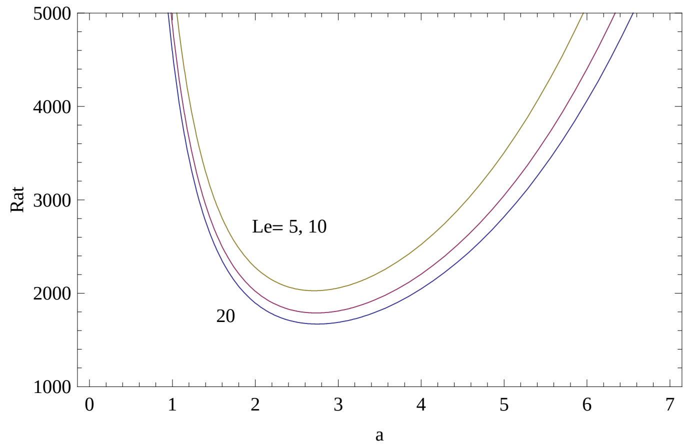


Figure 3.14: Oscillatory neutral stability curves for different values of L_e

3. The effect of Lewis number L_e has a stabilizing effect on the stationary and opposite effect has oscillatory convection.

4. The Soret parameter S_r has a stabilizing effect on the oscillatory and opposite effect has stationary convection.

5. The Darcy-Prandtl number Pr_D has a destabilize effect in case of oscillatory convection.

6. The Increasing the value of internal Rayleigh number R_i ,Soret number S_r and mechanical anisotropic parameter then increase the value of Nusselt number N_u i.e. increased heat transfer but increasing the value of Taylor number T_a ,solute Rayleigh number Ra_S, Lewis number L_e, thermal anisotropic parameter and couple stress parameter C decrease the value of Nusselt number N_u.

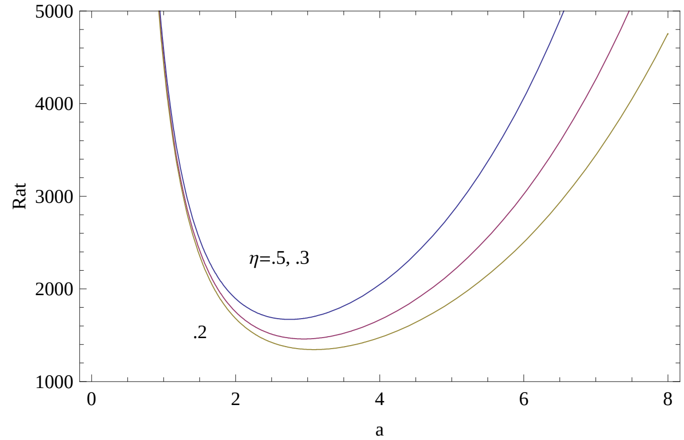


Figure 3.15: Oscillatory neutral stability curves for different values of η

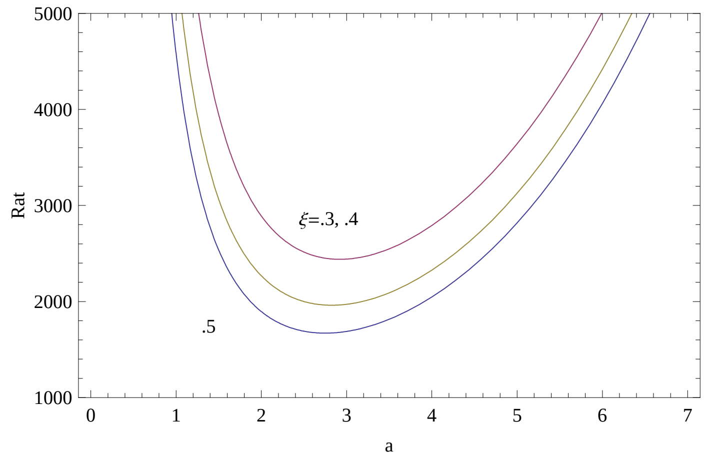


Figure 3.16: Oscillatory neutral stability curves for different values of ξ

7. Mass transfer increased with increasing the value of Soret number S_r and mechanical anisotropic parameter, Internal Rayleigh number R_i, Lewis number L_e i.e. stable and decrease with thermal anisotropic parameter, Taylor number T_a ,and couple stress parameter C, solute Rayleigh number Ra_S i.e. destable.

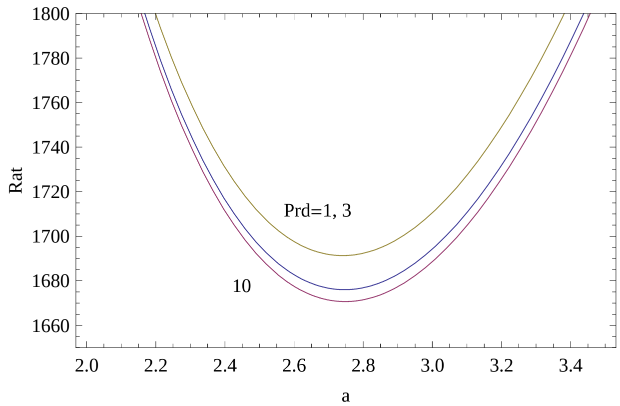
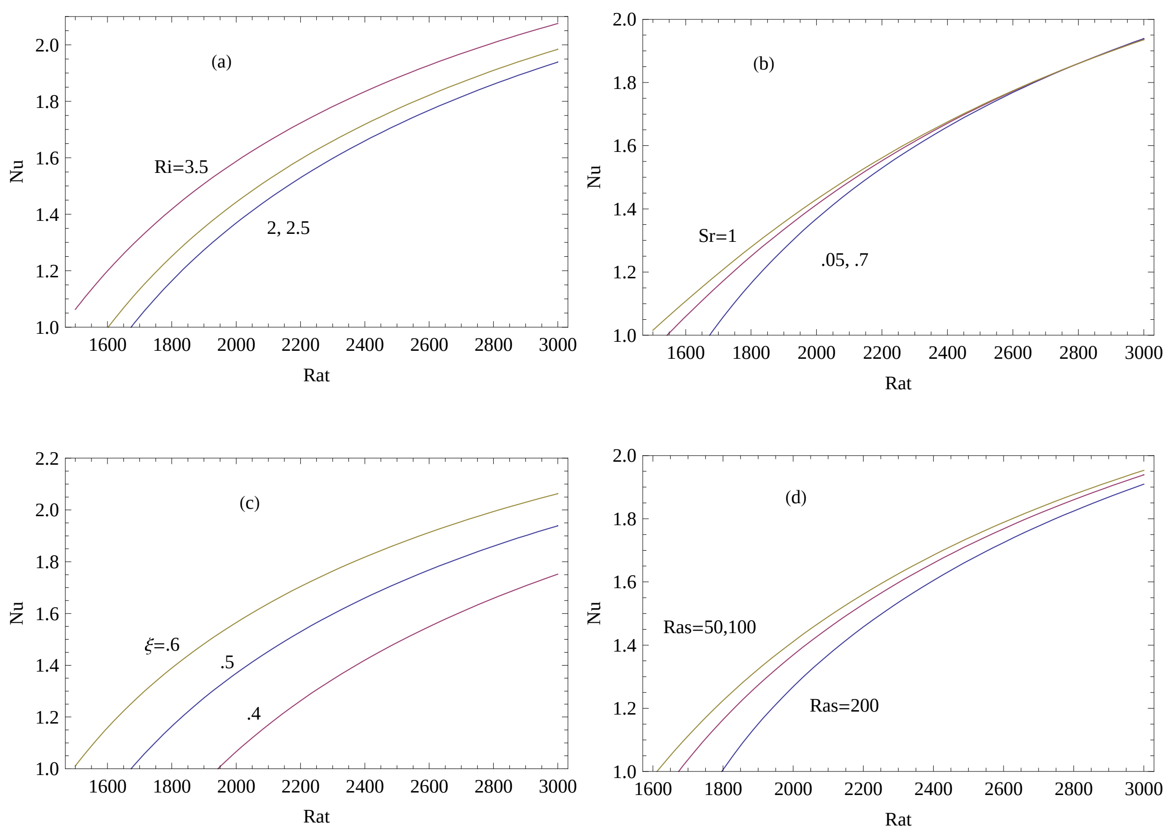


Figure 3.17: Oscillatory neutral stability curves for different values of Pr_D



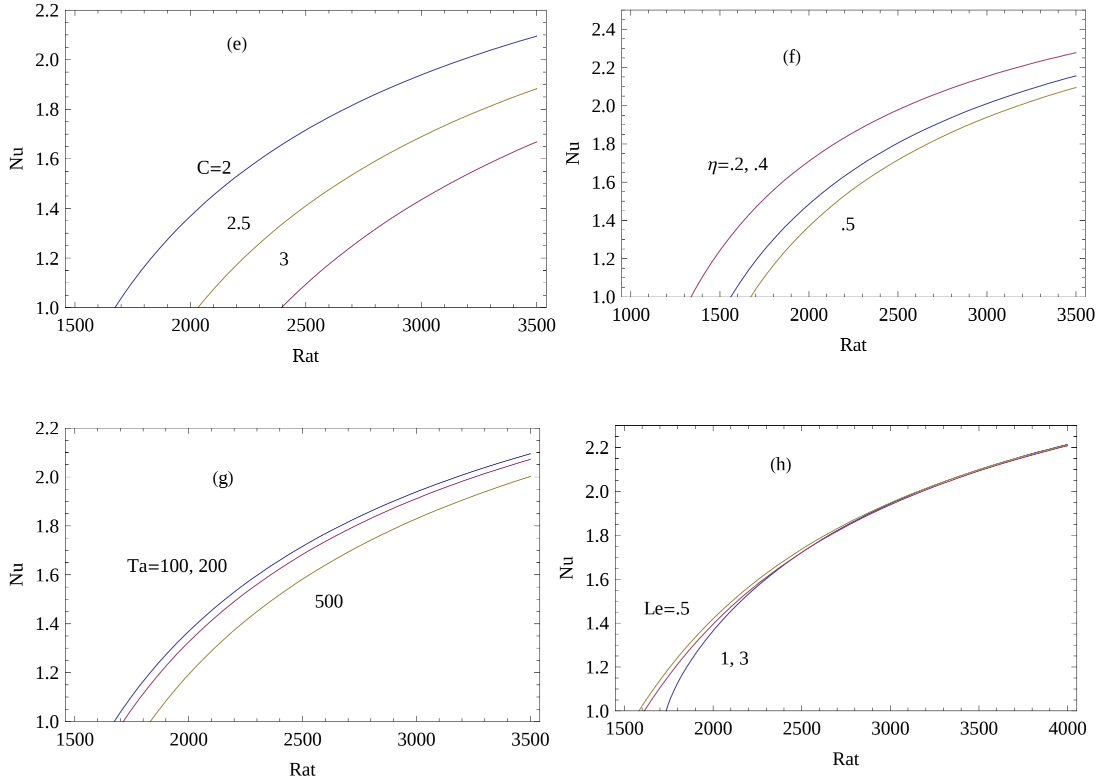
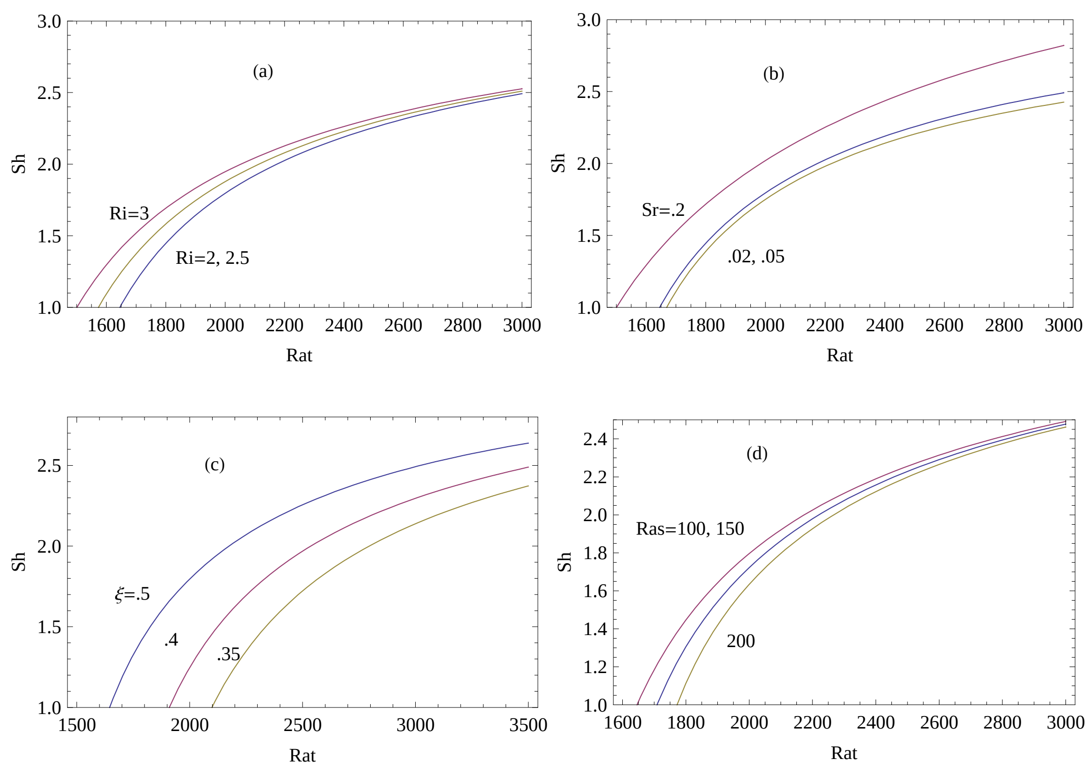


Figure 3.18: Variation of Nusselt number with Ra_T for different values of (a) R_i,(b) S_r,(c) ξ,(d) Ra_S,(e) C,(f) η,(g) T_a,(h) L_e



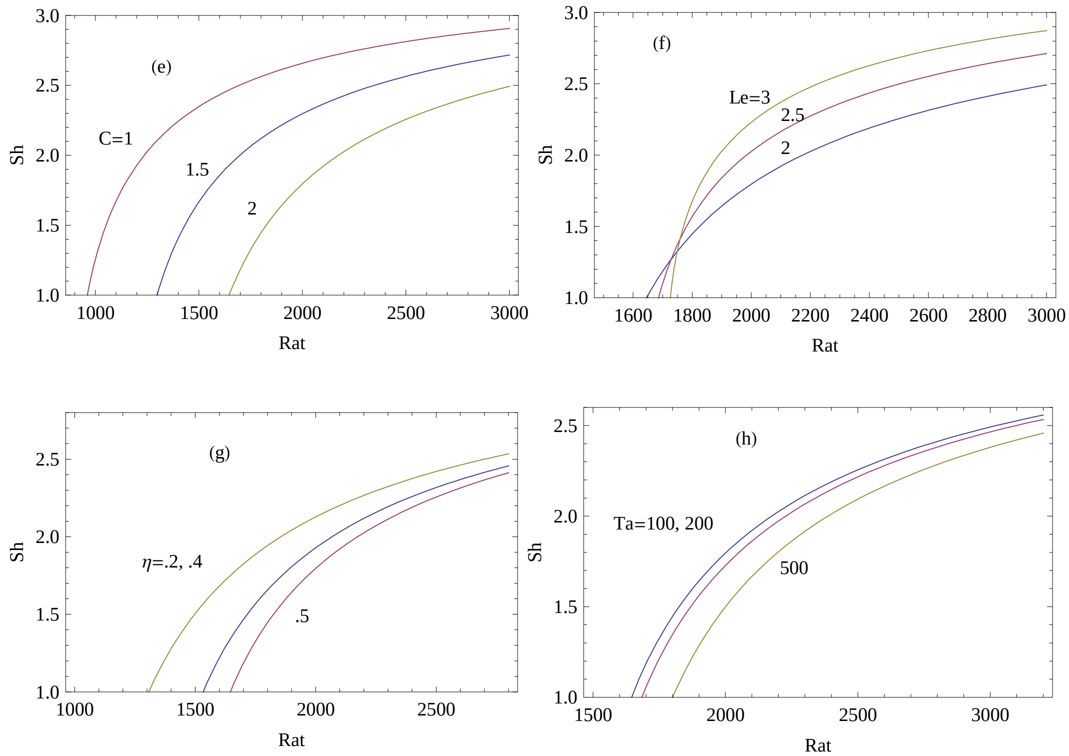


Figure 3.19: Variation of Sherwood number with Ra_T for different values of (a) R_i,(b) S_r,(c) ξ,(d) Ra_S,(e) C,(f) η,(g) T_a,(h) L_e

Chapter 4

Double diffusive convection in a viscoelastic fluid saturated rotating anisotropic porous layer with internal heat source

4.1 Introduction

Most of the studies in relevant area are mainly dealt with isotropic porous media; however there are many physical situations where thermal and mechanical anisotropy exists in porous matrix, one of such examples is our geothermal environment. Anisotropy is generally a consequence of preferential orientation of asymmetric geometry of porous matrix or fibers and is in fact encountered in numerous systems in industry and nature, also in artificial porous matrix anisotropy can be made deliberately according to applications. Srivastava et al.(2014) studied the effect of internal heating on double diffusive convection in a couple stress fluid saturated anisotropic porous medium.

This chapter is based on the research article:Double diffusive convection in a viscoelastic fluid saturated rotating anisotropic porous layer with internal heat source, published in **International Journal of Research in Advent Technology**,Vol.6, No.12,3524-3536 (2018).

There is large number of practical situations in which convection is driven by internal heat source. Internal heat generation arises in many important contexts, including reactor safety analyses, metal waste that is produced by spent nuclear fuel, fire and combustion studies, and the storage of radioactive materials. The study concerning internal heat source in porous media was done by Tveitereid (1977), performing thermal convection in a horizontal porous layer with internal heat source. Hill (2005) performed linear and nonlinear analyses on the double-diffusive convection in a porous layer with a concentration based internal heat source. Bhadauria (2012) studied the effect of internal heating on double diffusive convection in a fluid saturated anisotropic porous medium. The study of double diffusive convection in a rotating porous media is important due to both, its theoretical and practical applications in engineering. Some of the important areas of applications in engineering include the food and chemical process, solidification and centrifugal casting of metals, rotating machinery, petroleum industry and biomechanics problems. There are only few studies available on double diffusive convection in the presence of rotation.Chakrabarti and Gupta (1981) have analyzed the nonlinear thermohaline convection in a rotating porous medium. The effect of rotation on linear and nonlinear double diffusive convection in a sparsely packed porous medium was studied by Rudraiah et.al. (1986). Malashetty et al.(2013) studied the effect of rotation on the onset of double diffusive convection in a Darcy porous medium saturated with couple stress fluid.Malashetty and Heera (2008, 2009) studied the effect of rotation on the onset of double diffusive convection in a horizontal anisotropic porous layer. Gaikwad (2012) have done the linear stability analysis of double diffusive convection in a horizontal sparsely packed rotating anisotropic porous layer in presence of Soret effect. Sulochana et.al (2012) studied the onset of double diffusive convection in a couple stress fluid saturated rotating anisotropic porous layer. Bhadauria et al. (2013) studied cross diffusion convection in a Newtonian fluid-saturated rotating porous medium. The work published on natural convection of viscoelastic fluids in porous media is fairly limited. Convection in a viscoelastic fluid-saturated sparsely packed porous layer is studied by Rudraiah et al. (1990, 1982). Mardones et al. (2000, 2003) have investigated the Rayleigh-Benard convection for stationary convection in a binary viscoelastic fluid. Yoon et al. (2003, 2004), Kim et al. (2003), and Bertola and Cafaro (2006) studied the stability of a viscoelastic fluid where an existing constitutive model, which is rather simple, was employed to examine the effects of relaxation and retardation times on the stationary and oscillatory convection in a horizontal porous layer heated by a constant temperature. Park and Park (2004) studied Rayleigh-Benard convection of viscoelastic fluids in arbitrary finite domains. Convective instabilities in a viscoelastic-fluid-saturated porous medium with throughflow have been studied by Shivakumara and Sureshkumar (2007). Linear and nonlinear stability analyses of thermal convection for Oldroyd-B fluids in a porous media heated from below has been studied by Zhang et al.(2008). Malashetty et al. (2007) studied the onset of convection in a binary viscoelastic fluid-saturated porous layer. Kumar and Bhadauria (2011) have studied non-linear two-dimensional double diffusive convection in a rotating porous layer saturated by a viscoelastic fluid. Gaikwad et al. (2013) performed a binary viscoelastic fluid-saturated porous layer in internal heat source with onset of Darcy-Brinkman convection. Recently Srivastava et al.(2018) have studied linear and weak nonlinear double diffusive convection in a viscoelastic fluid saturated anisotropic porous medium with internal heat source. In the literature, no work is available on double diffusive convection in a rotating porous layer saturated by a viscoelastic fluid with an internal heat source. Therefore, in the present study stability analysis of internal heating effect on double diffusive convection in a rotating anisotropic porous medium saturated with a viscoelastic fluid has been done.

4.2 Mathematical Formulation

Consider a viscoelastic fluid saturated porous medium, confined between two infinitely extended horizontal planes at $z = 0$ and $z = d$, heated from below and cooled from above. Darcy model has been employed in the momentum equation. Further, an internal heat source term has been included in the energy equation. A Cartesian frame of reference is chosen so that the origin lies on the lower plane and the z-axis as vertical upward. The system is rotating about z-axis with a constant angular velocity Ω. An adverse temperature gradient is applied across the porous layer. The lower planes is kept at temperature $T_0 + \Delta T$, while upper planes is kept at temperature T_0, and concentration $S_0 + \Delta S$, and

S_0 respectively. The physical configuration of the model is reported in Figure 4.1. The

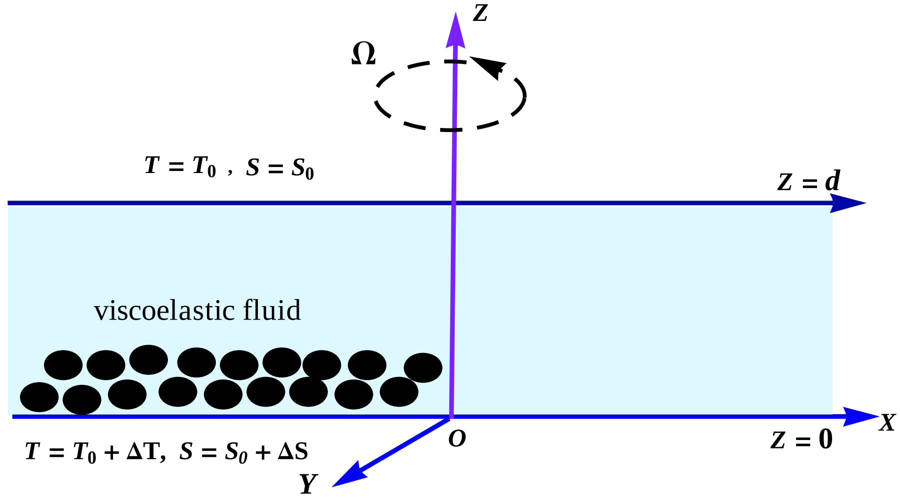


Figure 4.1: Physical configuration of the problem

governing equations are as given below

$$\nabla.\vec{q} = 0, \tag{4.2.1}$$

$$(1+\overline{\lambda_1}\frac{\partial}{\partial t})\frac{2\rho_0}{\phi}\Omega\times\vec{q}+\frac{\mu}{k}(1+\overline{\lambda_2}\frac{\partial}{\partial t})q = (1+\overline{\lambda_1}\frac{\partial}{\partial t})(-\nabla p+\rho g), \tag{4.2.2}$$

$$\gamma\frac{\partial T}{\partial t}+(\vec{q}.\nabla)T = \nabla(\kappa_T.\nabla T)+Q(T-T_0), \tag{4.2.3}$$

$$\phi\frac{\partial S}{\partial t}+(\vec{q}.\nabla)S = \kappa_S\nabla^2 S, \tag{4.2.4}$$

$$\rho = \rho_0[1-\beta_T(T-T_0)+\beta_S(S-S_0)] \tag{4.2.5}$$

The externally imposed thermal and solutal boundary conditions are given by

$$\begin{cases} T = T_0+\Delta T, & \text{at} \quad z=0 \quad \text{and} \quad T=T_0, & \text{at} \quad z=d, \\ S = S_0+\Delta S, & \text{at} \quad z=0 \quad \text{and} \quad S=S_0, & \text{at} \quad z=d, \end{cases} \tag{4.2.6}$$

4.3 Basic state

At this state, the velocity, pressure, temperature, concentration and density profiles are given by

$$\vec{q_b} = 0, p = p_b(z), T = T_b(z), S = S_b(z), \rho = \rho_b(z). \tag{4.3.1}$$

Putting Eq. (4.3.1) in Eq. (4.2.1)-(4.2.4), we get the following equations:

$$\frac{dp_b}{dz} = -\rho_b g, \tag{4.3.2}$$

$$\kappa_T \frac{d^2 T_b}{dz^2} + Q(T_b - T_0) = 0, \tag{4.3.3}$$

$$\frac{d^2 S_b}{dz^2} = 0, \tag{4.3.4}$$

The solution of Eq.(4.3.3) and Eq. (4.3.4), subject to the boundary conditions (4.2.6), are given by

$$T_b = T_0 + \Delta T \frac{\sin((\sqrt{\frac{Qd^2}{\kappa_T}})(1 - \frac{z}{d}))}{\sin\left(\sqrt{\frac{Qd^2}{\kappa_T}}\right)}. \tag{4.3.5}$$

$$S_b = S_0 + \Delta S(1 - \frac{z}{d}) \tag{4.3.6}$$

4.4 Perturbed equation

Now, superimpose a infinite amplitude disturbances on the basic state in the form:

$$\vec{q} = q_b + q', T = T_b + T', p = p_b + p', S = S_b + S', \rho = \rho_b + \rho', \tag{4.4.1}$$

and get the following equations

$$\begin{cases} \nabla.\vec{q'} = 0, \\ (1+\overline{\lambda_1}\frac{\partial}{\partial t})\frac{2\rho_0}{\phi}\Omega\times\vec{q'} + \frac{\mu}{k}(1+\overline{\lambda_2}\frac{\partial}{\partial t})q' = -(1+\overline{\lambda_1}\frac{\partial}{\partial t})(\nabla p - \rho_0(\beta_T T' - \beta_S S')g) \\ \gamma\frac{\partial T'}{\partial t} + (\vec{q'}.\nabla)T' + w'\frac{\partial T_b}{\partial z} = \kappa_{Tx}\nabla_1^2 T' + \kappa_{Tz}\frac{\partial^2 T'}{\partial z^2} + QT', \\ \frac{\partial S'}{\partial t} + (\vec{q'}.\nabla)S' + w'\frac{\partial S_b}{\partial z} = \kappa_S\nabla^2 S', \\ \rho' = -\rho_0(\beta_T T' - \beta_S S') \end{cases} \tag{4.4.2}$$

The resulting equations are non dimensionalized, using the following transformations:

$$(x', y', z') = (x^*, y^*, z^*)d, t' = t^*(\frac{d^2}{\kappa_{Tz}}), (u, v, w) = (u^*, v^*, w^*)(\frac{\kappa_{Tz}}{d}), \tag{4.4.3}$$

$$T' = (\Delta T)T^*, S' = (\Delta S)S^*, \lambda_1 = \frac{d^2}{\kappa_{Tz}}\lambda_1^*, \lambda_2 = \frac{d^2}{\kappa_{Tz}}\lambda_2^*, p' = \frac{\mu\kappa_{Tz}}{K_z}p^*$$

4.5 Non-Dimensionilized equation

The non-dimensionalized equations (on dropping the asterisks for simplicity) are obtained as,

$$\nabla.\vec{q} = 0, \tag{4.5.1}$$

$$(1+\lambda_1\frac{\partial}{\partial t})\sqrt{T_a}(\kappa\times\vec{q'}) + (1+\lambda_2\frac{\partial}{\partial t})q_a = -(1+\lambda_1\frac{\partial}{\partial t})\nabla p \tag{4.5.2}$$

$$+(1+\lambda_1\frac{\partial}{\partial t})(Ra_T T - \tau Ra_S S)k$$

$$\frac{\partial T}{\partial t} + w\frac{\partial T_b}{\partial z} = (\eta\nabla_1^2 + \frac{\partial^2}{\partial z^2})T + R_i T \tag{4.5.3}$$

$$\frac{\partial S}{\partial t} + w\frac{\partial S_b}{\partial z} = \tau\nabla^2 S \tag{4.5.4}$$

where $q_a = (\frac{1}{\xi}u, \frac{1}{\xi}v, w)$ is anisotropic modified velocity vector, $T_a = (\frac{2\Omega K_z}{\mu\phi})^2$ is Taylor number, $Ra_T = \frac{\beta_T g \Delta T K_z d}{\nu \kappa_{Tz}}$ is the thermal Rayleigh number, $Ra_S = \frac{\beta_S g \Delta S K_z d}{\nu \kappa_{Tz}}$ is the solute Rayleigh number, $R_i = \frac{Qd^2}{\kappa_{Tz}}$ is the internal heat source parameter, $\tau = \frac{\kappa_S}{\kappa_{Tz}}$ is diffusivity ratio, $\eta = \frac{\kappa_{Tx}}{\kappa_{Tz}}$ is thermal anisotropy parameter,$\xi = \frac{K_x}{K_z}$ is mechanical anisotropy parameter.

$$T_b = \frac{\sin\sqrt{R_i}(1-z)}{\sin\sqrt{R_i}}, \tag{4.5.5}$$

$$S_b = (1-z)$$

The above system will be solved by considering stress free and isothermal boundary conditions as given below:

$$w = \frac{\partial^2 w}{\partial z^2} = T = S = 0 \quad on \quad z = 0, z = 1 \tag{4.5.6}$$

The pressure term from eq.(4.5.2) is eliminated by taking curl of the momentum equation

$$(1+\lambda_2\frac{\partial}{\partial t})\frac{1}{\xi}\omega - (1+\lambda_1\frac{\partial}{\partial t})\sqrt{T_a}\frac{\partial q}{\partial z} = (1+\lambda_1\frac{\partial}{\partial t}) \tag{4.5.7}$$

$$\Big[Ra_T(\frac{\partial T}{\partial y}i - \frac{\partial T}{\partial x}j) - \tau Ra_S(\frac{\partial S}{\partial y}i - \frac{\partial S}{\partial x}j)\Big]$$

where $\omega = \nabla \times q$ is vorticity vector. On further taking curl,

$$(1+\lambda_2\frac{\partial}{\partial t})Q - (1+\lambda_1\frac{\partial}{\partial t})\sqrt{T_a}\frac{\partial \omega}{\partial z} = (1+\lambda_1\frac{\partial}{\partial t})\Big[Ra_T\nabla_1^2 T - \tau Ra_S \nabla_1^2 S\Big] \tag{4.5.8}$$

where $Q = (Q_1, Q_2, Q_3)$, $Q_1 = \frac{1}{\xi}\frac{\partial^2 v}{\partial y \partial x} + \frac{\partial^2 w}{\partial y \partial x} - (\frac{\partial^2 v}{\partial y^2} + \frac{1}{\xi}\frac{\partial^2 u}{\partial z^2})$, $Q_2 = \frac{1}{\xi}\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 w}{\partial y \partial z} - \frac{1}{\xi}(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial z^2})$, $Q_3 = -(\Delta_1^2 + \frac{1}{\xi}\frac{\partial^2}{\partial z^2})w$.

4.6 Linear stability Analysis

Linear equation are

$$(1+\lambda_2\frac{\partial}{\partial t})\frac{1}{\xi}\omega_z = (1+\lambda_1\frac{\partial}{\partial t})\sqrt{T_a}\frac{\partial \omega}{\partial z} \tag{4.6.1}$$

$$(1+\lambda_2\frac{\partial}{\partial t})(\nabla_1^2+\frac{1}{\xi}\frac{\partial^2}{\partial z^2})w+(1+\lambda_1\frac{\partial}{\partial t})\sqrt{T_a}\frac{\partial\omega_z}{\partial z} \tag{4.6.2}$$

$$=(1+\lambda_1\frac{\partial}{\partial t})\Big[Ra_T\nabla_1^2T-\tau Ra_S\nabla_1^2S\Big]$$

$$\Big(\frac{\partial}{\partial t}-\eta\nabla_1^2-\frac{\partial^2}{\partial z^2}-R_i\Big)T=w \tag{4.6.3}$$

$$\Big(\frac{\partial}{\partial t}-\tau\nabla^2\Big)S=w \tag{4.6.4}$$

By eliminating T, S, ω_z from above equations, it is obtained

$$\Big[(\frac{\partial}{\partial t}-\eta\nabla_1^2-\frac{\partial^2}{\partial z^2}-R_i)(\frac{\partial}{\partial t}-\tau\nabla^2)(1+\lambda_2\frac{\partial}{\partial t})^2(\nabla_1^2+\frac{1}{\xi}\frac{\partial^2}{\partial z^2})+\xi T_a(1+\lambda_1\frac{\partial}{\partial t})^2\frac{\partial^2}{\partial z^2} \tag{4.6.5}$$

$$-(1+\lambda_1\frac{\partial}{\partial t})(1+\lambda_2\frac{\partial}{\partial t})\nabla_1^2(\frac{\partial}{\partial t}-\tau\nabla^2)Ra_T-(\frac{\partial}{\partial t}-\eta\nabla_1^2-\frac{\partial^2}{\partial z^2}-R_i)\tau Ra_S\Big]w=0$$

$$w=\frac{\partial^2w}{\partial z^2}=\frac{\partial^4w}{\partial z^4}=0 \quad on \quad z=0,z=1. \tag{4.6.6}$$

Use Normal mode technique as

$$w=W(z)e^{(i(lx+my)+\sigma t)}\sin\pi z \tag{4.6.7}$$

where l, m are horizontal wave numbers and $\sigma=\sigma_r+i\sigma_j$ is the growth rate. Solving the above equations, the thermal Rayleigh number can be obtained as

$$Ra_T=\frac{(\sigma+\delta_2^2-R_i)(1+\lambda_2\sigma)\delta_1^2}{a^2(1+\lambda_1\sigma)}+\frac{T_a\pi^2\xi(1+\lambda_2\sigma)(\sigma+\delta_2^2-R_i)}{a^2(1+\lambda_1\sigma)} \tag{4.6.8}$$

$$+\tau Ra_S\frac{(\sigma+\delta_2^2-R_i)}{(\sigma+\tau\delta^2)},$$

where $a^2=l^2+m^2$, $\delta_1^2=\frac{\pi^2}{\xi}+a^2$, $\delta_2^2=\pi^2+\eta a^2$. The growth rate$\sigma$ is in general a complex quantity such that $\sigma=\sigma_r+i\sigma_j$. The system with $\sigma_r<0$ is always stable, while for $\sigma_r>0$ it will become unstable. For neutral stability state $\sigma_r=0$.

4.6.1 Stationary State

Now set σ_r=0 (i.e.σ_r =σ_j=0) in Eq.(4.6.8) at the margin of stability. The expression of the thermal Rayleigh number for stationary mode of convection is found as given below:

$$Ra_T^{st} = \frac{(\delta_2^2 - R_i)\delta_1^2}{a^2)} + \frac{T_a\pi^2\xi(\delta_2^2 - R_i)}{a^2} + \tau Ra_S\frac{(\delta_2^2 - R_i)}{\tau\delta^2} \tag{4.6.9}$$

It is important to note that the critical wave number $a = a_c^{St}$ depends on the Taylor number. In the absence of Taylor number, i.e. when $T_a = 0$, Eq.(4.6.9) gives

$$Ra_T^{st} = \frac{(\delta_2^2 - R_i)\delta_1^2}{a^2)} + \tau Ra_S\frac{(\delta_2^2 - R_i)}{\tau\delta^2} \tag{4.6.10}$$

For isotropic porous media, $\xi = \eta = 1$ and without internal-heating, i.e., $R_i = 0$ then

$$Ra_T^{st} = \frac{(\pi^2 + a^2)^2}{a^2} + Ra_S \tag{4.6.11}$$

which is the result given by Malashetty et al. (2009). For single component fluid, $Ra_S = 0$, then

$$Ra_T^{st} = \frac{(\pi^2 + a^2)^2}{a^2} \tag{4.6.12}$$

which has the critical value $Ra_c^{St} = 4\pi^2$ for $a_c^{St} = \pi$ are the classical results by Horton and Rogers (1945) and Lapwood (1948) for single component fluid in porous layer.

4.6.2 Oscillatory State

For the oscillatory mode of convection, set $\sigma = i\sigma_j$ in Eq.(4.6.8) and removing the complex quantities from the denominator, to obtain

$Ra_T^{osc} = \Delta_1 + i\sigma_i\Delta_2$.

where

$$\Delta_1 = \frac{(\tau\delta^2 + \sigma^2)(\delta_1^2 + \pi^2\xi T_a)[\delta_2^2 - R_i - \lambda_2\sigma^2 + \sigma^2\lambda_1(1 + \delta_2^2 - R_i)]}{a^2(1 + \sigma^2\lambda_1^2)} \tag{4.6.13}$$

$$+\frac{a^2(1+\sigma^2\lambda_1)\tau Ra_S[\tau\delta^2(\delta_2^2-R_i)+\sigma^2]}{(\tau\delta^2+\sigma^2)}$$

$$\Delta_2=\frac{(\tau\delta^2+\sigma^2)(\delta_1^2+\pi^2\xi T_a)[(1+\delta_2^2-R_i)-\lambda_1(\delta_2^2-R_i-\lambda_2\sigma^2)]}{a^2(1+\sigma^2\lambda_1^2)} \tag{4.6.14}$$

$$+\frac{a^2(1+\sigma^2\lambda_1)\tau Ra_S[\tau\delta^2-(\delta_2^2-R_i)]}{(\tau\delta^2+\sigma^2)}$$

For oscillatory mode $\Delta_2 = 0$ and ($\sigma_i \neq 0$, where σ is the oscillatory frequency which is not given for brevity. The necessary expression for oscillatory Rayleigh number is given by

$$Ra_T^{osc} = \Delta_1. \tag{4.6.15}$$

4.7 Nonlinear stability Analysis

In this section, nonlinear stability has been studied using minimal truncated Fourier series. For simplicity, only two dimensional rolls have been considered, so that all physical quantities do not dependent of y. Now introducing the stream function ψ as $u = \frac{\partial\psi}{\partial z}$, $w = \frac{-\partial\psi}{\partial x}$, then taking curl to eliminate pressure term from Eq.(4.2.2), to get

$$\Big(1+\lambda_2\frac{\partial}{\partial t}\Big)^2\Big(\frac{\partial^2}{\partial x^2}+\frac{1}{\xi}\frac{\partial^2}{\partial z^2}\Big)\psi+\Big(1+\lambda_1\frac{\partial}{\partial t}\Big)^2\xi T_a\frac{\partial^2\psi}{\partial z^2} \tag{4.7.1}$$

$$=\Big(1+\lambda_1\frac{\partial}{\partial t}\Big)\Big(1+\lambda_2\frac{\partial}{\partial t}\Big)\Big[\tau Ra_S\frac{\partial S}{\partial x}-Ra_T\frac{\partial T}{\partial x}\Big]$$

$$\Big(\frac{\partial}{\partial t}-\eta\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial z^2}-R_i\Big)T+\frac{\partial\psi}{\partial x}-\frac{\partial(\psi,T)}{\partial(x,z)}=0, \tag{4.7.2}$$

$$\Big(\frac{\partial}{\partial t}-\tau(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\Big))S+\frac{\partial\psi}{\partial x}-\frac{\partial(\psi,S)}{\partial(x,z)}=0 \tag{4.7.3}$$

It is to be noted that the effect of nonlinearity is to distort the temperature and concentration fields through the interaction of ψ and T ,ψ and S. As a result a component of the form $sin(2\pi z)$ will be generated. A minimal Fourier series which describes the finite amplitude convection is given by

$$\psi = M_0(t)sin(ax)sin(\pi z), \tag{4.7.4}$$

$$T = M_1(t)cos(ax)sin(\pi z) + M_2(t)sin(2\pi z), \tag{4.7.5}$$

$$S = M_3(t)cos(ax)sin(\pi z) + M_4(t)sin(2\pi z), \tag{4.7.6}$$

where the amplitudes $M_0(t)$, $M_1(t)$, $M_2(t)$,$M_3(t)$, $M_4(t)$ are functions of time and are to be determined. Substituting above expressions in Eqs. (4.7.1) - (4.7.3) and equating the like terms, the following set of nonlinear autonomous differential equations is obtained

$$\frac{dM_0}{dt} = D_1(t) \tag{4.7.7}$$

$$\frac{dD_1}{dt} = -\frac{1}{(\lambda_2^2\delta_1^2 + \pi^2\xi T_a\lambda_1^2)}\Big[(\delta_1^2 + \pi^2\xi T_a)M_0 + 2(\lambda_1^2\delta_1^2 + \pi^2\xi T_a\lambda_2^2)D_1 \tag{4.7.8}$$

$$+aRa_TM_1 - \tau aRa_SM_3 + aRa_T(\lambda_1 + \lambda_2)\frac{dM_1}{dt} - \tau aRa_S(\lambda_1 + \lambda_2)\frac{dM_3}{dt}$$

$$+aRa_T(\lambda_1\lambda_2)\frac{d^2M_1}{dt^2} - \tau aRa_S(\lambda_1\lambda_2)\frac{d^2M_3}{dt^2}\Big] = 0$$

$$\frac{dM_1}{dt} = -aM_0 - \pi aM_0M_2 + (R_i - \delta_2^2)M_1] \tag{4.7.9}$$

$$\frac{dM_2}{dt} = (R_i - 4\pi^2)M_2 + \frac{\pi a}{2}M_0M_1 \tag{4.7.10}$$

$$\frac{dM_3}{dt} = -aM_0 - \tau\delta^2M_3 - \pi aM_0M_2 \tag{4.7.11}$$

$$\frac{dM_4}{dt} = -4\pi^2\tau M_4 + \frac{\pi a}{2}M_0M_3 \tag{4.7.12}$$

Numerical method was used to solve the above nonlinear differential equation to find the amplitudes.

4.7.1 Steady finite amplitude convection

For steady state finite amplitude convection set left hand side of the Eqs.(4.7.7)-(4.7.12) equal to zero.

$$D_1(t) = 0 \tag{4.7.13}$$

$$(\delta_1^2 + \pi^2\xi T_a)M_0 + aRa_TM_1 - \tau aRa_SM_3 = 0 \tag{4.7.14}$$

$$aM_0 + \pi aM_0M_2 - (R_i - \delta_2^2)M_1 = 0 \tag{4.7.15}$$

$$(R_i - 4\pi^2)M_2 + \frac{\pi a}{2}M_0 M_1 = 0 \tag{4.7.16}$$

$$aM_0 + \tau\delta^2 M_3 + \pi a M_0 M_2 = 0 \tag{4.7.17}$$

$$\frac{\pi a}{2}M_0 M_3 - 4\pi^2\tau M_4 = 0 \tag{4.7.18}$$

On solving the above equation for the amplitudes, M_1, M_2, M_3, M_4 are obtained in terms of M_0 as

$$\begin{cases} M_3 = -\frac{8aM_0\tau}{a^2M_0^2+8\delta^2\tau^2}, \\ M_4 = -\frac{a^2M_0\tau}{\pi(a^2M_0^2+8\delta^2\tau^2)}, \\ M_1 = -\frac{2aM_0(4\pi^2-R_i)}{a^2M_0^2\pi^2+2(4\pi^2-R_i)(\delta_2^2-R_i)}, \\ M_2 = -\frac{a^2M_0^2\pi}{a^2M_0^2\pi^2+2(4\pi^2-R_i)(\delta_2^2-R_i)} \end{cases} \tag{4.7.19}$$

4.7.2 Steady Heat and Mass Transports

In the study of this type problem, quantification of heat and mass transport is very important in porous media. Nusselt number and Sherwood number which denote the rate of heat and mass transport per unit area for the fluid phase respectively, are given by

$$N_u = 1 + \left[\frac{\int_0^{2\pi/a}\frac{\partial T}{\partial z}dx,}{\int_0^{2\pi/a}\frac{\partial T_b}{\partial z}dx,}\right]_{z=0} \tag{4.7.20}$$

$$S_h = 1 + \left[\frac{\int_0^{2\pi/a}\frac{\partial S}{\partial z}dx,}{\int_0^{2\pi/a}\frac{\partial S_b}{\partial z}dx,}\right]_{z=0} \tag{4.7.21}$$

Substituting M_2 M_4 in eqs.(4.7.19), the expressions for N_u and S_h are obtained as

$$N_u = (1 - 2\pi M_2) \tag{4.7.22}$$

$$S_h = (1 - 2\pi M_4)$$

4.8 Results and Discussion

The effect of internal heat source on double diffusive convection in a viscoelastic fluid saturated rotating anisotropic porous layer has been studies using linear and nonlinear stability

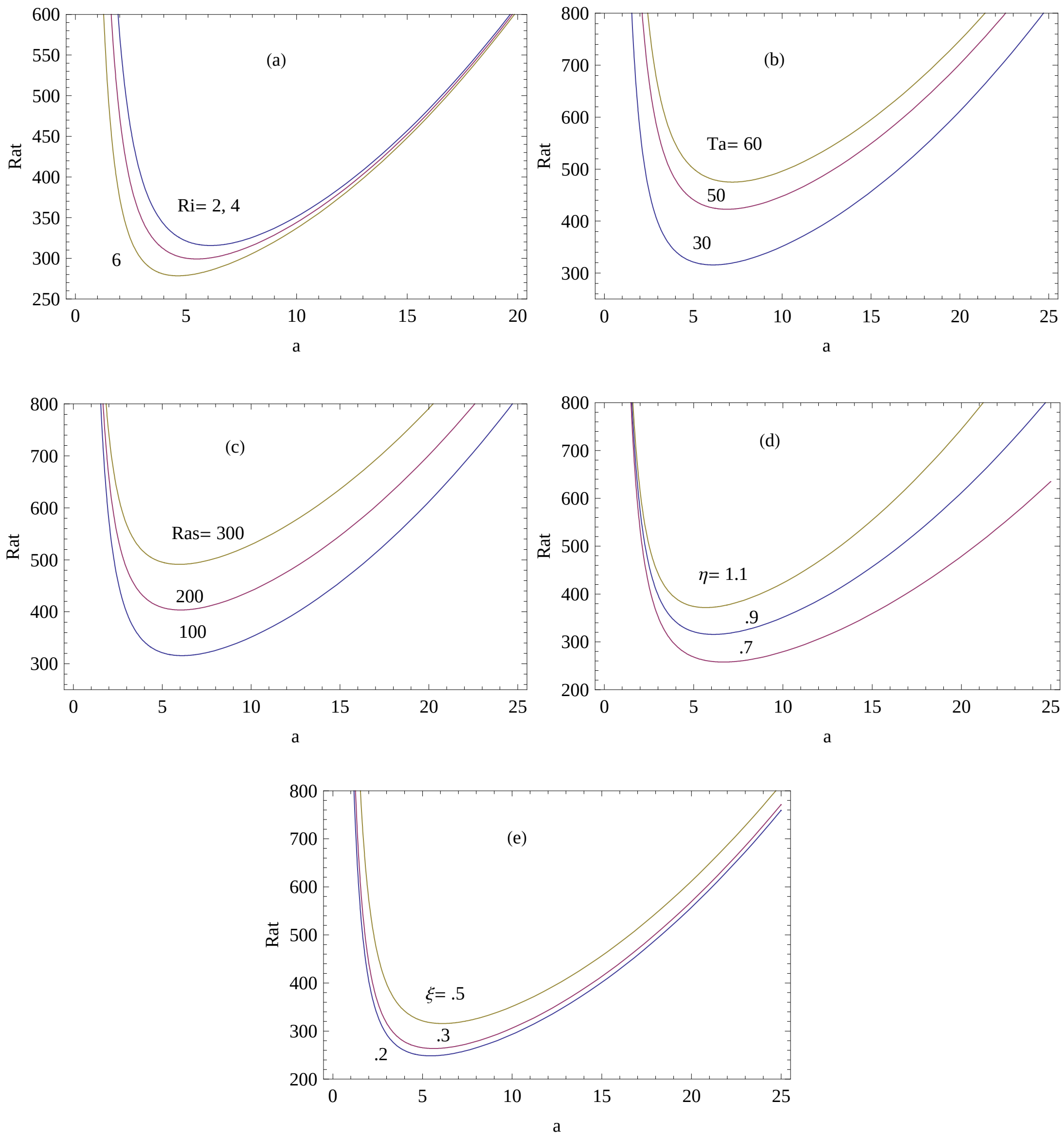


Figure 4.2: Stationary neutral stability curves for (a) R_i,(b) T_a,(c) Ra_S,(d) η, (e) ξ

analyses. In this section, the effects of various parameters on the onset of double diffusive convection has been obtained numerically and expressed them graphically. The numerical values of thermal Rayleigh number for stationary and oscillatory modes of convection for different values of the parameters such as Taylor number, relaxation and retardation parameters, solute Rayleigh number, and parameter are computed, and depicted in figures.

4.8.1 Linear analysis

The marginal stability curves in the $(Ra_T$, a) plane for the stationary and oscillatory modes are presented through graphs for different values of parameters.Figs. 4.2(a-e) are for sta-

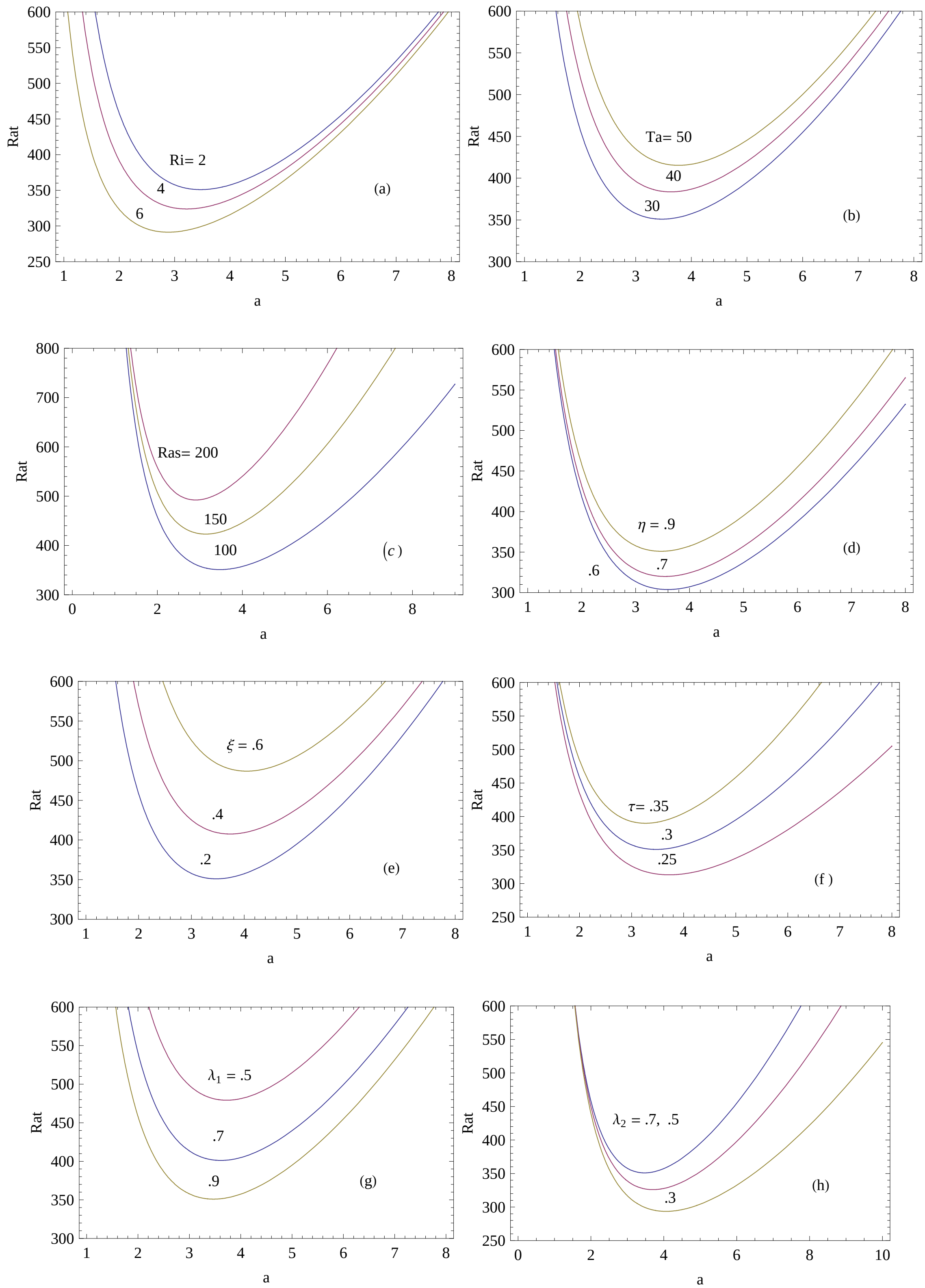
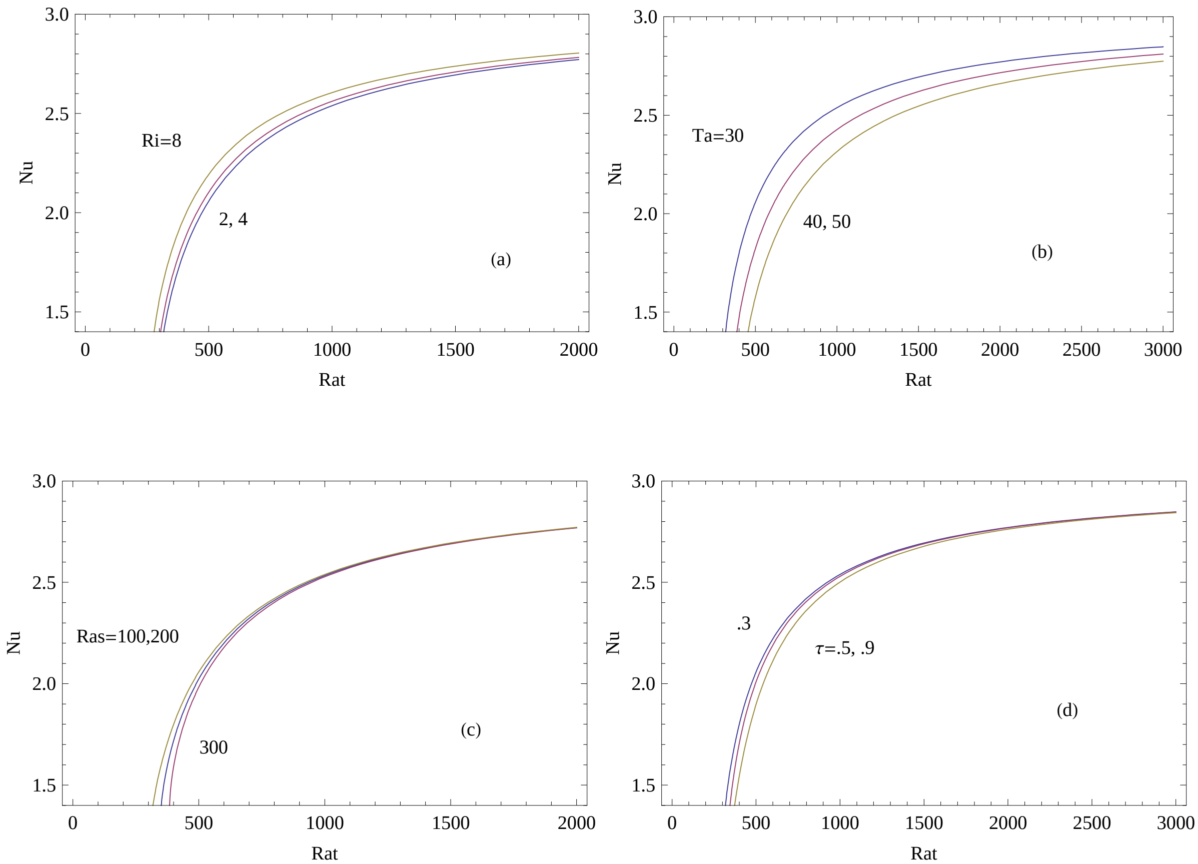


Figure 4.3: Oscillatory neutral stability curves for (a) R_i,(b) T_a,(c) Ra_S,(d) η ,(e)ξ , (f)τ, (g) λ_1, (h) λ_2

tionary mode,while Figs. 4.3(a-h)correspond to oscillatory mode of convection. We fix the values for the parameters as $T_a = 30$, $\xi = .5$,$Ra_S = 100$, τ =.3, λ_1=.5, λ_2=.9,η=.9 and R_i = 2, except the varying parameter.



From Fig. 4.2(a), Fig. 4.3(a), it is observed that on increasing the value of internal Rayleigh number R_i the critical values of stationary and oscillatory Rayleigh number decrease, thus destabilizing the system. This shows that the effect of an increment in the value of R_i, is to advance the onset of both stationary as well as oscillatory modes of convection. However, from Fig. 4.2(b), Fig. 4.3(b) for Taylor number T_a, Figs. 4.2(c), Fig. 4.3(c) for solutal Rayleigh number Ra_S, Figs. 4.2(d), Fig. 4.3(d) for thermal anisotropic parameter η and Fig. 4.2(e), Fig. 4.3(e) for mechanical anisotropic parameter ξ, it is observed respectively that on increasing the values of T_a, Ra_S, η, ξ the critical values of stationary and oscillatory Rayleigh numbers increase, thus stabilizing the system. This shows that the effect of increasing the values of T_a, Ra_S, η, ξ is to delay the onset of stationary and oscillatory convection. Further, it is found from Figs. 4.3(f), 4.3(h) that the effect of increasing the values of diffusivity ratio τ and the parameter λ_2 is to increase the critical value of the oscillatory Rayleigh number, thus delaying the onset of oscillatory convection.

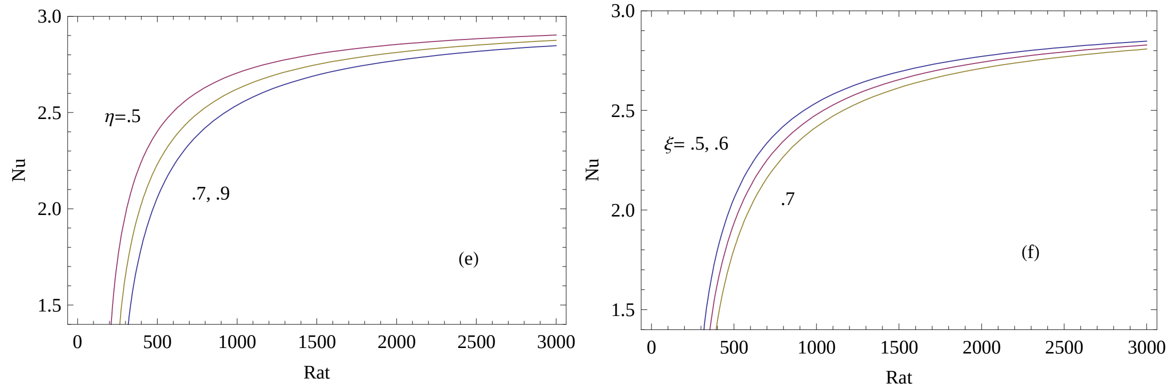
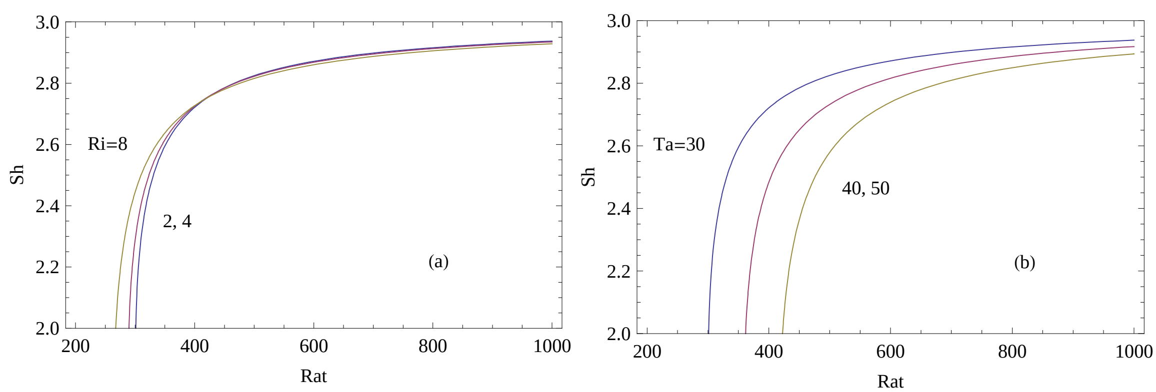


Figure 4.4: Graph between Nusselt number and Rayleigh number for different values of (a) R_i,(b) T_a,(c) Ra_S,(d) τ,(e) η, (f) ξ

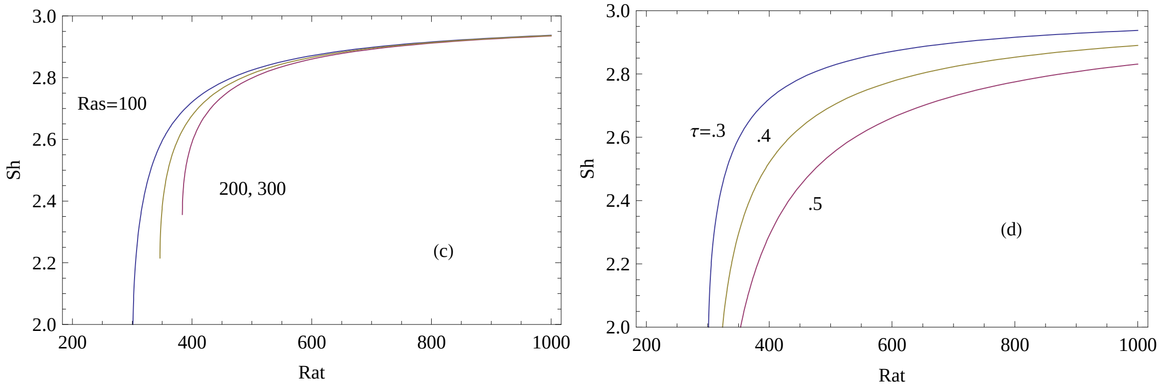


However opposite effect is found in Figs. 4.3(g),where an increment in the value of parameter λ_1 decreases the critical value of the oscillatory Rayleigh number, thus advancing the onset of oscillatory convection.

4.8.2 Nonlinear analysis

The effects of various parameters on the rate of heat and mass transfer are shown in Fig. 4.4 and Figs. 4.5 respectively.

Figs. 4.4(a),4.5(a) show that an increment in the value of the internal Rayleigh number R_i increases the values of both Nusselt number N_u and Sherwood number S_h, which is due to the fact that increasing the value of R_i advances the onset of convection. From Figs. 4.4(b),4.5(b) for Taylor number T_a, Figs. 4.4(c),4.5(c) for solute Rayleigh number



Ra_S, Figs. 4.4(d),4.5(d) for diffusivity ratio τ, Figs. 4.4(e), 4.5(e) for thermal anisotropic parameter η, Figs. 4.4(f),4.5(f) for mechanical anisotropic parameter ξ , it is observed that on increasing the values of T_a, Ra_S, η and ξ the values of both Nusselt number N_u and Sherwood number S_h decrease, thus stabilizing the system.

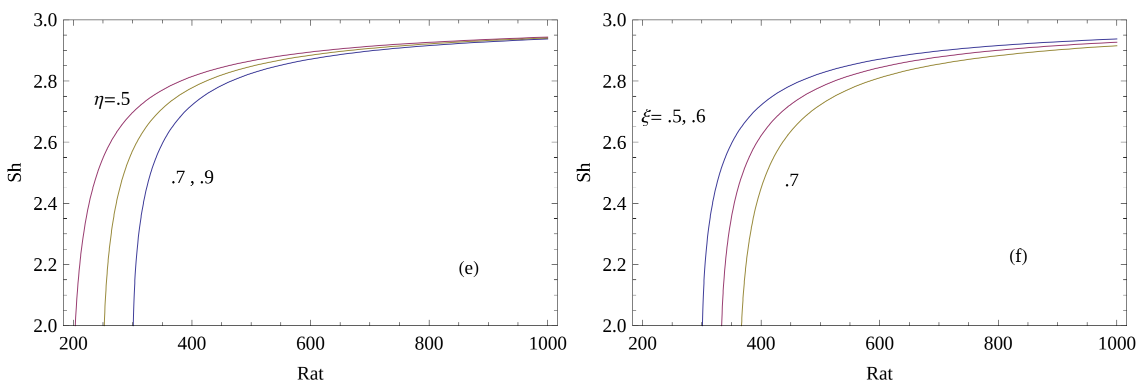


Figure 4.5: Graph between Sherwood and Rayleigh number for different values of (a) R_i,(b) T_a,(c) Ra_S,(d) τ,(e) η, (f) ξ

4.9 Conclusions

In this chapter, internal heating effect on double diffusive convection in a viscoelastic fluid saturated rotating anisotropic porous layer,which is heated and salted from below, is investigated. The problem has been solved analytically,performing linear and nonlinear analyses. Linear analysis is done using normal mode technique. Following conclusions are drawn:

1) The Taylor number T_a, mechanical anisotropic parameter ξ, solute Rayleigh number Ra_S and thermal anisotropic parameter η has a stabilizing effect on the both stationary and oscillatory convection.

2) The internal heat parameter R_i destabilizes the system in the stationary and oscillatory system.

3) The effect of diffusivity ratio τ, retardation parameter λ_2 has a stabilizing effect on the oscillatory convection.

4) The relaxation parameter λ_1 has a destabilizing effect on the oscillatory convection.

5) Increasing the value of internal Rayleigh number R_i, increases the value of Nusselt number N_u i.e. increases heat transfer but increasing the value of mechanical anisotropic parameter ξ, Taylor number T_a, solute Rayleigh number Ra_S, diffusivity ratio τ and thermal anisotropic parameter η decreases the value of Nusselt number N_u.

6) Mass transfer that is the value of Sherwood number increases on increasing the value of internal Rayleigh number R_i while decreases on increasing the values of mechanical anisotropic parameter Taylor number T_a, solute Rayleigh numberRa_S, diffusivity ratio τ and thermal anisotropic parameter η.

Chapter 5

Linear and Nonlinear Double Diffusive Convection in a Couple Stress Fluid Saturated Anisotropic Porous Layer with Soret Effect and Internal Heat Source

5.1 Introduction

The study of double diffusive convection in porous media plays a significant role in many areas, such as petroleum industry, solidification of binary mixtures, and migration of solutes in water-saturated soils. Other example include geophysics systems, crystal growth, electrochemistry, the migration of moisture through air contained in fibrous insulation, the Earth's oceans, and magma chambers. The problem of double diffusive convection in a

This chapter is based on the research article:Linear and Nonlinear Double Diffusive Convection in a Couple Stress Fluid Saturated Anisotropic Porous Layer with Soret Effect and Internal Heat Source, published as a book chapter **advances in mathematical methods and high performance computing**,(Springer),pp.429-448,(2019).

porous media has been presented by Ingham and Pop (2005), Nield and Bejan (2013), Vafai (2000,2005), and Vadasz (2008). The study was continued by Poulikakos (1986), Travison and Bejan (1986), and Momou (2002) among others. The first study of double diffusive convection in porous media was mainly concerned with linear stability analysis and was performed by Nield (1968) .

The growing importance of non-Newtonian fluids with suspended particles in modern technology and industries makes the investigation of such fluids desirable.These fluids are applied in the extrusion of polymer fluids in industry, exotic suspension, fluid film lubrication, solidification of liquid crystals, cooling of metallic plate in baths, and colloidal and suspension solutions. Non-Newtonian stress fluids have specific features, such as polar effect. The theory of polar fluids and related theories are models for fluids whose microstructure is mechanically significant. The theory for couple stress fluid was proposed by Stokes(1966), it is simpler polar fluid theory, that shows all the important features and effects of such fluids that occur inside a deforming continuum. The stabilizing effect of couple stress parameter is reported in the works of Sharma and Thakur (2000), who investigated thermal instability in an electrically conducting couple stress fluid with a magnetic field. Sunil et al.(2004) studied the effect of suspended particles on double diffusive convection in a couple stress fluid-saturated porous medium, Sharma and Sharma (2004) investigated the effect of suspended particles on couple stress fluid, heated from below, in the presence of rotation and a magnetic field. Malashetty et al.(2006) performed an analytical study of linear and nonlinear double diffusive convection with the Soret effect in couple stress liquids. Gaikwad and Kamble (2014) analyzed the effect of soret parameter on the onset of double diffusive convection in a Darcy porous medium saturated with couple stress fluid. Malashetty and Kollur (2011) investigated the onset of double diffusive convection in an anisotropic porous layer saturated with couple stress fluid. Shivkumara et al.(2011) analyzed the linear and non linear stability of double diffusive convection in a couple stress fluid-saturated porous layer. Recently Banyal (2013) has investigated a mathematical theorem on the onset of stationary convection in a couple stress fluid.

In the study of double diffusive convection with a Soret effect, some of the important areas of application and in engineering, in geophysics, oil reservoirs, and groundwater. Researchers have developed a great interest in these type of flows. In the presence of cross diffusion two transport properties are produced: the Soret effect and Dufour effect. The Soret effect describes the tendency of a solute to diffuse under the influence of a temperature gradient. There are only few studies available on double diffusive convection in the presence of Soret effect. The effect was described by Swiss scientist J.Soret, who was the first to study thermodiffusion (1879). Hurle and Jakeman (1971) conducted an experiment and theoretical, studied Soret-driven thermosolutal convection in a binary fluid mixture. Rudraiah and Malashetty (1986) discussed double diffusive convection in a porous medium in the presence of Soret and Dufour effects. Bahlol et al. (2003) studied double diffusive convection and Soret-induced convection in a shallow horizontal porous layer analytically and numerically. Malashetty and Birader (2012) carried out an analytical study of linear and nonlinear double diffusive convection in a fluid-saturated porous layer with Soret and Dufour effect. Bhadauria et al.(2014) performed linear and nonlinear double diffusive convection in a saturated anisotropic porous layer with couple stress fluid. Recently, Malashetty et al.(2016) performed an analytical study of linear and nonlinear double diffusive convection with Soret effect in couple stress liquids. Also in another study by Malashetty et al.(2013) investigated Soret effect on double diffusive convection in a darcy porous medium saturated with a couple stress fluids. A study concerning an internal heat source in porous media was provided by Tveitereid (1977), who performed thermal convection in a horizontal porous layer with internal heat sources. Hill (2005) performed linear and nonlinear analyses on the double-diffusive convection in a porous layer with a concentration based internal heat source. Srivastava et al.(2014) studied on double diffusive convection in a couple stress fluid-saturated anisotropic porous medium with internal heating. Govender (2007) investigated on the stability of centrifugally driven convection in a rotating anisotropic porous layer subject to gravity with Coriolis effect . Kapil (2015) studied at the onset of convection in a dusty couple stress fluid saturated porous medium under magnetics effect, and with variable gravity through.

The aim of this chapter was to study the Soret effect in a couple stress fluid under an internal heat source. Thus stability analysis of the Soret and internal heating effect on double diffusive convection in a anisotropic porous layer saturated with a couple stress fluid has been done.

5.2 Mathematical Formulation

We consider an infinitely extended horizontal porous medium saturated with a couple stress fluid, heated from below and cooled from above and confined between the planes at z=0 and z=d. The Darcy model has been employed in the momentum equation. Further, an internal heat source term has been included in the energy equation. A cartesian frame of reference is chosen in such a way that the origin lies on the lower plane and the z-axis is vertical upward. An adverse temperature gradient is applied across the porous layer. The lower planes is kept at temperature $T_0 + \Delta T$, while upper planes is kept at temperature T_0, with a concentration $S_0 + \Delta S$, and S_0 respectively. The physical configuration of the model is reported in Figure 5.1.

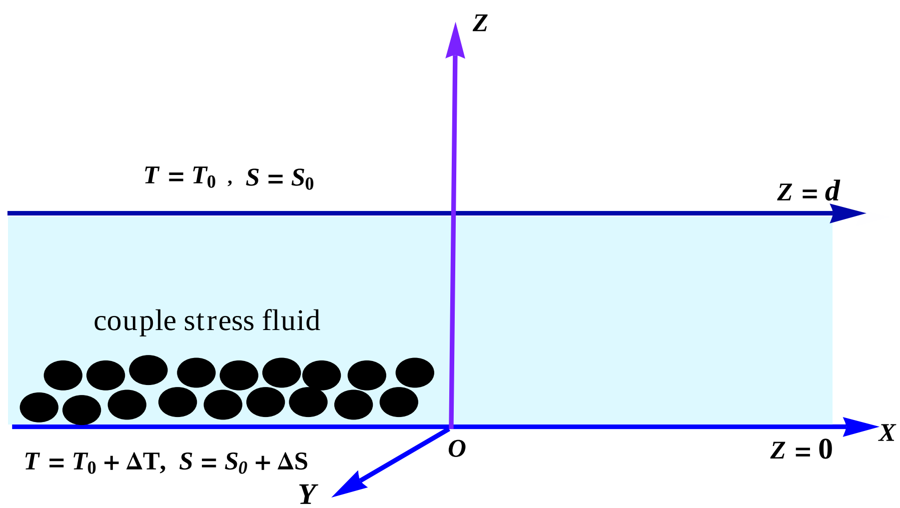


Figure 5.1: Physical configuration of the problem

The governing equations are given below

$$\begin{cases} \nabla.\vec{q} = 0, \\ \frac{\rho_0}{\varepsilon}(\frac{\partial q}{\partial t}) = -\nabla p + \rho g - \frac{1}{K}(\mu - \mu_c\nabla^2)\vec{q}, \\ \gamma\frac{\partial T}{\partial t} + (\vec{q}.\nabla)T = \nabla(\kappa_{Tz}.\nabla T) + Q(T - T_0), \\ \varepsilon\frac{\partial S}{\partial t} + (\vec{q}.\nabla)S = \kappa_s\nabla^2 S + D\nabla^2 T, \\ \rho = \rho_0[1 - \beta_T(T - T_0) + \beta_S(S - S_0)] \end{cases} \tag{5.2.1}$$

where the physical variables have their usual meanings as given in the nomenclature. The externally imposed thermal and solutal boundary conditions are given by

$$\begin{cases} T = T_0 + \Delta T, \quad \text{at} \quad z = 0 \quad \text{and} \quad T = T_0, \quad \text{at} \quad z = d, \\ S = S_0 + \Delta S, \quad \text{at} \quad z = 0 \quad \text{and} \quad S = S_0, \quad \text{at} \quad z = d, \end{cases} \tag{5.2.2}$$

5.2.1 Basic state

In this state, the velocity, pressure, temperature, concentration and density profiles are given by

$$\vec{q_b} = 0, p = p_b(z), T = T_b(z), S = S_b(z), \rho = \rho_b(z). \tag{5.2.3}$$

Putting Equation (5.2.3) in Equation (5.2.1), the following relations are obtained:

$$\frac{dp_b}{dz} = -\rho_b g, \tag{5.2.4}$$

$$\kappa_T\frac{d^2(T_b - T_0)}{dz^2} + Q(T_b - T_0) = 0, \tag{5.2.5}$$

$$\kappa_s\frac{d^2 S_b}{dz^2} + D\frac{d^2 T_b}{dz^2} = 0, \tag{5.2.6}$$

$$\rho_b = \rho_0[1 - \beta_T(T_b - T_0) + \beta_S(S_b - T_0)]. \tag{5.2.7}$$

The solution of Equation (5.2.5), subject to the boundary conditions (5.2.2), is given by

$$T_b = T_0 + \Delta T\frac{\sin\left((\sqrt{\frac{Qd^2}{\kappa_{Tz}}}(1 - \frac{z}{d})\right)}{\sin\left(\sqrt{\frac{Qd^2}{\kappa_{Tz}}}\right)}. \tag{5.2.8}$$

Use of the boundary conditions (5.2.2), in the solution of Equation (5.2.6),

$$S_b = S_0 + \Big(\Delta S + \frac{D\Delta T}{\kappa_s}\Big)(1 - \frac{z}{d}) - \frac{D\Delta T}{\kappa_s}\frac{\sin\Big(\big(\sqrt{\frac{Qd^2}{\kappa_{Tz}}}\big)(1-\frac{z}{d})\Big)}{\sin\Big(\sqrt{\frac{Qd^2}{\kappa_{Tz}}}\Big)} \tag{5.2.9}$$

Now, superimposing infinite amplitude disturbances on the basic state in the form:

$$\vec{q} = q_b + q', T = T_b + T', p = p_b + p', S = S_b + S', \rho = \rho_b + \rho', \tag{5.2.10}$$

The pressure term was eliminated by taking the curl twice of Equation (5.2.1) i.e. momentum equation. The resulting equations were non dimensionalized using the following transformations:

$$(x, y, z) = (x^*, y^*, z^*)d, \quad t = t^*(\frac{\gamma d^2}{\kappa_{Tz}}), \tag{5.2.11}$$

$$(u, v, w) = (u^*, v^*, w^*)(\frac{\kappa_{Tz}}{d}), \quad T = (\Delta T)T^*, S = (\Delta S)S^*$$

On dropping the asterisks for simplicity and using the stream function $u = \frac{\partial\psi}{\partial z}$, $w = -\frac{\partial\psi}{\partial x}$, it is obtained

$$\frac{1}{V_a}\frac{\partial}{\partial t}\nabla_1^2\psi + \Big(\frac{\partial^2}{\partial x^2} + \frac{1}{\xi}\frac{\partial^2}{\partial z^2}\Big)(1 - C\nabla_1^2)\psi = Ra_T\frac{\partial T}{\partial x} - Ra_S\frac{\partial S}{\partial x} = 0 \tag{5.2.12}$$

$$\Big[\frac{\partial}{\partial t} - \frac{\partial^2}{\partial z^2} - \eta\frac{\partial^2}{\partial x^2} - R_i\Big]T - f(z)\frac{\partial\psi}{\partial x} - \frac{\partial(\psi, T)}{\partial(x, z)} = 0 \tag{5.2.13}$$

$$\Big[\frac{\partial}{\partial t} - \frac{1}{L_e}(\frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial x^2})\Big]S - S_r\frac{Ra_T}{Ra_S}\nabla^2 T - b(z)\frac{\partial\psi}{\partial x} - \frac{\partial(\psi, S)}{\partial(x, z)} = 0 \tag{5.2.14}$$

where $V_a = \frac{\varepsilon P_r}{D_a}$ is Vadasz number, $Ra_T = \frac{\beta_T g\Delta T K_z d}{\nu\kappa_{Tz}}$ is the thermal Rayleigh number, $Ra_S = \frac{\beta_S g\Delta S K_z d}{\nu\kappa_{Tz}}$ is the solute Rayleigh number, $R_i = \frac{Qd^2}{\kappa_{Tz}}$ is the internal heat source parameter, $C = \frac{\mu_C}{\mu d^2}$ is the couple stress fluid, $L_e = \frac{\kappa_{Tz}}{\kappa_S}$is Lewis number , $\eta = \frac{\kappa_{Tx}}{\kappa_{Tz}}$ is thermal anisotropy parameter, $\xi = \frac{K_x}{K_z}$ is mechanical anisotropy parameter. consider the stress free and isothermal boundary conditions as given below will be solved the above system:

$$w = \frac{\partial^2 w}{\partial z^2} = T = S = 0 \text{ on } z = 0, z = 1. \tag{5.2.15}$$

T_b , S_b in dimensionless forms are given by

$$T_b = \frac{\sin\sqrt{R_i}(1-z)}{\sin\sqrt{R_i}}, \tag{5.2.16}$$

$$S_b = \frac{S_r L_e Ra_T \sin\left(\sqrt{R_i}(1-z)\right)}{Ra_S \sin\sqrt{R_i}} - \Big(\frac{S_r L_e Ra_T}{Ra_S} + 1\Big)(1-z)$$

5.3 Linear stability Analysis

To do linear stability analysis of the eigenvalue problem defined by Equations (5.2.12)-(5.2.14) subject to the boundary conditions (5.2.15),the time-dependent periodic disturbance in horizontal plane are considered as:

$$(w, T, S) = (W, \Theta, \phi)exp[i(lx + my) + \sigma t] \tag{5.3.1}$$

where l,m are horizontal wave numbers and $\sigma = \sigma_r + i\sigma_j$, is growth rate. Substituting equation (5.3.1) into the linearized equations (5.2.12)-(5.2.14), it is obtained

$$\Big[\frac{\sigma}{V_a}\delta^2 + \delta_1^2(1 - C\delta^2)\Big]W + aRa_T\Theta - aRa_S\phi = 0 \tag{5.3.2}$$

$$[\sigma + \eta_1 - R_i]\Theta - 2aFW = 0 \tag{5.3.3}$$

$$[\sigma + \frac{\delta^2}{L_e}]\phi - 2aBW + S_r\delta^2\frac{Ra_T}{Ra_S}\Theta = 0. \tag{5.3.4}$$

where $D = d/dz$ and $a^2 = l^2 + m^2$. The boundary conditions (5.2.15) are now read as W= $D^2W = \Theta = \phi = 0$ at $z = 0, 1$:

Assume the solutions of Equations (5.2.12)-(5.2.14) satisfying the boundary conditions (5.2.15) as

$(W(z), \Theta(z), \phi(z)) = (W_0, \Theta_0, \phi_0)sinn\pi z \quad (n = 1, 2, 3, ...)$

The thermal Rayleigh number can be obtained as

$$Ra_T = \frac{Ri - (\sigma + \eta_1)}{2a^2F}\Big[\frac{(\delta^2 + L_e\sigma)(\frac{\sigma}{V_a}\delta^2 + \delta_1^2(1 - C\delta^2)) - 2a^2BL_eRa_S}{\sigma + \delta^2 + \delta^2 S_r L_e}\Big] \tag{5.3.5}$$

where $a^2 = l^2 + m^2$, $\delta^2 = \pi^2 + a^2$, $\delta_1^2 = \frac{\pi^2}{\xi} + a^2$, $\eta_1 = \pi^2 + \eta a^2$, F=$\int_0^1 \frac{dT_b}{dz} sin^2(\pi z)dz$, B= $\int_0^1 \frac{dS_b}{dz} sin^2(\pi z)dz$, η is a representative viscosity of fluid. The growth rate σ is in general a complex quantity such that $\sigma = \sigma + i\sigma_i$. The system with σ_r <0 is always stable, while for σ_r >0 it will become unstable. For neutral stability state $\sigma_r = 0$.

5.3.1 Stationary State

The value of the thermal Rayleigh number for stationary mode of convection is given below:

$$Ra_T^{st} = \frac{Ri - \eta_1}{2a^2 F}\Big[\frac{\delta^2\delta_1^2(1 - C\delta^2) - 2a^2 B Ra_S L_e}{\delta^2(1 + L_e S_r)}\Big], \tag{5.3.6}$$

The critical wave number $a = a_c^{St}$, is the same as given by Malashetty et al.(2006). For single component fluid, $Ra_S = 0$ i.e. in the absence of solute, the Rayliegh number, is

$$Ra_T^{st} = \frac{(Ri - \eta_1)\delta_1^2(1 - C\delta^2)}{2a^2 F(1 + L_e S_r)} \tag{5.3.7}$$

For the system without internal heating, i.e., $R_i = 0$, we have F=-1/2 and

$$Ra_T^{st} = \frac{(\eta_1)\delta_1^2(1 - C\delta^2)}{a^2(1 + L_e S_r)} \tag{5.3.8}$$

which is the one obtained by Shivakumara et al. (2011). When C = 0 (i.e. Newtonian fluid case),

$$Ra_T^{st} = \frac{(\pi^2 + \eta^2 a^2)(a^2 + \frac{\pi^2}{\xi})}{a^2(1 + L_e S_r)} \tag{5.3.9}$$

In the case of no Soret effect

$$Ra_T^{st} = \frac{(\pi^2 + \eta^2 a^2)(a^2 + \frac{\pi^2}{\xi})}{a^2} \tag{5.3.10}$$

Lastly, in case of isotropic porous medium, put $\eta = \xi = 1$, then

$$Ra_T^{st} = \Big(\frac{\pi^2 + a^2}{a}\Big)^2 \tag{5.3.11}$$

which has the critical value $Ra_c^{St} = 4\pi^2$ for$a_c^S t = \pi$, the classical results obtained of Horton and Rogers (1945) and Lapwood (1948).

5.3.2 Oscillatory State

For oscillatory mode of convection, set $\sigma = i\sigma_i$ in Equation (5.3.5) and clear the complex quantities from the denominator, to obtain

$$Ra_T^{osc} = \Delta_1 + i\sigma_i\Delta_2.$$

where

$$\Delta_1 = \frac{1}{2a^2F}\frac{A_1B_1 + \sigma^2 A_2B_2}{B_1^2 + \sigma^2 B_2^2} \tag{5.3.12}$$

and

$$\Delta_2 = \frac{1}{2a^2F}\frac{A_2B_1 - A_1B_2}{B_1^2 + \sigma^2 B_2^2}, \tag{5.3.13}$$

$A_1 = (R_i - \eta_1)(\delta^2\delta_1^2(1 - C\delta^2) - \frac{\sigma^2}{V_a}L_e\delta^2) + \sigma^2(L_e\delta_1^2(1 - C\delta^2) + \frac{\delta^4}{V_a}) - (R_i - \eta_1)Ra_S 2a^2 BL_e$

$A_2 = (R_i - \eta_1)(L_e\delta_1^2(1 - C\delta^2) + \frac{\delta^4}{V_a}) - \delta^2\delta_1^2(1 - C\delta^2) + \frac{\sigma^2}{V_a}L_e\delta^2 + Ra_S 2a^2 BL_e$

$B_1 = \delta^2(1 + S_r L_e)$

$B_2 = 1$

For oscillatory onset $\Delta_2 = 0$ and $(\sigma_i \neq 0)$, where σ_i is the oscillatory frequency which is not given for brevity.

The necessary expression for the oscillatory Rayleigh number is:

$$Ra_T^{osc} = \Delta_1. \tag{5.3.14}$$

5.4 Nonlinear stability Analysis

In this section, nonlinear stability analysis using a minimal truncated Fourier series has been done. For simplicity only two dimensional rolls are considered so that all the physical quantities do not dependent of y. Consider the stream function ψ such that $u = \frac{\partial\psi}{\partial z}$, $w = -\frac{\partial\psi}{\partial x}$, then taking curl to eliminate pressure term from Equation (5.2.1), the resulting non-dimensional equations by using transformation given by Equation (5.2.11) and the following equation

$$\Big(\frac{1}{V_a}\frac{\partial}{\partial t}\nabla_1^2\psi + (\frac{\partial^2}{\partial x^2} + \frac{1}{\xi}\frac{\partial^2}{\partial z^2})(1 - C\nabla_1^2)\psi\Big) + Ra_T\frac{\partial T}{\partial x} - Ra_S\frac{\partial S}{\partial x} = 0, \tag{5.4.1}$$

$$\Big(\frac{\partial}{\partial t} - \frac{\partial^2}{\partial z^2} - \eta\frac{\partial^2}{\partial x^2} - R_i\Big)T - f(z)\frac{\partial \psi}{\partial x} - \frac{\partial(\psi, T)}{\partial(x, z)} = 0, \tag{5.4.2}$$

$$\Big[\frac{\partial}{\partial t} - \frac{1}{L_e}(\frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial x^2})\Big]S - \frac{\partial \psi}{\partial x}b(z) - \frac{\partial(\psi, S)}{\partial(x, z)} - S_r\frac{Ra_T}{Ra_S}\nabla^2 T = 0 \tag{5.4.3}$$

It should be noted that the effect of nonlinearity is to distort the temperature and concentration fields through the interaction of ψ and T, ψ and S. As a result, a component of the form $sin(2\pi z)$ will be generated, where V is zonal velocity induced by rotation. A minimal Fourier series that describes the finite amplitude convection is given by

$$\psi = A_1(t)sin(ax)sin(\pi z), \tag{5.4.4}$$

$$T = B_1(t)cos(ax)sin(\pi z) + B_2(t)sin(2\pi z), \tag{5.4.5}$$

$$S = C_1(t)cos(ax)sin(\pi z) + C_2(t)sin(2\pi z), \tag{5.4.6}$$

where the amplitudes $A_1(t)$, $B_1(t)$, $B_2(t)$,$C_1(t)$, $C_2(t)$ are functions of time and are to be determined. Substituting the above expressions in Equations. (5.4.1)-(5.4.3) and equating the coefficients like terms of the resulting equation, the following set of nonlinear autonomous differential equations were obtained

$$\frac{dA_1(t)}{dt} = \frac{-V_a}{\delta^2}(\delta 1^2(1 + C\delta^2)A_1 + aRa_T B_1 - aRa_S C_1) \tag{5.4.7}$$

$$\frac{dB_1(t)}{dt} = 2aFA_1 - \pi a A_1 B_2 + (R_i - \eta_1)B_1 \tag{5.4.8}$$

$$\frac{dB_2(t)}{dt} = \frac{\pi a}{2}A_1 B_1 + (R_i - 4\pi^2)B_2 \tag{5.4.9}$$

$$\frac{dC_1(t)}{dt} = 2aBA_1 - \delta^2 S_r\frac{Ra_T}{Ra_S}B_1 - \delta^2\frac{1}{L_e}C_1 - \pi a A_1 C_2 \tag{5.4.10}$$

$$\frac{dC_2(t)}{dt} = \pi\frac{a}{2}A_1C_1 - 4\pi^2 S_r\frac{Ra_T}{Ra_S}B_2 - \frac{4\pi^2}{L_e}C_2 \quad (5.4.11)$$

The numerical method was used to solve the above nonlinear differential equation to find the amplitudes.

5.4.1 Steady finite amplitude convection

For steady-state finite amplitude convection, the left hand side of the Equations (5.4.7)-(5.4.11) is set to be zero.

$$\delta1^2(1 + C\delta^2)A_1 + aRa_TB_1 - aRa_SC_1 = 0 \quad (5.4.12)$$

$$2aFA_1 - \pi aA_1B_2 + (R_i - \eta_1)B_1 = 0 \quad (5.4.13)$$

$$\frac{\pi a}{2}A_1B_1 + (R_i - 4\pi^2)B_2 = 0 \quad (5.4.14)$$

$$2aBA_1 - \delta^2 S_r\frac{Ra_T}{Ra_S}B_1 - \delta^2\frac{1}{L_e}C_1 - \pi aA_1C_2 = 0 \quad (5.4.15)$$

$$\pi\frac{a}{2}A_1C_1 - 4\pi^2 S_r\frac{Ra_T}{Ra_S}B_2 - \frac{4\pi^2}{L_e}C_2 = 0 \quad (5.4.16)$$

On solving for the amplitudes in terms of A_1, it is obtained

$$B_1 = \frac{4aF(z)(4\pi^2 - R_i)A_1}{a^2A_1^2\pi^2 - 8\pi^2R_i + 2R_i^2 + 8\pi^2\eta_1 - 2R_i\eta}, \; B_2 = \frac{2a^2F(z)\pi A_1^2}{a^2A_1^2\pi^2 - 8\pi^2R_i + 2R_i^2 + 8\pi^2\eta_1 - 2R_i\eta},$$

$$C_1 = \frac{\begin{array}{c}16\,(8A_1BL_e\pi^2Ra_SR_ia + 2A_1BL_eRa_SRi^2a + A_1^3BL_e\pi^2Ra_Sa^3 + A_1^3FL_e^2\pi^2Ra_Tsa^3 \\ -8A_1FL_e\pi^2Ra_TS_ra\delta^2 + 2A_1FL_eRa_TR_iS_ra\delta^2 + 8A_1BL_e\pi^2Ra_Sa\eta - 2A_1BL_eRa_SR_ia\eta_1)\end{array}}{Ra_S(A_1^2L_e^2a^2 + 8\delta^2)(-8\pi^2R_i + 2R_i^2 + A_1^2\pi^2a^2 + 8\pi^2\eta_1 - 2R_i\eta_1)},$$

$$C_2 = \frac{\begin{array}{c}2\,(8A_1^2BL_e^2\pi^2Ra_SR_ia^2 + 2A_1^2BL_e^2Ra_SR_i^2a^2 + A_1^4BL_e^2\pi^2Ra_Sa^4 - 8A_1^2FL_e\pi^2Ra_TSa^2\delta^2 \\ -8A_1^2FL_e^2\pi^2Ra_TSa^2\delta^2 + 2A_1^2FL_e^2Ra_TR_iSa^2\delta^2 + 8A_1^2BL_e^2\pi^2Ra_Sa^2\eta_1 - 2A_1^2BL_e^2Ra_SR_ia^2\eta_1)\end{array}}{Ra_S\pi(A_1^2L_e^2a^2 + 8\delta^2)(-8\pi^2R_i + 2R_i^2 + A_1^2\pi^2a^2 + 8\pi^2\eta_1 - 2R_i\eta_1)}.$$

To solve the above equation, a quadratic equation in $\frac{A_1^2}{8}$ is given by

$a_0x^2 + a_1x + a_2 = 0$

where $x = \frac{A_1^2}{8}$,

$a_0 = L_e^2 a^4 \pi^2 \delta_1^2 Ra_S(1 + C\delta^2)$

$a_1 = \frac{1}{4}\delta_1^2 Ra_S(1+C\delta^2)(R_i-\eta_1)L_e^2 a^2(R_i-4\pi^2)-\frac{1}{2}(R_i-4\pi^2)FRa_T Ra_S L_e^2 a^4-2L_e a^4\pi^2 Ra_S(B+L_eFS_r) + a^2\pi^2\delta^2\delta_1^2 Ra_S(1 + C\delta^2)$

$a_2 = \frac{(R_i-4\pi^2)}{4}(\delta^2\delta_1^2 Ra_S(1+C\delta^2)(R_i-\eta_1)-2L_e a^2 BRa_S(R_i-\eta_1)-2a^2\delta^2 FRa_S(L_eS_r+Ra_T))$

The required root of the above equation is

$x = \frac{-a_1+\sqrt{a_1^2-4a_oa_2}}{2a_0}$

5.4.2 Steady Heat and Mass Transports

In the study of this type of problem, quantification of heat and mass transport is very important in porous media. Let Nu and Sh denote the rate of heat and mass transport for the fluid phase, known as Nusselt number and Sherwood number respectively, and are defined by

$$Nu = 1 + \left[\frac{\int_0^{2\pi/a}\frac{\partial T}{\partial z}dx,}{\int_0^{2\pi/a}\frac{dT_b}{dz}dx,}\right]_{z=0} \tag{5.4.17}$$

$$Sh = 1 + \left[\frac{\int_0^{2\pi/a}\frac{\partial S}{\partial z}dx,}{\int_0^{2\pi/a}\frac{dS_b}{dz}dx,}\right]_{z=0} \tag{5.4.18}$$

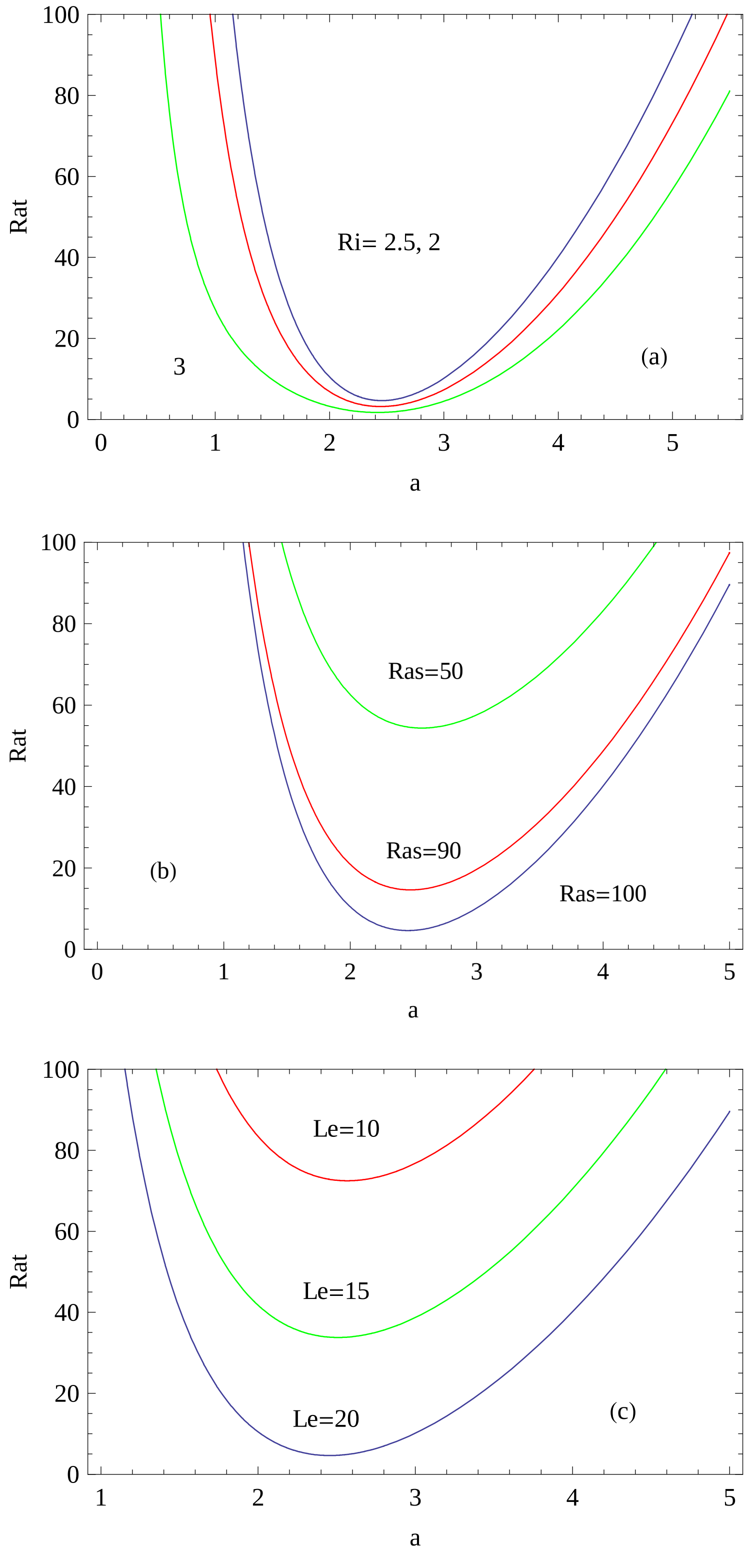
Substituting the values of T, T_b, S, S_b in eqs.(5.4.17)-(5.4.18), and simplifying

$$Nu = 1 - \frac{2\pi B_2}{\sqrt{R_i}\cot\sqrt{R_i}}, \tag{5.4.19}$$

$$Sh = 1 - \frac{2\pi C_2 Ra_S \sin\sqrt{R_i}}{-S_r Ra_T\cos\sqrt{R_i}\sqrt{R_i} + \sin\sqrt{R_i}Ra_s + \sin\sqrt{R_i}S_r Ra_T}$$

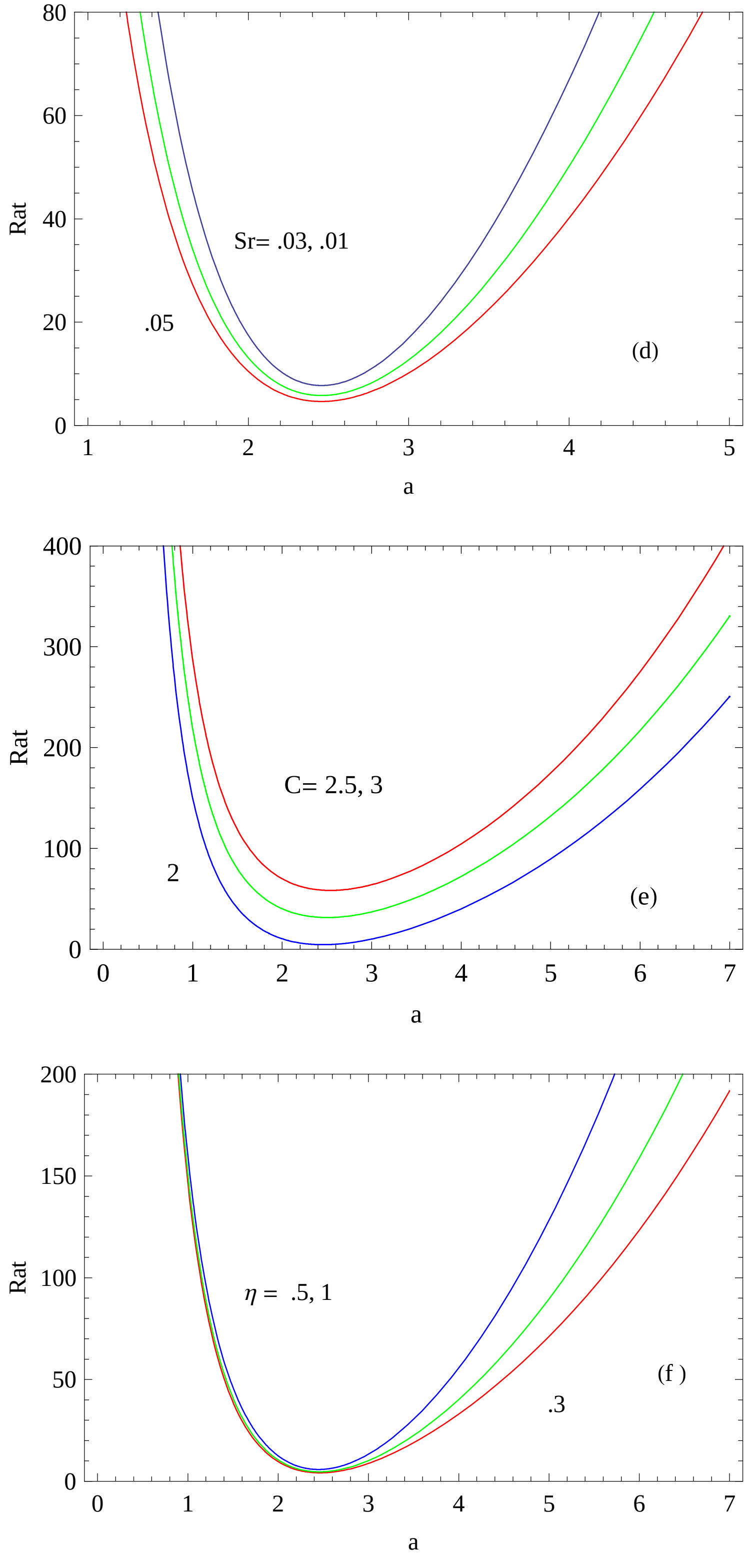
Substituting B_2, C_2 of Equations (5.4.1) into (5.4.19) gives

$$Nu, Sh \tag{5.4.20}$$



5.5 Results and Discussion

This chapter investigates the combined effect of internal heating and Soret effect on linear and nonlinear convection in a anisotropic porous medium saturated by a couple stress fluid.



In this section, effect of various parameters appearing in the governing equations is discussed on double diffusive convection. The stationary and oscillatory Rayleigh for different

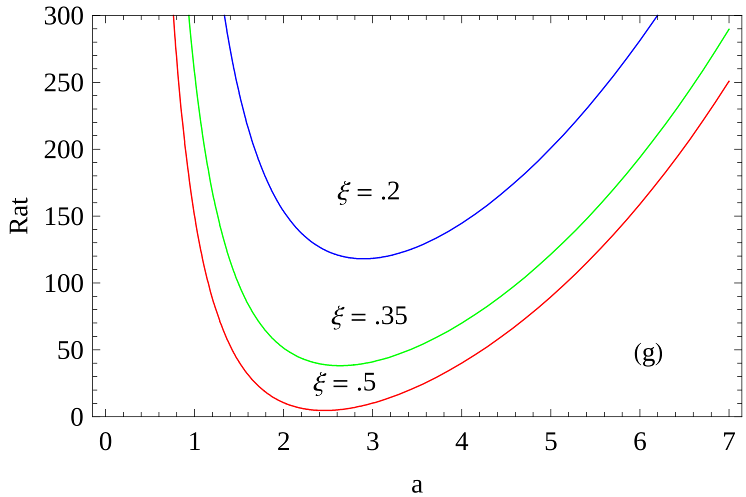
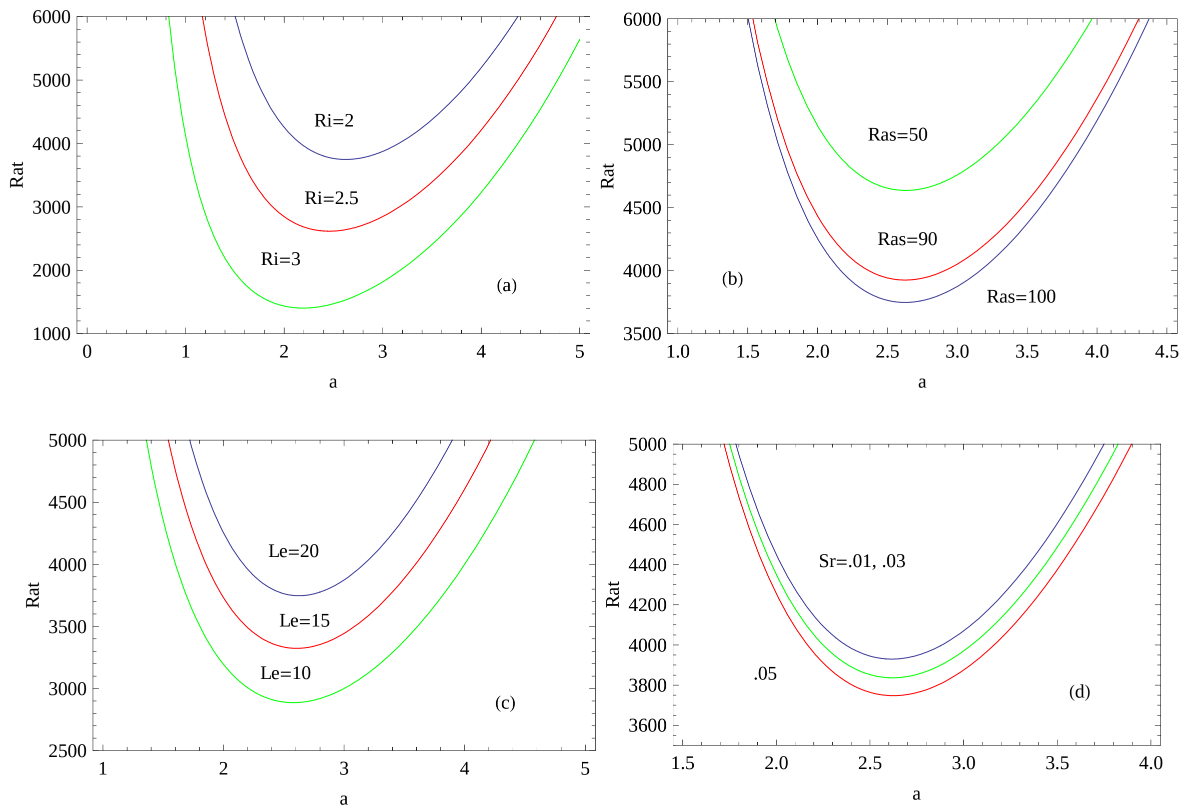


Figure 5.2: Stationary neutral stability curves for different values of (a) R_i, (b) Ra_s, (c) L_e, (d) S_r, (e) C, (f) η, (g) ξ



values of the parameters such as Vadasz number, the couple stress parameter, the solute Rayleigh number, the mechanical anisotropic parameter, and the thermal anisotropic parameter are computed, and depicted in figures. The neutral stability curves in plane (Ra_T,a) for various parameter values are presented in Figs.5.2(a-g) and Figs.5.3(a-h).

The values of the parameters as V_a = 5, C = 2, Ra_S = 100, L_e = 20, ξ=.5, η = .5,

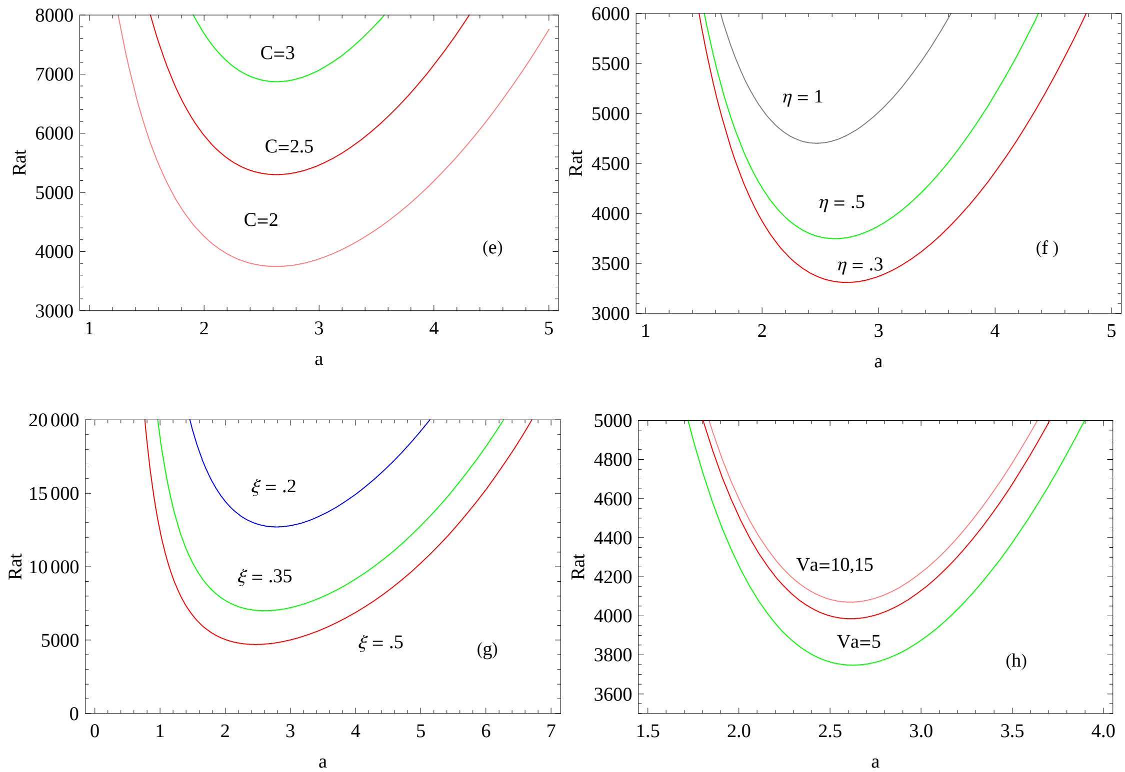
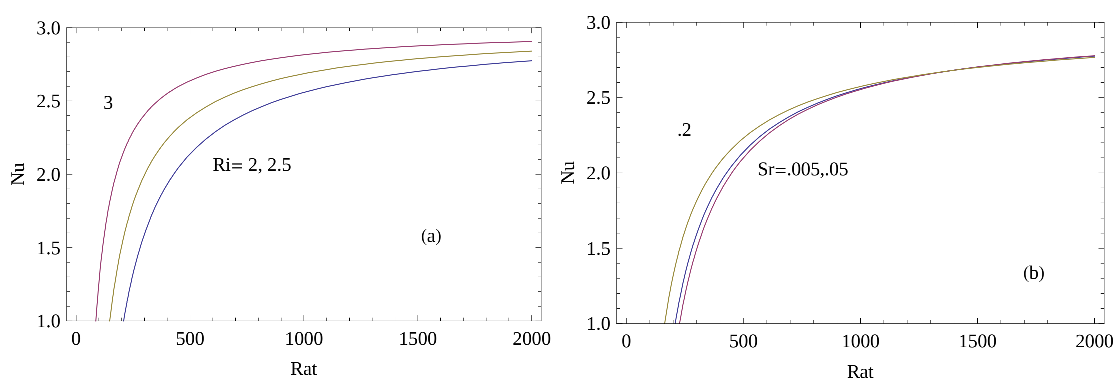
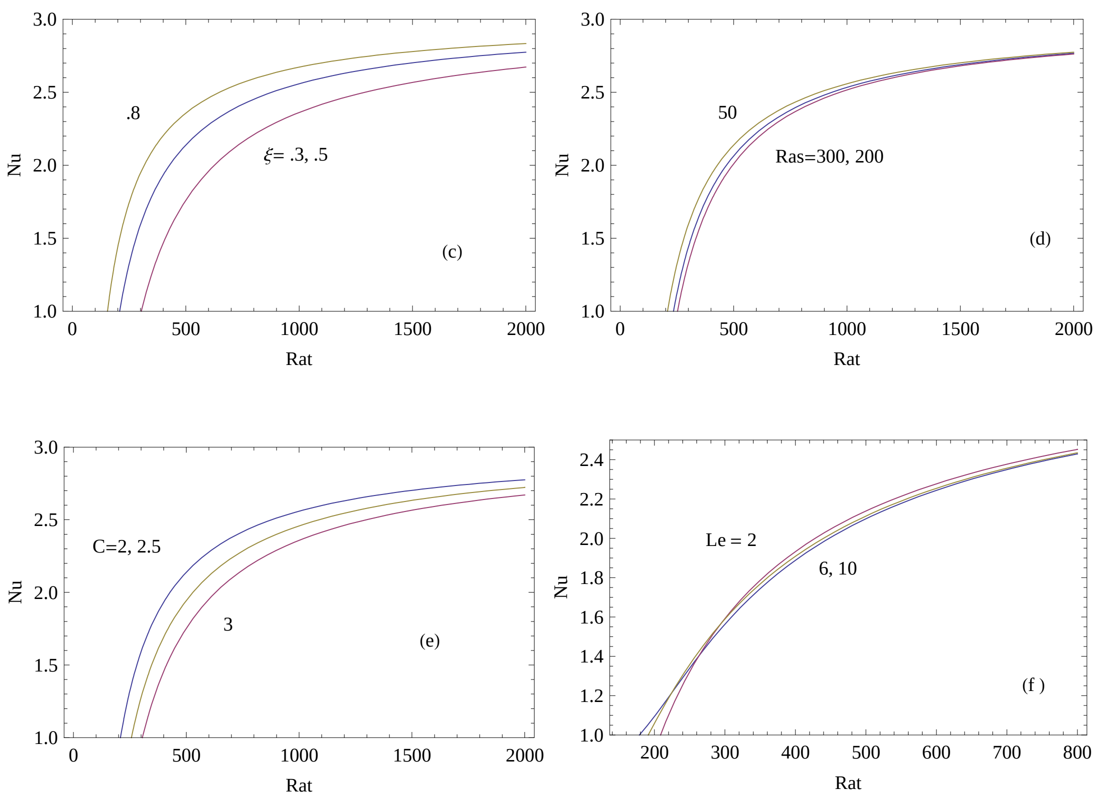


Figure 5.3: Oscillatory neutral stability curves for different values of (a) R_i, (b) Ra_s, (c) L_e, (d) S_r, (e) C, (f) η, (g) ξ, (h) V_a



$S_r = .05$ and $R_i = 2$, are fixed except for the varying parameter. From the Figs. 5.2,5.3 (a, b, d, g) it is found that on increasing the values of R_i, Ra_S, S_r and ξ, then values of stationary as well as of oscillatory Rayleigh numbers decrease, which shows that the effect of these parameters is to destabilize both stationary and oscillatory modes of convection. From Figs. 5.2,5.3(e) the effect of increasing the couple stress parameter C is to increase the value of both stationary and oscillatory Rayleigh numbers thus delaying the onset of convection for both stationary as well as of oscillatory modes of convection. In Fig. 5.2(c), it is found that the effect of increasing Lewis number L_e is to decrease the value of stationary Rayleigh number and from Fig. 5.3(c), to increase the value for oscillatory mode, thus to destabilize the stationary but stabilize the oscillatory mode of convection. Further, From Figs. 5.2,5.3(f), it is observed that an increment in the value of thermal anisotropic parameter η, increases stationary Rayleigh number but decreases oscillatory Rayleigh number. Thus stabilizes stationary mode but destabilizes oscillatory mode of convection. From Fig. 5.3(h) it is found that the minimum value of Rayleigh number for the oscillatory mode increases on increasing Vadasz number Va, indicating that the effect of the Vadasz number is to stabilize the system.



Considering the values of the parameters as C $= 2$, $Ra_S = 50$, $L_e = 2$, ξ=.5, $\eta = .5$, $S_r =$ .05 and $R_i = 2$, the graph for Nusselt and Sherwood are plotted and depicted respectively. In the Figs.5.4(a, g) and Figs.5.5(a, g) number. Figs.5.4(a, c) and Figs.5.5(a, c) show that an increment in the values of internal Rayleigh number R_i, mechanical anisotropic parameter ξ increases the rate of heat transfer but mass transfer first increases and then decreases as Ra_T increases. In Figs.5.4,5.5(b) it is noted that the effect of increasing the Soret parameter S_r is to increase the value of Nusselt number N_u and Sherwood number S_h . From Fig.5.4,5.5(d,g) it can be seen that increasing the value of solute Rayleigh number

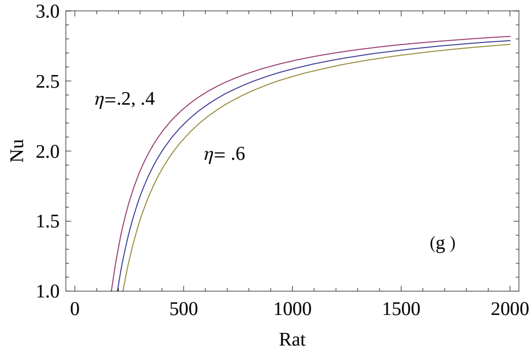
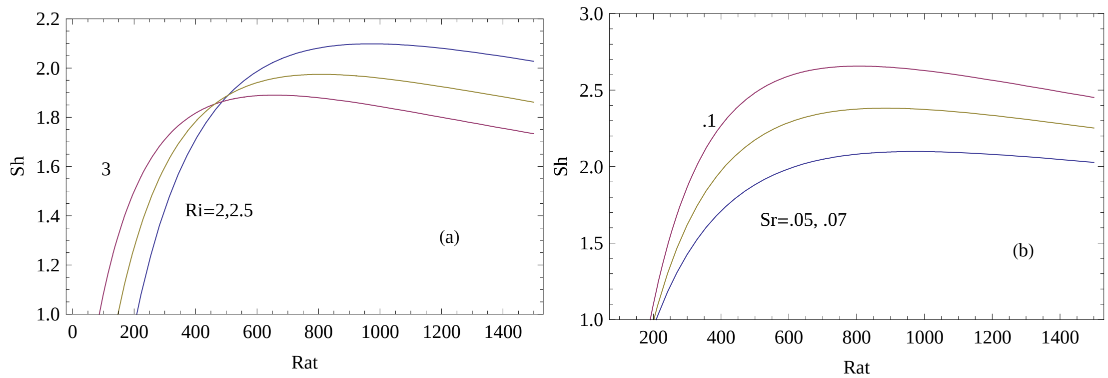
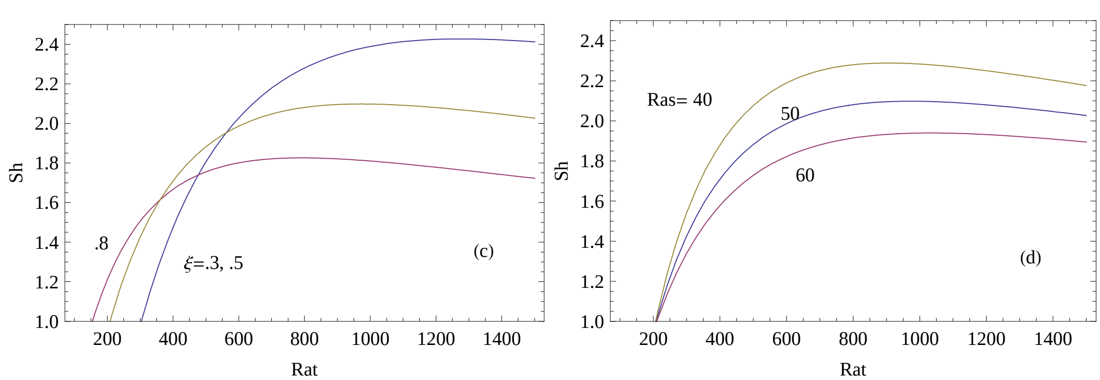


Figure 5.4: Nusselt number for different values of (a) R_i, (b) S_r, (c) ξ, (d) Ra_s, (e) C, (f) L_e, (g)η



Ra_S, thermal anisotropic parameter η is to decrease the value of Nusselt number N_u and Sherwood number S_h, thus reducing the heat and mass transfer. From Figs.5.4(e, f), it is clear that the Nusselt number decreases on increasing the value of couple stress parameter C and the Lewis number L_e. However Sherwood number first decreases and then increases for C but increases and then decreases first Lewis number as Ra_T increases.



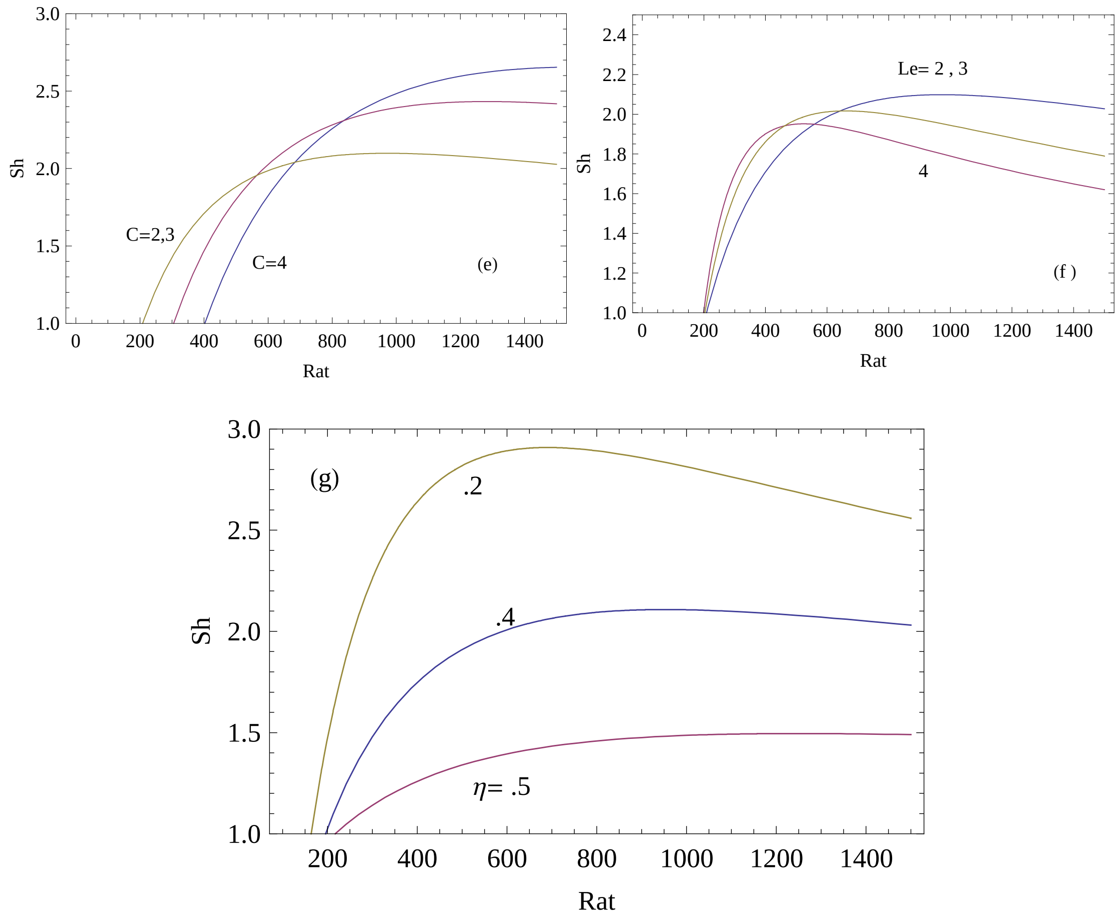


Figure 5.5: Graph between Sherwood number and Rayleigh number for different values of (a) R_i, (b) S_r, (c) ξ, (d) Ra_s, (e) C, (f) L_e, (g)η

5.6 Conclusions

The Soret effect and the internal heating effect on double diffusive convection in a anisotropic porous medium saturated with a couple stress fluid which is heated and salted from below is investigated using linear and nonlinear stability analyses. The linear analysis is carried out using the normal mode technique. The following conclusions were drawn:

1) The Vadasz number Va has a stabilizing effect on the oscillatory convection.

2) The Internal heat parameter R_i, solute Rayleigh number Ra_S, the Soret parameter S_r, and the mechanical anisotropic parameter ξ destabilize the system in the stationary and oscillatory modes.

3) The Couple stress fluid C, the normalized porosity parameter η have a stabilizing effect on the both stationary and oscillatory convection.

4) The Lewis number L_e has a destabilizing effect in case of stationary and opposite oscillatory convection.

5) On increasing the value of internal Rayleigh number R_i, the mechanical anisotropic parameter ξ, the Soret parameter S_r, the value of Nusselt number N_u increases, but on increasing the value of the couple stress C, the normalized porosity parameter η, the solutal Rayleigh number Ra_S and Lewis number L_e the value of Nusselt number N_u decreases.

6) Mass transfer S_h increases with the increasing value of the internal Rayleigh number R_i , the mechanical anisotropic parameter ξ, the Soret parameter S_r and decreases with normalized porosity parameter η, the Lewis number L_e, couple stress C and the solutal Rayleigh number Ra_S.

Chapter 6

Cross diffusion effects on thermal instability in a Maxwell fluid saturated rotating anisotropic porous medium

6.1 Introduction

Anisotropy is generally a consequence of preferential orientation of asymmetric geometry of porous matrix or fibers and is in fact encountered in numerous systems in industry and nature. Anisotropy is particularly important in a geological context, since sedimentary rocks generally have a layered structure; the permeability in the vertical direction is often much less than in the horizontal direction. Anisotropy can also be a characteristic of artificial porous materials like pelleting used in chemical engineering process and fiber material used in insulating purpose. The review of research on convective flow through anisotropic porous medium has been well documented by McKibbin (1992) and Storesletten (1998,

This chapter is based on the research article:Cross diffusion effects on thermal instability in a Maxwell fluid saturated rotating anisotropic porous medium, communicated.

2004). Bhadauria et al.(2011) have studied natural convection in a rotating anisotropic porous layer with internal heat source. Gaikwad et al.(2014) performed linear stability analysis of double diffusive convection in a horizontal sparsely packed rotating anisotropic porous layer in the presence of Soret effect. Malashetty and Kollur (2011) investigated the onset of double diffusive convection in a couple stress fluids saturated anisotropic porous layer. Gaikwad et al. (2016) have carried out cross-diffusion effects on the onset of double diffusive convection in a couple stress fluid saturated rotating anisotropic porous layer.

The studies of double diffusive convection in porous media plays very significant roles in many areas such as in petroleum industry, solidification of binary mixture, migration of solutes in water saturated soils. Other examples include geophysics system, crystal growth, electrochemistry, the migration of moisture through air contained in fibrous insulation, Earth's oceans, magma chambers etc. The problem of double diffusive convection in a porous media has been presented by Ingham and Pop (2005), Nield and Bejan (2013), Vadasz(2008) and Vafai (2000,2005). Very first study on double diffusive convection in porous media mainly concerns with linear stability analysis, was performed by Nield (1968). Thermal convection in a binary fluid driven by the Soret and Dufour effects has been investigated by Knobloch(1980). He has shown that equations are identical to the thermosolutal problem except regarding a relation between the thermal and solute Rayleigh numbers. The double-diffusive convection in a porous medium in the presence of Soret and Dufour coefficients has been studied by Rudraiah and Malashetty (1986) for a Darcy porous medium using linear analysis, which was extended to include weak nonlinear analysis by Rudraiah and Siddheshwar (1998). Linear and nonlinear study of double diffusive convection in a fluid saturated porous layer with cross-diffusion effects has been carried out by Malashetty and Biradar (2011). The onset of double diffusive convection in a binary Maxwell fluid saturated porous layer with cross-diffusion effects has been carried out by Malashetty and Biradar (2012).Gaikwad and Dhanraj (2012) analyses the onset of double diffusive convection in a couple stress fluid saturated anisotropic porous layer with cross-diffusion effects. Recently, as some new technologically significant materials are discovered acting like non-Newtonian fluids, therefore, mathematicians, physicists and engineers are actively conducting research in rheology. Maxwell fluids can be considered as a special case of a Jeffreys-Oldroyd B fluid, which contain relaxation and retardation time coefficients. Maxwells constitutive relation can be recovered from that corresponding to Jeffreys-Oldroyd B fluids by setting the retardation time to be zero. Several fluids such as glycerin, crude oils or some polymeric solutions, behave as Maxwell fluids. Viscoelastic fluid flow in porous media has attracted considerable attention due to the large demands of such fluids in diverse fields such as biorheology, geophysics, chemical industries, and petroleum industries.Wang et al.(2008) have made the stability analysis of double diffusive convection of Maxwell fluid in a porous medium. The onset of double diffusive convection in a viscoelastic saturated porous layer has been considered by many researchers as Gaikwad et al. (2009), Kim et al.(2003), Malashetty et al.(2011), Srivastava and Singh(2018). Bhadauria et al.(2011) have studied nonlinear two dimensional double diffusive convection through viscoelastic fluid in a rotating porous layer. Narayana et al.(2012) performed linear and nonlinear stability analysis of binary Maxwell fluid convection in a porous medium.

Recently, Zhao at al.(2014) have done linear and nonlinear stability analysis of double diffusive convection in a Maxwell fluid saturated porous layer with internal heat source.Gaikwad et al.(2013) used the onset of double diffusive convection in a Maxwell fluid saturated porous layer and also Gaikwad et al.(2016) have done onset of convection in a Maxwell fluid saturated anisotropic porous layer use of Darcy-Brinkman model. Ram et al.(2015) have studied onset instability of thermosolutal convective flow of viscoelastic Maxwell fluid thorough porous medium with linear heat source effect. More recently, Awad et al.(2010) have studied linear stability of a Maxwell fluid with cross-diffusion and double-diffusive convection used the Darcy-Brinkman-Maxwell model. They found that the effect of relaxation time is to decrease the critical Darcy-Rayleigh number. Altawallbeh et al. (2017) have done linear stability analysis of double diffusive convection in a viscoelastic fluid saturated porous layer with cross diffusion effects and internal heat source.

However, to the best of my knowledge no literature is available in which linear and nonlinear stability analysis for cross diffusive convection has been done in an anisotropic porous media saturated with viscoelastic fluid of type Oldroyd-B. Therefore, in the present chapter stability analysis of Maxwell fluid saturated rotating anisotropic porous layer has been done on double diffusive convection in the presence of cross diffusion. Our objective is to study how the onset criterion for oscillatory convection is affected by Maxwell fluid and the other parameters, and also to find heat and mass transports. We can obtain, some previously published results in the limiting cases of the present results.

6.2 Mathematical Formulation

Consider an infinitely extended horizontal anisotropic porous layer saturated with Maxwell fluid mixture, confined between the planes at z=0 and z=d, which is heated from below and cooled from above. Darcy model that includes the Coriolis term is used for the momentum equation. A Cartesian frame of reference is chosen with x -and y -axes at the lower boundary plane and z -axis directed vertically upwards in the gravity field. The axis of rotation is assumed to coincide with the z -axis, rotating with a constant angular velocity Ω. An adverse temperature gradient is applied across the porous layer. The lower planes is kept at temperature $T_0 + \Delta T$, while upper planes is kept at temperature T_0, with concentration $S_0 + \Delta S$, and S_0 respectively. The governing equations are as given below

$$\nabla.\vec{q} = 0, \tag{6.2.1}$$

$$(1+\overline{\lambda_1}\frac{\partial}{\partial t})\Big[\frac{\rho_0}{\varepsilon}\frac{\partial q}{\partial t} + 2\frac{\rho_0}{\varepsilon}\Omega \times \vec{q}\Big] = (1+\overline{\lambda_1}\frac{\partial}{\partial t})(-\nabla p + \rho g) + \mu(K.\vec{q}) \tag{6.2.2}$$

$$\gamma\frac{\partial T}{\partial t} + (\vec{q}.\nabla)T = \kappa_T\nabla^2 T + K_{TS}\nabla^2 S, \tag{6.2.3}$$

$$\varepsilon\frac{\partial S}{\partial t} + (\vec{q}.\nabla)S = \kappa_S\nabla^2 S + K_{ST}\nabla^2 T, \tag{6.2.4}$$

$$\rho = \rho_0[1 - \beta_T(T - T_0) + \beta_S(S - S_0)] \tag{6.2.5}$$

where the physical variables have their usual meanings as given in the nomenclature. The externally imposed thermal and solutal boundary conditions are as given by

$$\begin{cases} T = T_0 + \Delta T, & \text{at} \quad z = 0 \quad \text{and} \quad T = T_0, \quad \text{at} \quad z = d, \\ S = S_0 + \Delta S, & \text{at} \quad z = 0 \quad \text{and} \quad S = S_0, \quad \text{at} \quad z = d, \end{cases} \tag{6.2.6}$$

6.3 Basic state

At this state the velocity, pressure, temperature, concentration and density profiles are given by

$$\vec{q_b} = 0, p = p_b(z), T = T_b(z), S = S_b(z), \rho = \rho_b(z). \tag{6.3.1}$$

Putting the Eq. (6.3.1) in Eq. (6.2.1)-(6.2.5), the following equations are obtained

$$\frac{dp_b}{dz} = -\rho_b g, \tag{6.3.2}$$

$$\frac{d^2T_b}{dz^2} = 0, \tag{6.3.3}$$

$$\frac{d^2S_b}{dz^2} = 0, \tag{6.3.4}$$

$$\rho_b = \rho_0[1 - \beta_T(T_b - T_0) + \beta_S(S_b - S_0)]. \tag{6.3.5}$$

Now, superimpose infinite amplitude disturbances on the basic state in the form:

$$\vec{q} = q_b + q', T = T_b + T', p = p_b + p', S = S_b + S', \rho = \rho_b + \rho'. \tag{6.3.6}$$

Substituting Eqs.(6.3.6) into Eqs.(6.2.1)-(6.2.5), using Eqs.(6.3.2)-(6.3.5), to get

$$\nabla.\vec{q'} = 0, \tag{6.3.7}$$

$$(1 + \overline{\lambda_1}\frac{\partial}{\partial t})\left[\frac{\rho_0}{\varepsilon}\frac{\partial q'}{\partial t} + 2\frac{\rho_0}{\varepsilon}\Omega \times \vec{q'}\right] + \mu(K.\vec{q'}) = (1 + \overline{\lambda_1}\frac{\partial}{\partial t})(-\nabla p' - \rho_0 g\beta_T T' + \beta_S S') \tag{6.3.8}$$

$$\gamma\frac{\partial T'}{\partial t} + (\vec{q'}.\nabla)T + w'\frac{\partial T_b}{\partial z} = \kappa_T.\nabla^2 T' + K_{TS}.\nabla^2 S', \tag{6.3.9}$$

$$\varepsilon\frac{\partial S'}{\partial t} + (\vec{q'}.\nabla)S' + w'\frac{\partial S_b}{\partial z} = \kappa_S.\nabla^2 S' + K_{ST}.\nabla^2 T' \tag{6.3.10}$$

$$\rho' = -\rho_0[\beta_T T' + \beta_S S'] \tag{6.3.11}$$

The pressure term was eliminate by taking curl of the Eq. (6.3.8). Once again taking the curl of the resulting equation, the resulting equation and the Eq.(6.3.9) - Eq.(6.3.10) are then non-dimensionalized using the following transformations:

$$(x,y,z)=(x^*,y^*,z^*)d,\quad t=t^*(\frac{\gamma d^2}{\kappa_T}),p=(\frac{\mu\kappa_T}{\kappa})p^* \tag{6.3.12}$$

$$(u,v,w)=(u^*,v^*,w^*)(\frac{\kappa_T}{d}),\quad T=(\Delta T)T^*,S=(\Delta S)S^*,\quad \lambda_1=\lambda_1^*(\frac{d^2}{\kappa_T}).$$

The non dimensional equations (on dropping the asterisks for simplicity) are obtained as

$$(1+\lambda_1\frac{\partial}{\partial t})\Big[\frac{1}{V_a}\frac{\partial q}{\partial t}+\sqrt{T_a}\hat{k}\times q\Big]q_a=-(1+\lambda_1\frac{\partial}{\partial t})(\nabla p+\hat{k}Ra_T T-\hat{k}Ra_S S) \tag{6.3.13}$$

$$\Big[\frac{\partial}{\partial t}-\nabla^2+(q.\nabla)\Big]T-w=D_f\frac{Ra_S}{Ra_T}\nabla^2 S \tag{6.3.14}$$

$$\Big[\lambda\frac{\partial}{\partial t}-\nabla^2\frac{1}{L_e}+(q.\nabla)\Big]S-w=S_r\frac{Ra_T}{Ra_S}\nabla^2 T \tag{6.3.15}$$

where $T_a=(\frac{2\Omega\kappa_T}{\nu\varepsilon})^2$ is Taylor number, $V_a=\frac{\varepsilon\gamma\nu d^2}{K_z\kappa_T}$ is Vadasz number, $Ra_T=\frac{\beta_T g\Delta T K_z d}{\nu\kappa_T}$ is the thermal Rayleigh number, $Ra_S=\frac{\beta_S g\Delta S K_z d}{\nu\kappa_T}$ is the solute Rayleigh number, $\lambda=\frac{\epsilon}{\gamma}$ is normalized parameter, $L_e=\frac{\kappa_T}{\kappa_S}$is Lewis number, $g=(0,0,g)$ is gravitational accelaration, $S_r=\frac{K_{ST}\beta_S}{\kappa_T\beta_T}$ the soret parameter, $D_f=\frac{K_{TS}\beta_T}{\kappa_T\beta_S}$ the Dufour parameter, $\xi=\frac{K_x}{K_z}$ is mechanical anisotropic parameter, $q_a=(\frac{1}{\xi}u,\frac{1}{\xi}v,w)$ is anisotropic modified velocity vector.

$$(1+\lambda_1\frac{\partial}{\partial t})\Big[\frac{1}{V_a}\frac{\partial\omega}{\partial t}-\sqrt{T_a}\frac{\partial\omega}{\partial z}\Big]+\frac{1}{\xi}\omega=(1+\lambda_1\frac{\partial}{\partial t})\Big[(\frac{\partial T}{\partial y}i-\frac{\partial T}{\partial x}j)Ra_T-\frac{1}{L_e}(\frac{\partial S}{\partial y}i-\frac{\partial S}{\partial x}j)Ra_S\Big] \tag{6.3.16}$$

where $\omega=\nabla\times q$ is vorticity vector

$$(1+\lambda_1\frac{\partial}{\partial t})\Big[\frac{1}{V_a}\frac{\partial}{\partial t}(\nabla^2 q)+\sqrt{T_a}\frac{\partial\omega}{\partial z}\Big]-Q=(1+\lambda_1\frac{\partial}{\partial t})\Big[\nabla_1^2 T Ra_T-\frac{1}{L_e}\nabla_1^2 S Ra_S\Big] \tag{6.3.17}$$

where $Q=(Q_1,Q_2,Q_3)$

$$Q_1=\frac{1}{\xi}\frac{\partial^2 v}{\partial y\partial x}+\frac{\partial^2 w}{\partial x\partial z}-(\frac{\partial^2 v}{\partial y^2}+\frac{1}{\xi}\frac{\partial^2 u}{\partial z^2}) \tag{6.3.18}$$

$$Q_2=\frac{1}{\xi}\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 w}{\partial y\partial z}-\frac{1}{\xi}(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial z^2})$$

$$Q_3 = -(\nabla_1^2 + \frac{1}{\xi}\frac{\partial^2}{\partial z^2})w$$

consider the stress free and isothermal boundary conditions as given below will be solved the above system:

$$\psi = \frac{\partial^2 \psi}{\partial z^2} = T = S = 0 \ \ on \ \ z = 0, z = 1. \tag{6.3.19}$$

6.4 Linear stability Analysis

In order to study linear stability, the eigenvalue problem defined by Eqs.(6.3.16)-(6.3.17) is solved. Taking vertical component the equation are

$$\omega_z = (1 + \lambda_1 \frac{\partial}{\partial t})\xi\sqrt{T_a}\frac{\partial w}{\partial z} \tag{6.4.1}$$

$$(\nabla_1^2 + \frac{1}{\xi}\frac{\partial^2}{\partial z^2})w + (1 + \lambda_1\frac{\partial}{\partial t})\sqrt{T_a}\frac{\partial \omega_z}{\partial z} = (1 + \lambda_1 \frac{\partial}{\partial t})\Big[\nabla_1^2 T Ra_T - \frac{1}{L_e}\nabla_1^2 S Ra_S\Big] \tag{6.4.2}$$

$$(\nabla_1^2 + \frac{1}{\xi}\frac{\partial^2}{\partial z^2})w + (1 + \lambda_1\frac{\partial}{\partial t})^2 T_a \frac{\partial^2 w}{\partial z^2} = (1 + \lambda_1 \frac{\partial}{\partial t})\Big[\nabla_1^2 T Ra_T - \frac{1}{L_e}\nabla_1^2 S Ra_S\Big], \tag{6.4.3}$$

where $\omega_z = \nabla \times w$. For linear stability analysis, normal mode technique is used to solve the eigenvalue problem defined by Eq. (6.4.3), (6.3.14) and (6.3.15), by using time periodic disturbance in horizontal plane. For fundamental mode

$$\begin{pmatrix} w \\ T \\ S \end{pmatrix} = \begin{pmatrix} w_0 \\ \Theta_0 \\ \phi_0 \end{pmatrix} exp[i(lx + my) + \sigma t] Sin(\pi z) \tag{6.4.4}$$

where l, m are the horizontal wave numbers, $a^2 = l^2 + m^2$ and $\sigma = \sigma_r + i\sigma_j$ is the growth rate. Substituting expression (6.4.4) into the linearized eqs. (6.3.13)-(6.3.14), it is obtained

$$\Big[\delta_1^2 + (1 + \lambda_1\sigma)^2 \xi T_a \pi^2\Big] w_0 - (1 + \lambda_1\sigma)a^2 Ra_T \Theta_0 + \frac{1}{L_e}(1 + \lambda_1\sigma)a^2 Ra_S \phi_0 = 0 \tag{6.4.5}$$

$$[\sigma + \delta^2]\Theta_0 - w_0 + \delta^2 D_f \frac{Ra_S}{Ra_T}\phi_0 = 0 \tag{6.4.6}$$

$$[\lambda\sigma + \frac{\delta^2}{L_e}]\phi_0 - w_0 + \delta^2 S_r \frac{Ra_T}{Ra_S}\Theta_0 = 0. \tag{6.4.7}$$

where $\delta^2 = (\pi^2 + a^2)$, $\delta_1^2 = a^2 + \frac{1}{\xi}\pi^2$. The above equations can be expressed in matrix form as

$$\begin{bmatrix} \delta_1^2 + (1+\lambda_1\sigma)^2\xi T_a\pi^2 & -(1+\lambda_1\sigma)a^2 Ra_T & +\frac{1}{L_e}(1+\lambda_1\sigma)a^2 Ra_S \\ -1 & \sigma+\delta^2 & \delta^2 D_f \frac{Ra_S}{Ra_T} \\ -1 & \delta^2 S_r \frac{Ra_T}{Ra_S} & \lambda\sigma + \frac{\delta^2}{L_e} \end{bmatrix} \begin{bmatrix} w_0 \\ \Theta_0 \\ \phi_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

For non zero solution, the determinant of the matrix has to be zero, therefore, the thermal Rayleigh number can be obtained as

$$Ra_T = \frac{Le}{a^2(1+\lambda_1\sigma)(L_e\lambda\sigma + \delta^2 + \delta^2 S_r)}\Big[\Big(\delta_1^2 + (1+\lambda_1\sigma)^2\xi T_a\pi^2\Big) \tag{6.4.8}$$

$$\Big((\sigma+\delta^2)(\lambda\sigma + \frac{\delta^2}{L_e}) - \delta^4 D_f S_r\Big) + a^2 Ra_S\Big(1+\lambda_1\sigma)(\delta^2 D_f + \frac{1}{L_e}(\sigma+\delta^2)\Big)\Big].$$

The growth rate σ is in general a complex quantity such that $\sigma = \sigma_r + i\sigma_j$. The system with σ_r <0 is always stable, while for σ_r >0 it will become unstable. For neutral stability state $\sigma_r = 0$.

6.4.1 Stationary State

The steady onset corresponds to σ=0 (i.e.σ_r =σ_j=0) and becomes the Eq.(6.4.8). The value of the thermal Rayleigh number of the system for a stationary mode of convection is as given below:

$$Ra_T^{st} = \frac{1}{a^2\delta^2(1+S_r)}\Big[(\delta_1^2 + \xi T_a\pi^2)(\delta^4 - \delta^4 D_f S_r L_e) + a^2 Ra_S(\delta^2 L_e D_f + \delta^2)\Big], \tag{6.4.9}$$

6.4.2 Oscillatory State

Now set $\sigma_r = 0$ and $\sigma_j \neq 0$ (i.e.$\sigma = i\sigma_j$) in Eq.(6.4.8) and removing the complex quantities from the denominator, to obtain

$Ra_T^{osc} = \Delta_1 + i\sigma_j\Delta_2$.

$$\Delta_1 = \frac{(Ad_1) + \sigma^2(Bd_2)}{a^2(A^2 + \sigma^2 B^2)} \tag{6.4.10}$$

$$\Delta_2 = \frac{(Ad_2) - (Bd_1)}{a^2(A^2 + \sigma^2 B^2)}, \tag{6.4.11}$$

where, $a_1 = \delta_1^2 + (1 - \sigma^2\lambda_1)\xi T_a \pi^2$,

$a_2 = 2\xi T_a \pi^2$

$a_3 = \frac{\delta^4}{L_e} - \delta^4 S_r D_f - \lambda\sigma^2$

$a_4 = \lambda\delta^2 + \frac{\delta^2}{L_e}$

$a_5 = (\delta^2 D_f + \frac{\delta^2}{L_e} - \frac{1}{L_e}\sigma^2\lambda_1)$

$a_6 = (\lambda_1\delta^2 D_f + \frac{1}{L_e} + \frac{\delta^2}{L_e}\lambda_1)$

$A = \frac{\delta^2}{L_e} + \frac{\delta^2 S_r}{L_e} - \sigma^2\lambda\lambda_1$

$B = \lambda + \lambda_1(\frac{\delta^2 S_r}{L_e} + \frac{\delta^2}{L_e})$

$d_1 = (a_1 a_3 - \sigma^2 a_2 a_4 + a_5 a^2 Ra_S)$

$d_2 = (a_2 a_3 + a_1 a_4 + a_6 a^2 Ra_S)$

for oscillatory mode $\Delta_2 = 0$ and $(\sigma_i \neq 0)$. The Rayleigh number Ra_T^{osc} has to be real, therefore, The expression for oscillatory Rayleigh number is obtained as

$$Ra_T^{osc} = \Delta_1. \tag{6.4.12}$$

6.5 Nonlinear stability Analysis

In this section, nonlinear stability analysis has been done using minimal truncated Fourier series. For simplicity, only two dimensional rolls are considered, so that all the physical quantities do not dependent of y. Taking curl to eliminate the pressure term from Eq. (6.2.1), introducing the stream function ψ by using $u = \frac{\partial\psi}{\partial z}$, $w = -\frac{\partial\psi}{\partial x}$ and then for steady state assuming $\frac{\partial}{\partial t} = 0$, it is obtained

$$\Big(\frac{\partial^2}{\partial x^2} + \frac{1}{\xi}\frac{\partial^2}{\partial z^2} + (T_a)\xi\frac{\partial^2}{\partial z^2}\Big)\psi + \Big(Ra_T\frac{\partial T}{\partial x} - \frac{1}{L_e}Ra_S\frac{\partial S}{\partial x}\Big) = 0, \tag{6.5.1}$$

$$\nabla^2 T + D_f\frac{Ra_S}{Ra_T}\nabla^2 S - \frac{\partial\psi}{\partial x} + \frac{\partial(\psi, T)}{\partial(x, z)} = 0, \tag{6.5.2}$$

$$\frac{\nabla^2}{L_e}S + S_r\frac{Ra_T}{Ra_S}\nabla^2 T - \frac{\partial\psi}{\partial x} + \frac{\partial(\psi, S)}{\partial(x, z)} = 0. \tag{6.5.3}$$

It is to be noted that the effect of nonlinearity is to distort the temperature and concentration fields through the interaction of ψ and T, ψ and S. As a result a component of the form $sin(2\pi z)$ will be generated. A minimal Fourier series which describes the finite amplitude convection is given by

$$\psi = A_1(t)sin(ax)sin(\pi z), \tag{6.5.4}$$

$$T = B_1(t)cos(ax)sin(\pi z) + B_2(t)sin(2\pi z), \tag{6.5.5}$$

$$S = C_1(t)cos(ax)sin(\pi z) + C_2(t)sin(2\pi z), \tag{6.5.6}$$

where the amplitudes $A_1(t)$, $B_1(t)$, $B_2(t)$,$C_1(t)$, $C_2(t)$ are functions of time and are to be determined.

6.5.1 Steady finite amplitude motions

Substituting above expressions (6.5.4)-(6.5.6) in Eqs. (6.5.1)-(6.5.3) and equating the like terms, the following set of nonlinear autonomous differential equations were obtained

$$\left(\delta_1^2 + T_a\xi\pi^2\right)A_1 + aRa_TB_1 - \frac{1}{L_e}aRa_SC_1 = 0 \tag{6.5.7}$$

$$\delta^2B_1 + aA_1 + \pi aA_1B_2 + \delta^2D_f\frac{Ra_S}{Ra_T}C_1 = 0 \tag{6.5.8}$$

$$\pi aA_1B_1 - 8\pi^2B_2 - 8\pi^2D_f\frac{Ra_S}{Ra_T}C_2 = 0 \tag{6.5.9}$$

$$\frac{1}{L_e}\delta^2C_1 + aA_1 + \pi aA_1C_2 + \delta^2S_r\frac{Ra_T}{Ra_S}B_1 = 0 \tag{6.5.10}$$

$$\frac{8\pi^2}{L_e}C_2 - \pi aA_1C_1 + 8\pi^2S_r\frac{Ra_T}{Ra_S}B_2 - = 0 \tag{6.5.11}$$

Numerical method is used to solve the above nonlinear differential equation to find the amplitudes. On solving for the amplitudes in terms of A_1, B_1, B_2, C_1, C_2 can be obtained.

6.5.2 Steady Heat and Mass Transports

In the study of this type problem, quantification of heat and mass transport is very important. Let Nu and Sh denote the rate of heat and mass transport for the fluid phase. The Nusselt number and Sherwood number are defined by

$$Nu = 1 + \left[\frac{\int_0^{2\pi/a} \frac{\partial T}{\partial z} dx}{\int_0^{2\pi/a} \frac{\partial T_b}{\partial z} dx}\right]_{z=0} \tag{6.5.12}$$

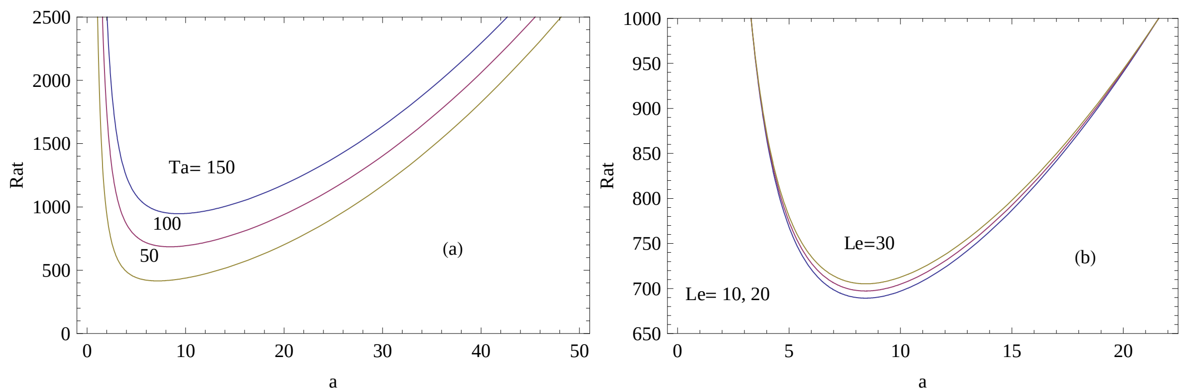
$$Sh = 1 + \left[\frac{\int_0^{2\pi/a} \frac{\partial S}{\partial z} dx}{\int_0^{2\pi/a} \frac{\partial S_b}{\partial z} dx}\right]_{z=0} \tag{6.5.13}$$

Substituting the expressions of T, T_b, S and S_b in Eqs.(6.5.12)-(6.5.13), it is obtained

$$Nu = (1 - 2\pi B_2) + D_f \frac{Ra_S}{Ra_T}(1 - 2\pi C_2), \tag{6.5.14}$$

$$Sh = (1 - 2\pi C_2) + S_r L_e \frac{Ra_T}{Ra_S}(1 - 2\pi B_2).$$

Further, using the expressions of B_2, C_2 into (6.5.14), the final expressions of Nu and Sh can be obtained in terms of various parameters governing the system.



6.6 Results and Discussion

This paper investigates the effects of rotation and cross diffusion on stationary and oscillatory convection in a Maxwell fluid saturated anisotropic porous medium. The effects of various parameters such as mechanical anisotropy parameter, solute Rayleigh number,

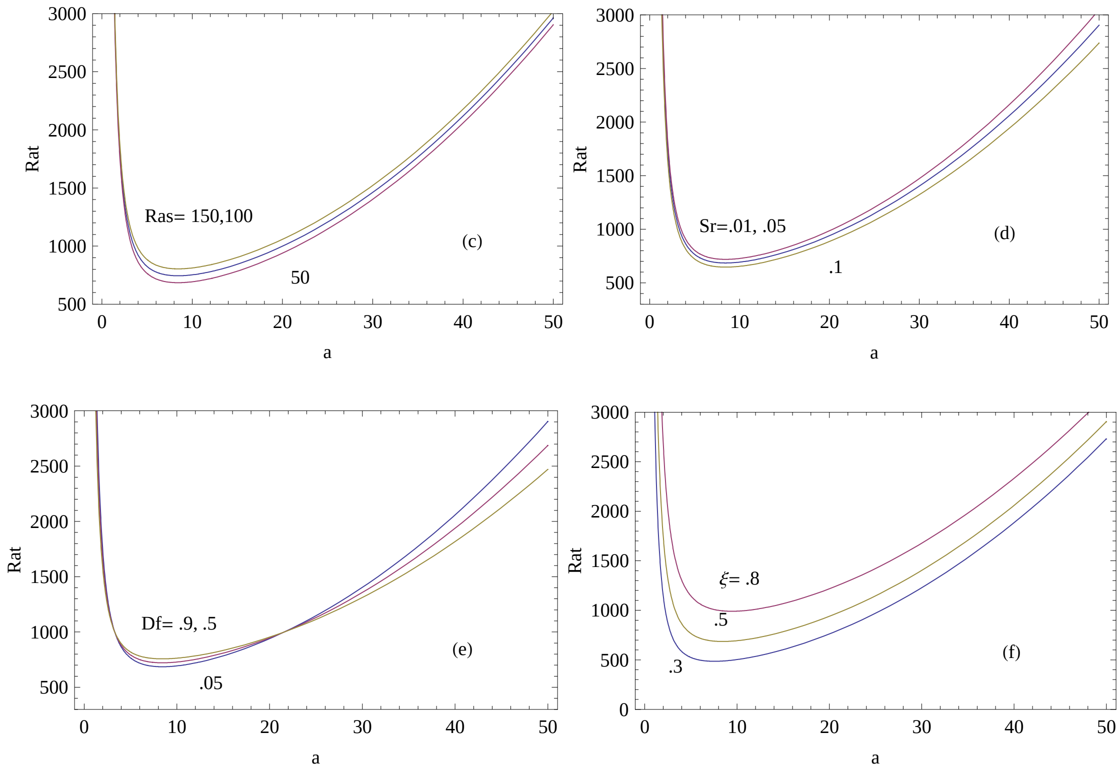
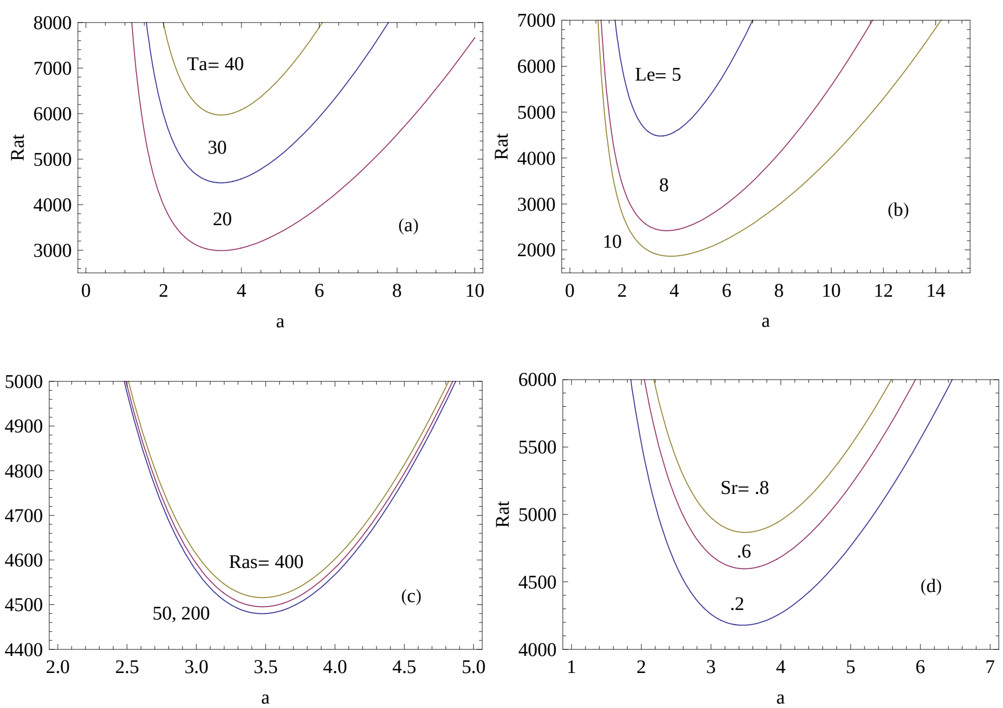


Figure 6.1: Stationary neutral stability curves for different values of (a) T_a, (b) L_e, (c) Ra_s, (d) S_r, (e) D_f, (f) ξ



Lewis number, relaxation parameter, Soret and Dufour parameters are computed and the results are depicted in figures.

The neutral stability curves in the (Ra_T,a) plane for various parameter values are as

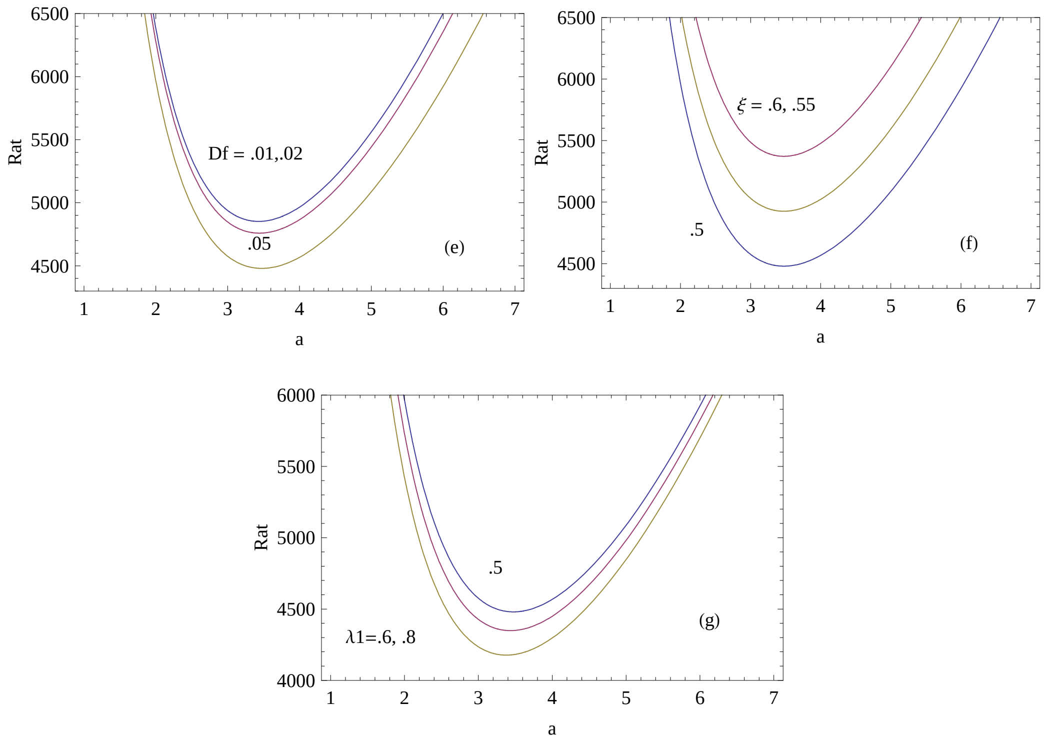
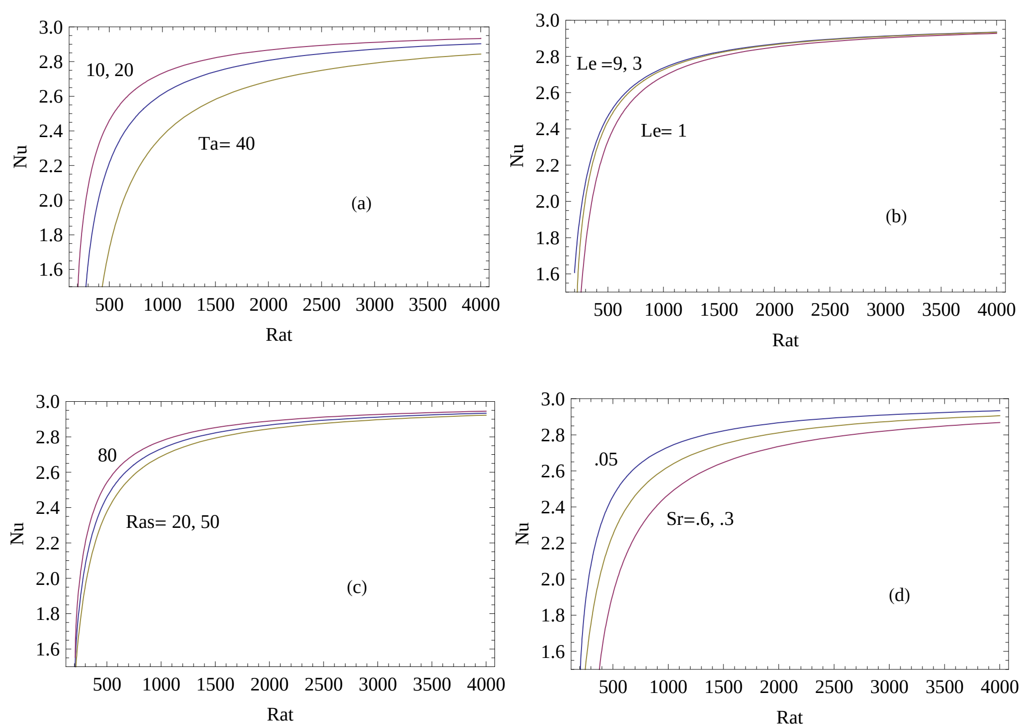


Figure 6.2: Oscillatory neutral stability curves for different values of (a) T_a, (b) L_e, (c) Ra_s, (d) S_r, (e) D_f, (f) ξ, (g) λ_1



shown in Figures. The values for the parameters are fined as $T_a = 100$, $\lambda_1 = .5$, $\lambda = .5$, $Ra_S = 50$, $L_e = 5$, S_r=.05, D_f=.05 and $\xi = .5$, except the varying parameter. Figs. 6.1(a-f) are for stationary mode of convection, while Figs. 6.2(a-g) are for oscillatory mode

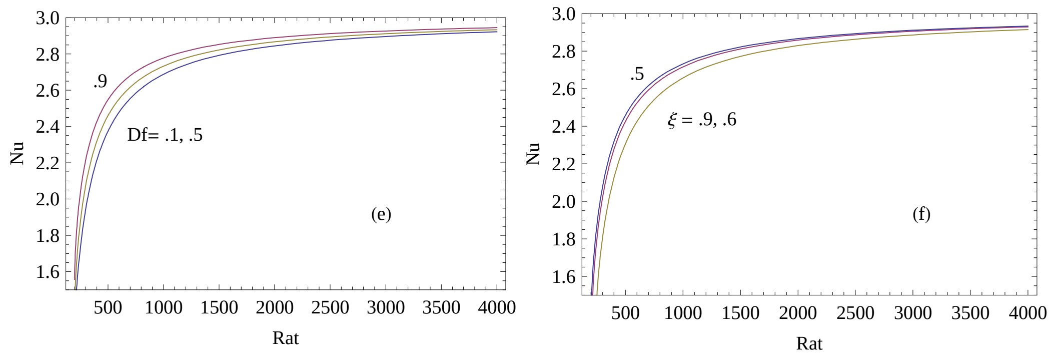


Figure 6.3: Nusselt number for different values of (a) T_a, (b) L_e, (c) Ra_s, (d) S_r, (e) D_f, (f) ξ

of convection.

In the Figs. 6.1 (a, c, f) and Figs. 6.2 (a,c,f),effects of the parameters; Taylor number T_a, solute Rayleigh number Ra_S and mechanical anisotropy parameter ξ are depicted respectively. From these figures, it is observed that an increment in the values of these parameters, increases the values of stationary and oscillatory Rayleigh number, thus stabilize the system, means onset of convection will take place at later point. In Figs. 6.1,6.2(b, e), it is shown that the effect of increasing Lewis number L_e and Dufour parameter D_f is to increase the value of Rayleigh number for stationary mode but decrease the value for oscillatory mode, thus to stabilize the stationary and destabilize the oscillatory mode of convection. Further,Figs. 6.1,6.2(d), show the effect of Soret parameter, respectively for both stationary and oscillatory modes. It is found that increasing the value of the Soret parameter S_r, decreases the value of stationary Rayleigh number, thus destabilizing the onset of stationary mode of convection but increases the value of oscillatory Rayleigh number, thus stabilizing the system. Also Fig.6.2(g) shows the effect of relaxation parameter on the onset of convection. It is observed that the oscillatory Rayleigh number decreases on increasing the value of the relaxation parameter λ_1, indicating that the effect of relaxation parameter is to destabilize the system, as the oscillatory convection takes place little early.

Now, the quantity of heat and mass transfer across the porous medium is computed in terms of the thermal Nusselt number and Sherwood number as functions of various parameters, which are fixed at $T_a = 10$, $Ra_S = 50$, $L_e = 5$, S_r=.05, D_f=.5 and $\xi = .5$ with varying one of the parameters. The results are depicted in the Figs.6.3(a-f) through

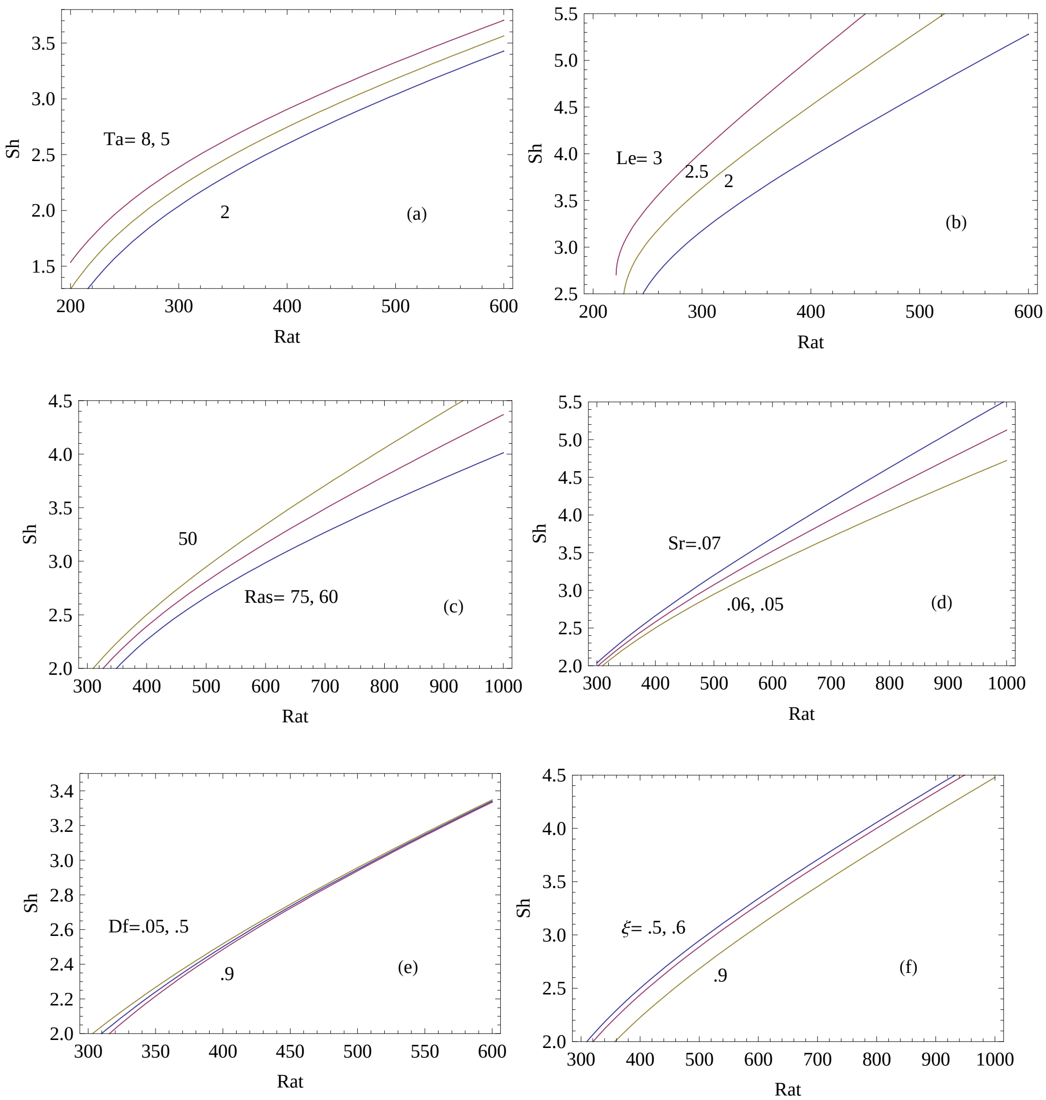


Figure 6.4: Graph between Sherwood number and Rayleigh number for different values of (a) T_a, (b) L_e, (c) Ra_s, (d) S_r, (e) D_f, (f) ξ

the Rayleigh-Nusselt number plane and in the Figs.6.4(a-f) through the Rayleigh-Sherwood number plane. It is found from the Figs.6.3,6.4 (a,d),that the value of Nusselt number N_u decreases, while that of S_h increases on increasing the values of Taylor number T_a and of the Soret parameter S_r. This shows that the effect of Taylor number and Soret parameter is to decrease the heat transport, while increase the mass transport in the system. Figs.6.3,6.4(b) show that heat and mass transports increase on increasing the value of Lewis number L_e, thus destabilizing the system. it is found from the Figs.6.3, 6.4(c,e) that the effects of increasing Ra_S, D_f have a stabilizing effect on the system as heat transport increase and mass transport decrease on increasing the values of these parameters. Further, it is found from the Figs.6.3, 6.4(f) that the effects of increasing ξ has a destabilizing effect on the system as heat and mass transport decrease on increasing the values of these parameters.

6.7 Conclusions

Soret effect and Dufour effect on cross diffusion convection in a rotating anisotropic porous medium saturated with a Maxwell fluid which is heated and salted from below, is investigated analytically using linear and nonlinear stability analyses. The linear analysis is done using normal mode technique while the nonlinear analysis is based on a minimal representation of double Fourier series. Following conclusions are drawn:

1) The Taylor number T_a, solute Rayleigh number Ra_S, and mechanical anisotropy parameter ξ have stabilizing effect on the system in both stationary and oscillatory modes of convection.

2) The Soret parameter S_r has destabilizing effect on stationary mode while stabilizing effect on oscillatory mode of convection.

3) The Lewis number L_e and Dufour parameter D_f have a stabilizing effect on stationary convection while opposite effect on oscillatory convection.

4) The relaxation parameter λ_1 has destabilizing effect on oscillatory convection.

5) The Lewis number L_e increases both Nusselt number and the Sherwood number.

6) Increments in Solute Rayleigh number Ra_S and Dufour parameter D_f increase the Nusselt number and decrease the Sherwood number.

6) Increments in mechanical anisotropy parameter ξ decrease both Nusselt number and the Sherwood number.

7) Increments in Taylor number T_a and Soret parameter S_r decrease the Nusselt number and increase the Sherwood number.

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List of Publications

Published

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Double Diffusive Convection in a Viscoelastic Fluid Saturated rotating anisotropic porous layer with internal heat source

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Abstract-In this paper, the effect of internal heating on double diffusive convection in a rotating anisotropic porous medium saturated with a viscoelastic fluid,which is heated and salted from below,is studied analytically. Linear stability analysis has been performed by using Normal mode technique and nonlinear theory is based on minimal representation of Fourier series up to two terms. The modified Darcy model,which includes the time derivative and Coriolis terms has been employed in the momentum equation. The effects of Taylor number,solute Rayleigh number,internal heat source parameter, diffusivity ratio,relaxation and retardation parameters, thermal and mechanical anisotropy parameters on the stationary and oscillatory convection are obtained and shown graphically. Also, heat and mass transports have been obtained in terms of the Nusselt number and Sherwood number respectively and presented through Figs.

Index Terms-Viscoelastic fluid; Double diffusive convection; Rotation; Internal heat source; Porous media.

1. INTRODUCTION

Most of the studies in relevant area are mainly dealt with isotropic porous media; however there are many physical situations where thermal and mechanical anisotropy exists in porous matrix, one of such examples is our geothermal environment. Anisotropy is generally a consequence of preferential orientation of asymmetric geometry of porous matrix or fibers and is in fact encountered in numerous systems in industry and nature, also in artificial porous matrix anisotropy can be made deliberately according to applications. Srivastava et al. [5] studied the effect of internal heating on double diffusive convection in a couple stress fluid saturated anisotropic porous medium.

There is large number of practical situations in which convection is driven by internal heat source. Internal heat generation arises in many important contexts, including reactor safety analyses, metal waste that is produced by spent nuclear fuel, fire and combustion studies, and the storage of radioactive materials. The study concerning internal heat source in porous media is provided by Tveitereid [27], performing thermal convection in a horizontal porous layer with internal heat source. Hill [3] performed linear and nonlinear analyses on the double-diffusive convection in a porous layer with a concentration based internal heat source. Bhadauria et al.[9] studied the effect of internal heating on double diffusive convection in a couple stress fluid saturated anisotropic porous medium. Govender [14] investigated the Corioliseffect on the stability of centrifugally driven convection in a rotating anisotropic porous layer subject to gravity.

The studies of double diffusive convection in porous media plays very significant roles in many areas such as in petroleum industry, solidification of binary mixture, migration of solutes in water saturated soils. Other example includes geophysics system, crystal growth, electrochemistry, the migration of moisture through air contained in fibrous insulation, Earth's oceans, magma chambers etc. The onset of thermal instability in a horizontal porous layer was first studied extensively by Horton and Rogers [15] and Lapwood [18]. However, Nield [28] was first to investigate double diffusive generalization of the Horton–Rogers–Lapwood problem, performing only linear stability analysis. Some other researchers who have worked on double diffusive convection in a porous medium are Taunton et al.[37], Patil and Vaidyanathan [31,32], Griffith [13]. The onset of double diffusive convection in a horizontal porous layer has been investigated by Rudraiah et al. [34] using a weak non-linear theory. The problem of double diffusive convection in a porous media has been presented by Ingham and Pop [16], Nield and Bejan [19] and Vafai [39,40], Vadasz[41,42]. The study was continued by Poulikakos [30], Travison and Bejan [38], Momou [22] etc.

The study of double diffusive convection in a rotating porous media is important due to both, its theoretical and practical applications in engineering. Some of the important areas of applications in engineering include the food and chemical process, solidification and centrifugal casting of metals, rotating machinery, petroleum industry and biomechanics problems. There are only few studies available on double diffusive convection in the presence of rotation. Chakrabarti and Gupta [4] have analyzed the nonlinear thermohaline convection in a rotating porous medium. The effect of rotation on linear and nonlinear double diffusive convection in a sparsely packed porous medium was studied by Rudraiah et.al. [34]. Malashetty et al. [23] studied the effect of rotation on the onset of double diffusive convection in a Darcy porous medium saturated with couple stress fluid. Malashetty and Heera [24, 25] studied the effect of rotation on the onset of double diffusive convection in a horizontal anisotropic porous layer. Gaikwad [12] have done the linear stability analysis of double diffusive convection in a horizontal sparsely packed rotating anisotropic porous layer in presence of Soret effect. Sulochana et.al [10] studied the onset of double diffusive convection in a couple stress fluid saturated rotating anisotropic porous layer. Bhadauria et al. [7] studied cross diffusion convection in a Newtonian fluid-saturated rotating porous medium.

The work published on natural convection of viscoelastic fluids in porous media is fairly limited. Convection in a viscoelastic fluid-saturated sparsely packed porous layer is studied by Rudraiah et al. [33, 35]. Mardones et al. [20, 21] have investigated the Rayleigh-Benard convection for stationary convection in a binary viscoelastic fluid. Yoon et al. [43, 44], Kim et al. [17], and Bertola and Cafaro [6] studied the stability of a viscoelastic fluid where an existing constitutive model, which is rather simple, was employed to examine the effects of relaxation and retardation times on the stationary and oscillatory convection in a horizontal porous layer heated by a constant temperature. Park and Park [29] studied Rayleigh-Benard convection of viscoelastic fluids in arbitrary finite domains. Convective instabilities in a viscoelastic-fluid-saturated porous medium with throughflow have been studied by Shivakumara and Sureshkumar [36]. Linear and nonlinear stability analyses of thermal convection for Oldroyd-B fluids in a porous media heated from below has been studied by Zhang et al. [45]. Malashetty et al. [26] studied the onset of convection in a binary viscoelastic fluid-saturated porous layer. Kumar and Bhadauria [1] have studied non-linear two-dimensional double diffusive convection in a rotating porous layer saturated by a viscoelastic fluid. Gaikwad et al. [11] performed onset of Darcy-Brinkman convection in a binary viscoelastic fluid-saturated porous layer with internal heat source. Recently Srivastava et al. [2] have studied linear and weak nonlinear double diffusive convection in a viscoelastic fluid saturated anisotropic porous medium with internal heat source.

In the present literature, no work is available on double diffusive convection in a rotating porous layer saturated by a viscoelastic fluid with an internal heat source. Therefore, in the present study stability analysis of internal heating effect on double diffusive convection in a rotating anisotropic porous medium saturated with a viscoelastic fluid has been done.

2. GOVERNING EQUATION

Consider a viscoelastic fluid saturated porous medium, confined between two infinitely extended horizontal planes at $z=0$ and $z=d$, heated from below and cooled from above. Darcy model has been employed in the momentum equation. Further, an internal heat source term has been included in the energy equation. A Cartesian frame of reference is chosen in such a way that the origin lies on the lower plane and the z-axis as vertical upward. The system is rotating about z-axis with a constant angular velocity Ω. An adverse temperature gradient is applied across the porous layer and the lower and upper planes are kept at temperature $T_0+\Delta T$ and T_0, and concentration $S_0+\Delta S$ and S_0 respectively. The physical configuration of the model is reported in the Fig.A.

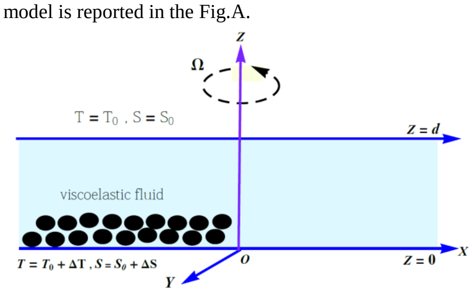


Fig .A: Physical configuration of the problem

$$\nabla .q=0 \tag{1}$$

$$\left(1+\overline{\lambda}_1\frac{\partial}{\partial t}\right)\frac{2\rho_0}{\phi}(\Omega\times q)+\frac{\mu}{\kappa}\left(1+\overline{\lambda}_2\frac{\partial}{\partial t}\right)=\left(1+\overline{\lambda}_1\frac{\partial}{\partial t}\right)$$
$$(-\nabla p+\rho_0 g) \tag{2}$$

$$\gamma\frac{\partial T}{\partial t}+(q\cdot\nabla)T=\nabla(\kappa_T\cdot\nabla T)+Q(T-T_0) \tag{3}$$

$$\phi\frac{\partial S}{\partial t}+(q\cdot\nabla)S=\kappa_S\nabla^2 S \tag{4}$$

$$\rho=\rho_0\left[1-\beta_T(T-T_0)-\beta_S(S-S_0)\right] \tag{5}$$

$$T=T_0+\Delta T;\ \text{ at } z=0 \text{ and } T=T_0;\ \text{ at } z=d; \tag{6}$$
$$S=S_0+\Delta S;\ \text{ at } z=0 \text{ and } \mathrm{S}=S_0;\ \text{ at } z=d;$$

2.1 Basic Solution

At this state, the velocity, pressure, temperature and density profiles are given by
$q_b = 0,\ p = p_b(z),\ T = T_b(z),\ S = S_b(z),\ \rho = \rho_b(z).$ (7)
Substituting Eq. (7) in Eq. (1-4), we get the following equations:

$$\frac{dp_b}{dz} = -\rho_b g, \quad (8)$$

$$\kappa_T \frac{d^2 T_b}{dz^2} + QT = 0, \quad (9)$$

$$\frac{d^2 S_b}{dz^2} = 0, \quad (10)$$

The solution of Eq. (9) and Eq. (10) subject to the boundary conditions (6), are given by

$$T_b = T_0 + \Delta T \frac{Sin\sqrt{R_i}\left(1-\frac{z}{d}\right)}{Sin\sqrt{R_i}}. \quad (11)$$

$$S_b = S_0 + \Delta S(1-\frac{z}{d}) \quad (12)$$

2.2 Perturbed Equation

Now, we superimpose finite amplitude perturbations on the basic state in the form:

$q = q_b + q',\ T = T_b + T',\ p = p_b + p',\ S = S_b + S',\ \rho = \rho_b + \rho',$ (13)
and get the following equations

$$\nabla . q' = 0 \quad (14)$$

$$\left(1+\overline{\lambda_1}\frac{\partial}{\partial t}\right)\frac{2\rho_0}{\phi}(\Omega \times q') + \frac{\mu}{\kappa}\left(1+\overline{\lambda_2}\frac{\partial}{\partial t}\right)q' = -\left(1+\overline{\lambda_1}\frac{\partial}{\partial t}\right)\left(\nabla p - \rho_0 g\left(\beta_T T' - \beta_S S'\right)\right) \quad (15)$$

$$\gamma\frac{\partial T'}{\partial t} + (q'\cdot\nabla)T' = \left(\kappa_{Tx}\cdot\nabla^2 T'\right) + QT' - w'\frac{\partial T_b}{\partial z} \quad (16)$$

$$\frac{\partial S'}{\partial t} + (q'\cdot\nabla)S' + w'\frac{\partial S_b}{\partial z} = \kappa_S \nabla^2 S \quad (17)$$

$$\rho' = -\rho_0\left[\beta_T T' - \beta_S S'\right] \quad (18)$$

The resulting equations are non-dimensionalized using the following transformations;

$$(x', y', z') = (x^*, y^*, z^*)d,\ t' = t^*\left(\frac{d^2}{\kappa_{Tz}}\right),$$

$$q' = \frac{\kappa_{Tz}}{d}q^*,\ \lambda_1 = \frac{d^2}{\kappa_{Tz}}\lambda_1^*$$

$$\lambda_2 = \frac{d^2}{\kappa_{Tz}}\lambda_2^*,\ \ (u,v,w) = (u^*,v^*,w^*)\left(\frac{\kappa_{Tz}}{d}\right),$$

$$T' = (\Delta T)T^*,\ S' = (\Delta S)S^*,\ \ p' = \frac{\mu\ \kappa_{Tz}}{K_z}p^* \quad (19)$$

2.3 Non-Dimensionilized Equation

The non-dimensionalized equations (on dropping the asterisks for simplicity) are obtained as,

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\sqrt{T_a}\left(\kappa \times q'\right) + \left(1+\lambda_2\frac{\partial}{\partial t}\right)q_a + \left(1+\lambda_1\frac{\partial}{\partial t}\right)\nabla p = \left(1+\lambda_1\frac{\partial}{\partial t}\right)\left(Ra_T T - \tau Ra_S S\right)k \quad (20)$$

$$\frac{\partial T}{\partial t} + w\frac{\partial T_b}{\partial z} = \left(\eta\nabla_1^{\ 2} + \frac{\partial^2}{\partial z^2}\right) + R_i T \quad (21)$$

$$\frac{\partial S}{\partial t} + w\frac{\partial S_b}{\partial z} = \tau\ \nabla^2 S \quad (22)$$

where $q = \left(\frac{1}{\xi}u, \frac{1}{\xi}v, w\right)$ is anisotropic modified velocity vector, $Pr_D = \frac{\phi\gamma v d^2}{\kappa_T k}$ is Darcy-Prandtl number, $Ra_T = \frac{\beta_T g \Delta T K_z d}{v\kappa_{Tz}}$ is the thermal Rayleigh number, $Ra_S = \frac{\beta_S g \Delta S K_z d}{v\kappa_{Tz}}$ is the solute Rayleigh number, $R_i = \frac{Qd^2}{\kappa_{Tz}}$ is the internal Rayleigh parameter, $T_a = \left(\frac{2\Omega K_z}{\mu\phi}\right)^2$ is Taylor number, $\eta = \frac{\kappa_{Tx}}{\kappa_{TZ}}$ is thermal anisotropy parameter, $L_e = \frac{\kappa_{TZ}}{\kappa_S}$ is Lewis number, $\tau = \frac{\kappa_S}{\kappa_{Tz}}$ is diffusivity ratio, $S_r = \frac{K_{21}\Delta T}{\kappa_{TZ}\Delta S}$ is the Soret parameter, $\xi = \frac{K_x}{K_z}$ mechanical anisotropy parameter. The above system will be solved by considering stress free and isothermal boundary conditions as given below:

$$w = \frac{\partial^2 w}{\partial z^2} = T = S = 0 \ \text{ on } \ z = 0, z = 1. \quad (23)$$

The pressure term from Eq. (20) is eliminated by taking curl of the momentum equation.

$$\left(1+\lambda_2\frac{\partial}{\partial t}\right)\frac{1}{\xi}\omega-\left(1+\lambda_1\frac{\partial}{\partial t}\right)\sqrt{T_a}\frac{\partial q}{\partial z}=\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left[Ra_T\left(\frac{\partial T}{\partial y}i-\frac{\partial T}{\partial x}j\right)-\tau Ra_S\left(\frac{\partial S}{\partial y}i-\frac{\partial S}{\partial x}j\right)\right]$$

where $\omega=\nabla\times q$ is vorticity vector. On further taking curl, we get

$$\left(1+\lambda_2\frac{\partial}{\partial t}\right)Q-\left(1+\lambda_1\frac{\partial}{\partial t}\right)\sqrt{T_a}\frac{\partial \omega}{\partial z}=\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left[Ra_T\nabla_1^2T-\tau Ra_S\nabla_1^2S\right] \quad (24)$$

where $Q=(Q_1,Q_2,Q_3)$,

$$Q_1=\frac{1}{\xi}\frac{\partial^2 v}{\partial y\partial x}+\frac{\partial^2 w}{\partial y\partial x}-\left(\frac{\partial^2 v}{\partial y^2}+\frac{1}{\xi}\frac{\partial^2 u}{\partial y^2}\right),$$

$$Q_2=\frac{1}{\xi}\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 w}{\partial y\partial z}-\frac{1}{\xi}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial z^2}\right),$$

$$Q_3=-\left(\nabla_1^2+\frac{1}{\xi}\frac{\partial^2}{\partial z^2}\right)w$$

3. LINEAR STABILITY ANALYSIS

Linear equations are

$$\left(1+\lambda_2\frac{\partial}{\partial t}\right)\frac{1}{\xi}\omega_z=\left(1+\lambda_1\frac{\partial}{\partial t}\right)\sqrt{T_a}\frac{\partial w}{\partial z} \quad (25)$$

$$\left(1+\lambda_2\frac{\partial}{\partial t}\right)\left(\nabla_1^2+\frac{1}{\xi}\frac{\partial^2}{\partial z^2}\right)w+\left(1+\lambda_1\frac{\partial}{\partial t}\right)\sqrt{T_a}\frac{\partial \omega_z}{\partial z}=\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left[Ra_T\nabla_1^2T-\tau Ra_S\nabla_1^2S\right] \quad (26)$$

$$\left(\frac{\partial}{\partial t}-\eta\nabla_1^2-\frac{\partial^2}{\partial z^2}-R_i\right)T=w \quad (27)$$

$$\left(\frac{\partial}{\partial t}-\tau\nabla^2\right)S=w \quad (28)$$

By eliminating T,S,ω_z from above equation, we obtain

$$\left[\begin{array}{l}\left(\frac{\partial}{\partial t}-\eta\nabla_1^2-\frac{\partial^2}{\partial z^2}-R_i\right)\left(\frac{\partial}{\partial t}-\tau\nabla^2\right)\left(1+\lambda_2\frac{\partial}{\partial t}\right)^2\\ \left(\nabla_1^2+\frac{1}{\xi}\frac{\partial^2}{\partial z^2}\right)+\xi T_a\left(1+\lambda_1\frac{\partial}{\partial t}\right)^2\frac{\partial^2}{\partial z^2}-\left(1+\lambda_1\frac{\partial}{\partial t}\right)\\ \left(1+\lambda_2\frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial t}-\tau\nabla^2\right)Ra_T\nabla_1^2-\\ \left(\frac{\partial}{\partial t}-\eta\nabla_1^2-\frac{\partial^2}{\partial z^2}-R_i\right)\tau Ra_S\end{array}\right]w=0 \quad (29)$$

$$w=\frac{\partial^2 w}{\partial z^2}=\frac{\partial^4 w}{\partial z^4}\text{ on } z=0,\ z=1.$$

Use normal mode technique

$$w=W(z)exp\left(i\left(lx+my\right)+\sigma t\right)\text{Sin}\,\pi z \quad (30)$$

where l, m are horizontal wave numbers and $\sigma=\sigma_r+i\sigma_i$, is the growth rate. Solve the above equation, the thermal Rayleigh number can be obtained as

$$Ra_T=\frac{(\sigma+\delta_2^2-R_i)(1+\lambda_2\sigma)\delta_1^2}{a^2(1+\lambda_1\sigma)}+\frac{T_a\pi^2\xi(1+\lambda_2\sigma)(\sigma+\delta_2^2-R_i)}{(1+\lambda_1\sigma)}+\frac{(\sigma+\delta_2^2-R_i)}{(\sigma+\tau\delta^2)}\tau Ra_S \quad (31)$$

where $a^2=l^2+m^2$, $\delta^2=\pi^2+a^2$, $\delta_1^2=\frac{\pi^2}{\xi}+a^2$, $\delta_2^2=\pi^2+\eta a^2$. The growth rate σ is in general a complex quantity such that $\sigma=\sigma_r+i\ \sigma_i$. The system with $\sigma_r<0$ is always stable, while for $\sigma_r>0$ it will become unstable. For neutral stability state $\sigma_r=0$

3.1 Stationary State

Now we set $\sigma=0$ in Eq. (31) at the margin of stability. The expression of the thermal Rayleigh number for stationary mode of convection is found as given below:

$$Ra_T^{st}=\frac{(\sigma+\delta_2^2-R_i)\delta_1^2}{a^2}+\frac{T_a\pi^2\xi(\delta_2^2-R_i)}{a^2}+\frac{(\delta_2^2-R_i)}{\tau\delta^2}\tau Ra_S \quad (32)$$

It is important to note that the critical wave number $a=a_c^{St}$ depends on the couple stress parameter and Taylor number. In the absence of Taylor number i.e. $T_a=0$ Eq. (32) gives

$$Ra_T^{st} = \frac{(\sigma+\delta_2^2-R_i)\delta_1^2}{a^2}+\frac{(\delta_2^2-R_i)}{\tau\ \delta^2}\tau Ra_S$$

For isotropic porous media when $\xi=\eta=1$ and the system without internal-heating, i.e., $R_i=0,$ we get

$$Ra_T^{st} = \frac{(\pi^2+a^2)^2}{a^2}+Ra_S$$

which is the result given by Malashetty et al. [25]. For single component fluid, $Ra_S=0,$ we obtain

$$Ra_T^{st} = \frac{(\pi^2+a^2)^2}{a^2} \quad (33)$$

which has the critical value $Ra_T^{st}=4\pi^2$ for $a_c^{st}=\pi$ are the classical results obtained by Horton and Roger [16] and Lapwood [19] for single component fluid in porous layer.

3.2 Oscillatory State

For the oscillatory mode of convection, we set $\sigma=i\sigma_i$ in Eq. (27) and clear the complex quantities from the denominator, to obtain

$$Ra_T^{osc}=\Delta_1+i\sigma_i\Delta_2 \quad (34).$$

where

$$\Delta_1=\frac{\left(\tau\delta^2+\sigma^2\right)\left(\delta_1^2+\pi^2\xi T_a\right)\begin{bmatrix}\delta_2^2-R_i-\lambda_2\sigma^2+\lambda_1\sigma^2\\ \left(1+\delta_2^2-R_i\right)\end{bmatrix}}{a^2\left(1+\lambda_1^2\sigma^2\right)} + \frac{a^2\left(1+\lambda_1\sigma^2\right)\tau Ra_s\left(\delta^2\tau\left(\delta_2^2-R_i\right)\sigma^2\right)}{\delta^2\tau+\sigma^2} \quad (35)$$

$$\Delta_2=\frac{\left(\tau\delta^2+\sigma^2\right)\left(\delta_1^2+\pi^2\xi T_a\right)\left[\left(1+\delta_2^2-R_i\right)-\lambda_1\left(\delta_2^2-R_i-\lambda_2\sigma^2\right)\right]}{a^2\left(1+\lambda_1^2\sigma^2\right)} + \frac{a^2\left(1+\lambda_1\sigma^2\right)\tau Ra_s\left(\delta^2\tau-\left(\delta_2^2-R_i\right)\right)}{\delta^2\tau+\sigma^2}$$

For oscillatory mode $\Delta_2=0$ and $\sigma_i\neq 0$, where σ is the oscillatory frequency which is not given for brevity.

We have the necessary expression for oscillatory Rayleigh number as:

$$Ra_T^{osc}=\Delta_1 \quad (36)$$

4. NONLINEAR STABILITY ANALYSIS

In this section, nonlinear stability has been studied using minimal truncated Fourier series. For simplicity, we consider only two dimensional rolls, so that all the physical quantities are independent of y. We introduce the stream function ψ as $u=\frac{\partial\psi}{\partial z}$, $w=-\frac{\partial\psi}{\partial x}$, then taking curl to eliminate pressure term from Eq.(2), to get

$$\left(1+\lambda_2\frac{\partial}{\partial t}\right)^2\left(\frac{\partial^2}{\partial x^2}+\frac{1}{\xi}\frac{\partial^2}{\partial z^2}\right)\psi+\left(1+\lambda_1\frac{\partial}{\partial t}\right)^2\xi T_a\frac{\partial^2\psi}{\partial z^2}=\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left(1+\lambda_2\frac{\partial}{\partial t}\right)\left[\tau Ra_S\frac{\partial S}{\partial x}-Ra_T\frac{\partial T}{\partial x}\right] \quad (37)$$

$$\left(\frac{\partial}{\partial t}-\left(\eta\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}-R_i\right)\right)T-\frac{\partial(\psi,T)}{\partial(x,z)}+\frac{\partial\psi}{\partial x}=0 \quad (38)$$

$$\left(\frac{\partial}{\partial t}-\tau\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)\right)S-\frac{\partial(\psi,S)}{\partial(x,z)}+\frac{\partial\psi}{\partial x}=0 \quad (39)$$

It is to be noted that the effect of nonlinearity is to distort the temperature and concentration fields through the interaction of ψ, with T and S. As a result a component of the form $Sin\,(2\pi z)$ will be generated. A minimal Fourier series which describes the finite amplitude convection is given by

$$\psi=M_0(t)\sin(ax)\sin(\pi z) \quad (40)$$

$$T=M_1(t)\cos(ax)\sin(\pi z)+M_2(t)\sin(2\pi z) \quad (41)$$

$$S=M_3(t)\cos(ax)\sin(\pi z)+M_4(t)\sin(2\pi z) \quad (42)$$

where the amplitudes $M_0(t)$, $M_1(t)$, $M_2(t)$, $M_3(t)$, $M_4(t)$ are functions of time and are to be determined. Substituting above expressions in Eqs. (37) - (39) and equating the like terms, the following set of nonlinear autonomous differential equations is obtained

$$\frac{dM_0}{dt}=D_1(\mathrm{t}) \quad (43)$$

$$\frac{dD_1}{dt}=-\frac{1}{\left(\lambda_2^2\delta_1^2+\pi^2\xi T_a\lambda_1^2\right)}\begin{bmatrix}\left(\delta_1^2+\pi^2\xi T_a\right)M_0+2\left(\lambda_1^2\delta_1^2+\pi^2\xi T_a\lambda_2^2\right)D_1\\ +aRa_TM_1-\tau aRa_SM_3+aRa_T\left(\lambda_1+\lambda_2\right)\frac{dM_1}{dt}\\ -\tau aRa_S\left(\lambda_1+\lambda_2\right)\frac{dM_3}{dt}+aRa_T\left(\lambda_1\lambda_2\right)\frac{d^2M_1}{dt^2}\\ -\tau aRa_S\left(\lambda_1\lambda_2\right)\frac{d^2M_3}{dt^2}\end{bmatrix} \quad (44)$$

$$\frac{dM_1}{dt} = -\left[aM_0 + \pi a M_0 M_2 + \left(\delta_2{}^2 - R_i\right)M_1\right] \quad (45)$$

$$\frac{dM_2}{dt} = -\left[\left(4\pi^2 - R_i\right)M_2 - \frac{\pi a}{2}M_0 M_1\right] \quad (46)$$

$$\frac{dM_3}{dt} = -\left[aM_0 + \pi a M_0 M_2 + \delta^2 \tau M_3\right] \quad (47)$$

$$\frac{dM_4}{dt} = -\left[4\pi^2 \tau M_4 - \frac{\pi a}{2}M_0 M_3\right] \quad (48)$$

Numerical method was used to solve the above nonlinear differential equation to find the amplitudes.

4.1 Steady Finite Amplitude Motions

For steady state finite amplitude convection, we have to set left hand side of the Eq. (43-48) equal to zero.

$$D_1(t) = 0 \quad (49)$$

$$\left(\delta_1{}^2 + \pi^2 \xi T_a\right)M_0 + aRa_T M_1 - \tau a Ra_S M_3 = 0 \quad (50)$$

$$aM_0 + \pi a M_0 M_2 + \left(\delta_2{}^2 - R_i\right)M_1 = 0 \quad (51)$$

$$\left(4\pi^2 - R_i\right)M_2 - \frac{\pi a}{2}M_0 M_1 = 0 \quad (52)$$

$$aM_0 + \pi a M_0 M_2 + \delta^2 \tau M_3 = 0 \quad (53)$$

$$4\pi^2 \tau M_4 - \frac{\pi a}{2}M_0 M_3 = 0 \quad (54)$$

On solving the above equations for the amplitudes, we obtain M_1, M_2, M_3, M_4 in terms of M_0 as

$$M_1 = -\frac{2a\left(4\pi^2 - R_i\right)M_0}{a^2 M_0{}^2 \pi^2 + 2\left(4\pi^2 - R_i\right)\left(\delta_2{}^2 - R_i\right)} \quad (55a)$$

$$M_2 = -\frac{a^2 \pi M_0{}^2}{a^2 M_0{}^2 \pi^2 + 2\left(4\pi^2 - R_i\right)\left(\delta_2{}^2 - R_i\right)} \quad (55b)$$

$$M_3 = -\frac{\left(8aM_0\tau\right)}{\left(a^2 M_0{}^2 + 8\delta^2\tau^2\right)} \quad (55c)$$

$$M_4 = -\frac{\left(a^2 M_0 \tau\right)}{\pi\left(a^2 M_0{}^2 + 8\delta^2\tau^2\right)} \quad (55d)$$

4.2 Steady Heat And Mass Transports

In the study of this type problem, quantification of heat and mass transport is very important in porous media. Let N_u and S_h be denoted as the rate of heat and mass transports per unit for the fluid phase known as Nusselt number and Sherwood number respectively, defined by

$$Nu = 1 + \left[\frac{\int_0^{\frac{2\pi}{a}} \frac{\partial T}{\partial z}dx}{\int_0^{\frac{2\pi}{a}} \frac{\partial T_b}{\partial z}dx}\right]_{z=0} \quad (56)$$

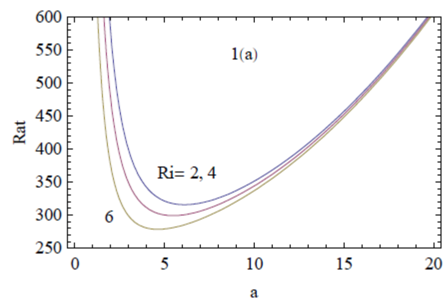
$$Sh = 1 + \left[\frac{\int_0^{\frac{2\pi}{a}} \frac{\partial S}{\partial z}dx}{\int_0^{\frac{2\pi}{a}} \frac{\partial S_b}{\partial z}dx}\right]_{z=0}$$

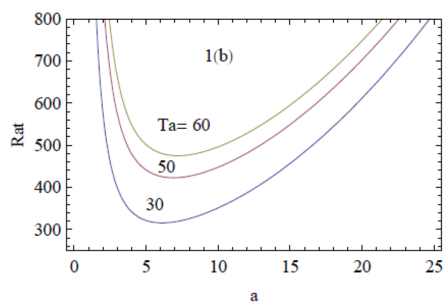
Substituting $M_2,\ M_4$ in (55a, 55b, 55c, 55d), the expressions for N_u and S_h are obtained as

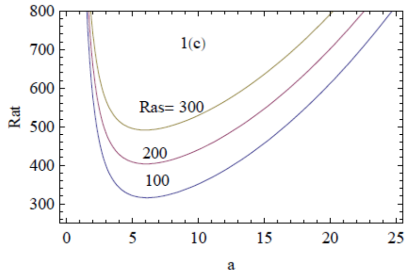
$$N_u = 1 - 2\pi M_2$$

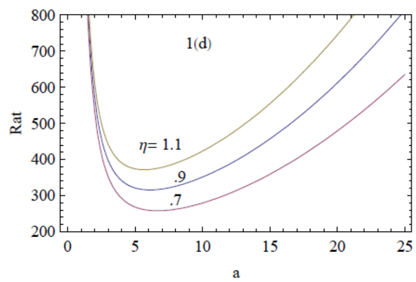
$$S_h = 1 - 2\pi M_4.$$

Figures









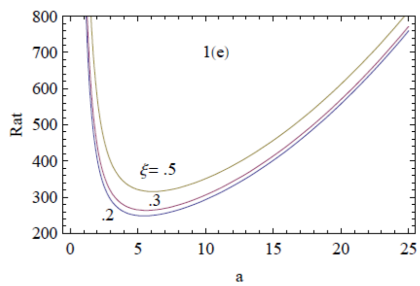
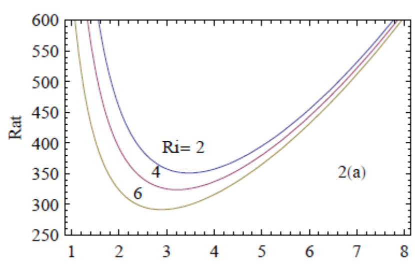
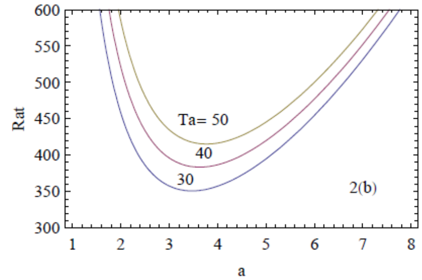
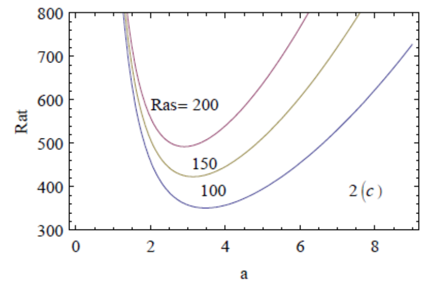
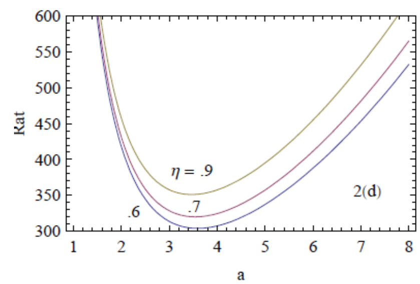


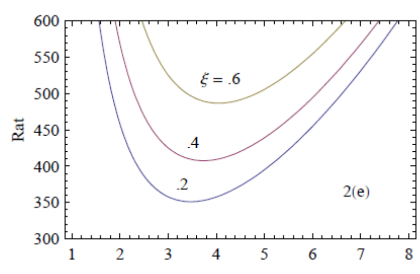
Fig1. Neutral stability curves for different values of different parameter











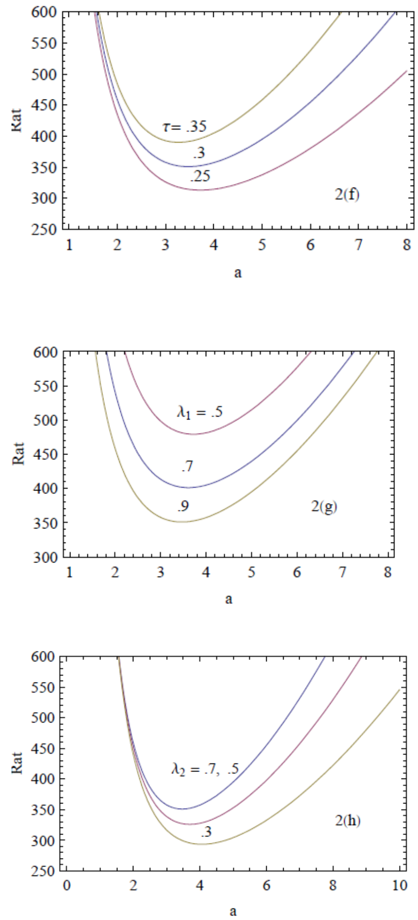
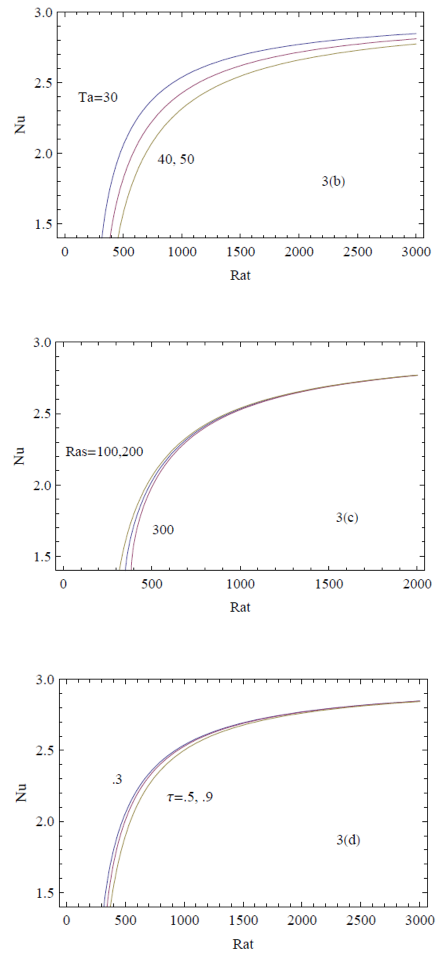
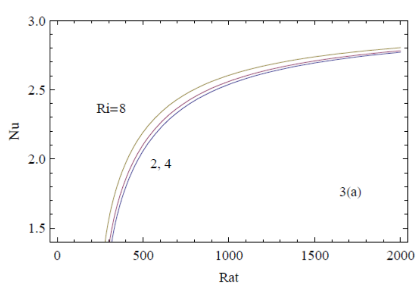
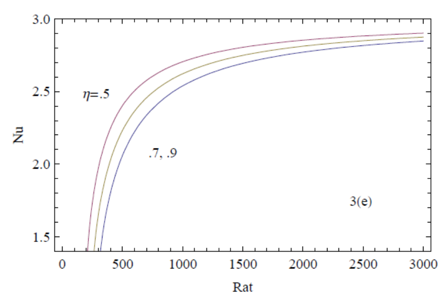


Fig 2. Ocsllatory stability curves for different values of different parameter







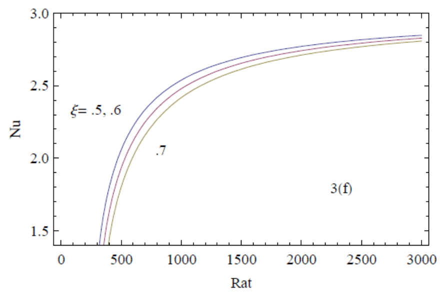
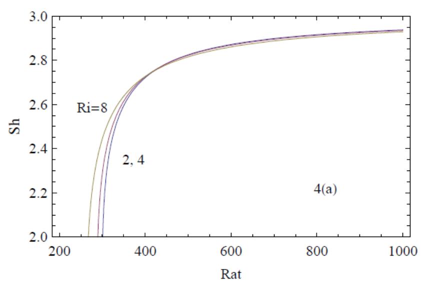
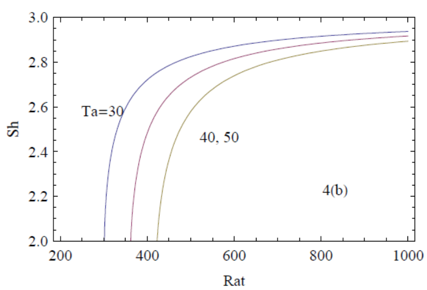
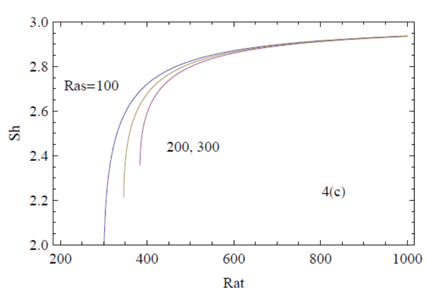
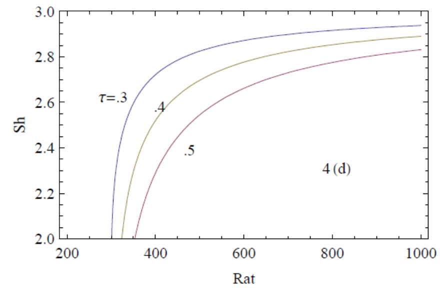


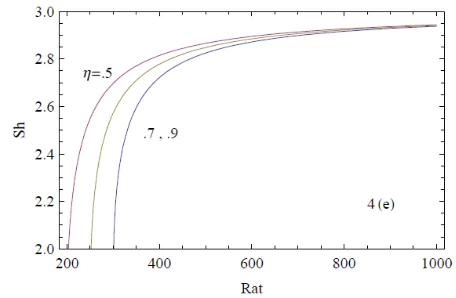
Fig 3.Nusselt number curves for different values of different parameter











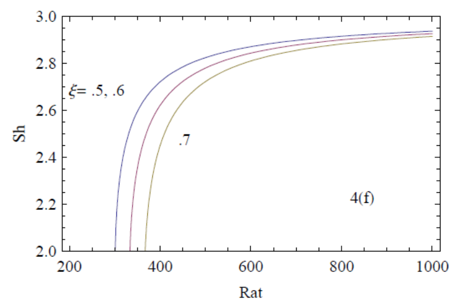


Fig 4.Sherwood number curves for different values of different parameter

5. RESULTS AND DISCUSSION

We have studied the effect of internal heat source on double diffusive convection in a viscoelastic fluid saturated rotating anisotropic porous layer using linear and nonlinear stability analyses. In this section, we obtain the effects of various parameters in the governing equations on the onset of double diffusive convection numerically and express them graphically. The numerical values of thermal Rayleigh number for stationary and oscillatory modes of convection for different values of the parameters such as Taylor number, relaxation and retardation parameters, solute

Rayleigh number, and parameter are computed, and depicted in figures.

Linear Stability

The marginal stability curves in the (Ra_T, a) plane for the stationary and oscillatory modes are presented through graphs for different values of the parameters. Figs 1(a-e) are for stationary mode, while Figs 2(a-h) correspond to oscillatory mode of convection. We fix the values for the parameters as $T_a = 30, \xi = .5$, $Ra_S = 100$, $\tau = .3$, $\lambda_2 = .9$, $\lambda_1 = .5$, $\eta = .9$ and $R_i = 2$, except the varying parameter.

From Figs 1(a), 2(a), it is observed that on increasing the value of internal Rayleigh number R_i, the critical values of stationary and oscillatory Rayleigh number decrease, thus destabilizing the system. This shows that the effect of an increment in the value of R_i, is to advance the onset of both stationary as well as oscillatory modes of convection. However, from Figs 1(b), 2(b) for Taylor number T_a, Figs 1(c), 2(c) for solutal Rayleigh number Ra_S, Figs 1(d), 2(d) for thermal anisotropic parameter η and Figs 1(e), 2(e) for mechanical anisotropic parameter ξ, it is observed respectively that on increasing the values of T_a, Ra_S, η, ξ the critical values of stationary and oscillatory Rayleigh numbers increase, thus stabilizing the system. This shows that the effect of increasing the values of T_a, Ra_S, η, ξ is to delay the onset of stationary and oscillatory convection.

Further, it is found from Figs 2(f, h) that the effect of increasing the values of diffusivity ratio τ and the parameter λ_2 is to increase the critical value of the oscillatory Rayleigh number, thus delaying the onset of oscillatory convection. However opposite effect is found in Fig 2(g), where an increment in the value of parameter λ_1 decreases the critical value of the oscillatory Rayleigh number, thus advancing the onset of oscillatory convection.

Nonlinear Stability

The effects of various parameters on the rate of heat and mass transfer are shown in Fig 3 and Fig 4 respectively. Figs 3(a) and 4(a) show that an increment in the value of the internal Rayleigh number R_i increases the values of both Nusselt number N_u and Sherwood number S_h, which is due to the fact that increasing the value of R_i advances the onset of convection. From Figs 3(b) and 4(b) for Taylor number T_a, Figs 3(c), 4(c) for solute Rayleigh number Ra_S, Figs 3(d), 4(d) for diffusivity ratio τ, Figs 3(e), 4(e) for thermal anisotropic parameter η, Figs 3(f), 4(f) for mechanical anisotropic parameter ξ, it is observed that on increasing the values of T_a, Ra_S, τ, η and ξ, the values of both Nusselt number N_u and Sherwood number S_h decrease, thus stabilizing the system.

6. CONCLUSIONS

In this paper, internal heating effect on double diffusive convection in a viscoelastic fluid saturated rotating anisotropic porous layer, which is heated and salted from below, is investigated. The problem has been solved analytically, performing linear and nonlinear analyses. Linear analysis is done using normal mode technique. Following conclusions are drawn:

1) The Taylor number T_a, mechanical anisotropic parameter ξ, solute Rayleigh number Ra_S and thermal anisotropic parameter η has a stabilizing effect on the both stationary and oscillatory convection.
2) The internal heat parameter R_i destabilizes the system in the stationary and oscillatory system.
3) The effects of diffusivity ratio τ and retardation parameter λ_2 have stabilizing effect on the oscillatory convection.
4) The relaxation parameter λ_1 has a destabilizing effect on the oscillatory convection.
5) The increasing the value of internal Rayleigh number R_i then increase the value of Nusselt number N_u i.e. increased heat transfer but increasing the value of mechanical anisotropic parameter ξ,Taylor number T_a ,solute Rayleigh number Ra_S, diffusivity ratio τ and thermal anisotropic parameter η decreases the value ofNusselt number N_u.
6) Mass transfer that is the value of Sherwood number increases on increasing the value of internal Rayleigh number R_i, while decreaseson increasing the values of mechanical anisotropic parameter ξ, Taylor

number T_a ,solute Rayleigh number Ra_S, diffusivity ratio τ and thermal anisotropic parameter η.

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Appendix

Latin symbols

a	wave number
d	depth of porous layer
g	Acceleration due to gravity
τ	Diffusivity ratio $\tau = \frac{\kappa_S}{\kappa_{Tz}}$
Ra_T	Thermal Rayleigh number $Ra_T = \frac{\beta_T g \Delta T K_z d}{\nu \kappa_{Tz}}$
Ra_S	Solute Rayleigh number $Ra_S = \frac{\beta_S g \Delta S K_z d}{\nu \kappa_{Tz}}$
K	permeability
T	temperature
S	solute concentration
ΔT	Temperature difference across the porous layer
ΔS	Solute difference across the porous layer
t	time
p	reduced pressure
q	Fluid velocity (u,v,w)
Pr_D	Prandtl number $Pr_D = \frac{\phi \gamma \nu d^2}{\kappa_T k}$
R_i	Internal Rayleigh number $R_i = \frac{Qd^2}{\kappa_T}$
T_a	Taylor number $T_a = \left(\frac{2\Omega K_z}{\mu \phi}\right)^2$
Q	Internal heat source
N_u	Nusselt number

S_h	Sherwood number
(x,y,z)	Space co-ordinates

Greek symbols

κ_T	Effective thermal diffusivity $\kappa_{Tx}(ii+jj)+\kappa_{Tz}(kk)$
κ_{Tx}	Effective thermal diffusivity in x-direction
κ_{Tz}	Effective thermal diffusivity in direction
β_T	Coefficient of thermal expansion
β_S	Coefficient of solute expansion
$\overline{\lambda_1}$	Stress-relaxation time
$\overline{\lambda_2}$	Strain-retardation time
λ_1	Relaxation parameter $\left(\frac{\kappa_{Tz}}{\gamma d^2}\right)\overline{\lambda_1}$
λ_2	Retardation parameter $\left(\frac{\kappa_{Tz}}{\gamma d^2}\right)\overline{\lambda_2}$
T_0	Reference temperature
S_0	Reference concentration
σ	Growth rate
μ	Dynamic viscosity of the fluid
μ_c	Effective viscosity of the fluid
ϕ	Porosity
γ	Heat capacities ratio $\frac{(\rho c_p)_m}{(\rho c_p)_f}$
ν	Kinematic viscosity $\frac{\mu}{\rho_0}$
ρ	Fluid density
ρ_0	Reference density

Other symbols

∇_1^2	$\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$
∇^2	$\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$

Subscripts

b	basic state
c	critical
0	reference value

Superscripts

'	perturbed quantity
*	Dimensionless quantity
osc	oscillatory
st	stationary

Internal heating and Soret effect on Darcy - Brinkman convection in a binary viscoelastic fluid saturated porous layer

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***Abstract*—Linear and nonlinear analyses have been done and the combine effect of internal heating and Soret effect on Darcy - Brinkman convection in a binary viscoelastic fluid saturated porous layer, heated and salted from below, has been studied, analytically. Linear stability analysis has been performed by using normal mode technique and nonlinear analysis is done using truncated Fourier series. The modified Darcy-Brinkman-Oldroyd model, including the time derivative term, is employed for the momentum equation. The effects of Darcy number, Soret parameter, relaxation and retardation parameters, solute Rayleigh number, internal heat source, Lewis number and Darcy-Prandtl number on stationary and oscillatory convection are shown graphically. Also heat and mass transports are calculated in terms of the Nusselt number and Sherwood number and presented graphically.**

***Index Terms*— Double diffusive convection; Viscoelastic fluid; Internal heat source; Soret parameter; Porous media.**

I. INTRODUCTION

There is large number of practical situations in which convection is driven by internal heat source in a porous medium. The wide applications of such convection occur in nuclear reactions, nuclear heat cores, nuclear energy, nuclear waste disposals, oil extractions, and crystal growth. The study concerning internal heat source in porous media is provided by Tveitereid [1], who obtained the steady solution in the form of hexagons and two dimensional rolls for convection in a horizontal porous layer with internal heat source. Horton and Rogers [2] and Lapwood [3] were the first to obtained analytically the expression for critical Rayleigh number for the onset of convection in a fluid-saturated porous layer heated from below. Bejan [4] studied analytically the buoyancy induced convection with internal heat source, Parthiban and Patil [5] studied the effect of non-uniform boundaries temperature on thermal instability in a porous medium with internal heat source and predicted that internal heat source parameter advances the onset of convection. Hill [6] performed linear and nonlinear analyses on the double-diffusive convection in a porous layer with a concentration based internal heat source. Bhadauria et al. [7]-[8] studied effect of internal heating on double diffusive convection in a couple stress fluid saturated anisotropic porous medium and also natural convection in a rotating anisotropic porous layer with internal heat source. Khan and Aziz [9] studied transient heat transfer in a heat-generating fin with radiation and convection with temperature-dependent heat transfer coefficient.

Further, there are many studies available on the effect of cross-diffusion on onset of double-diffusive convection in a porous medium. Thermal convection in a binary fluid driven by the Soret and Dufour effects has been investigated by Knobloch [10]. Hurle and Jakeman [11] performed a theoretical study of Soret driven thermosolutal convection in a binary fluid mixture. Linear and nonlinear analyses of double diffusive convection in a fluid saturated porous layer with cross-diffusion effects has been carried out by Malashetty and Biradar [12]. Rudraiah and Malashetty [13] carried out a study on double diffusive convection in a porous medium in the presence of Soret and Dufour effects, while Gaikwad et. al. [14]-[16] performed a linear and nonlinear double diffusive convection in a fluid-saturated anisotropic porous layer with cross-diffusion and obtained the effect of cross diffusion coefficients. Bhadauria, Hashim et al. [17] investigated the double diffusive convection in a fluid saturated anisotropic porous layer with Soret effect and internal heat source. Rudraiah, Siddheshwar [18] did a weak nonlinear stability analysis of double diffusive convection with cross diffusion in a fluid saturated porous medium and obtained some very interesting results.

Convection in binary fluids is a complex process. The presence of concentration currents as well as thermal currents leads to linear and nonlinear behavior. In a binary fluid, the density depends on both temperature and solute concentration. This leads to a competition between heat diffusion and solute diffusion, and consequently oscillatory motions may occur. The oscillatory convective instability in binary fluid mixtures is well understood by Platten and Legros [19]. Taslim and Narusawa [20] investigated binary fluid composition and double diffusive convection in a porous medium. Further, the studies of double diffusive convection in porous media plays very significant roles in many areas such as in petroleum industry, solidification of binary mixture, migration of solutes in water saturated soils. Other examples include; geophysics system, crystal growth, electrochemistry, the migration of moisture through air contained in fibrous insulation, Earth's oceans, magma chambers etc. The studies on double diffusive convection in a porous media has been presented in details by Ingham and Pop [21], Nield and Bejan [22] and vafai [23]-[24] and Vadasz [25] in their books. Further, it was performed by many other researchers, namely;

Poulikakos [26], Travison and Bejan [27], Momou [28] etc. The very first study on double diffusive convection in porous media was mainly concerned with linear stability analysis, and was performed by Nield [29].

It is well known that the Darcy's law is not valid for non-Newtonian fluid flows in porous media. Swamy et al [30] studied the onset of Darcy-Brinkman convection in a binary viscoelastic fluid saturated porous layer, where the modified Darcy-Brinkman-Oldroyd model has been developed. However, published works on thermal convection of viscoelastic fluids in porous media are fairly limited. Rudraiah et al. [31] have studied the thermal stability of a viscoelastic fluid saturated sparsely packed porous layer. Kim et al. [32] studied the thermal instability of viscoelastic fluids in a porous medium by performing linear and nonlinear analyses. Yoon et al. [33] analyzed the onset of thermal convection in a horizontal porous layer saturated with a viscoelastic liquid using a linear theory. Zhang et al. [34] carried out linear and nonlinear analyses of thermal convection for Oldroyd-B fluids in porous media, heated from below. Gaikwad and Kouser [35] investigated the onset of Darcy-Brinkman convection in a binary viscoelastic fluid saturated porous layer with internal heat source. Gaikwad and Dhanraj [36] studied Soret effect on Darcy-Brinkman convection in a binary viscoelastic fluid-saturated porous layer and studied the cross diffusion effects on convective instability. Stability analysis of Soret-driven double diffusive convection of a Maxwell fluid in a porous medium has been investigated by Wang and Tan [37]. Narayana et al. [38] performed linear and nonlinear stability analysis of binary Maxwell fluid convection in a porous medium with Soret and Dufour effects. Rudraiah et al. [39] have studied the stability of a viscoelastic fluid saturated sparsely packed porous layer. Malashetty et al. [40] have investigated the onset of convection in a binary viscoelastic fluid saturated porous layer. Kumar and Bhadauria [41] performed stability analysis to study thermal instability in a rotating anisotropic porous layer saturated by a viscoelastic fluid.

Malashetty et al. [42] did an analytical study of linear and nonlinear double diffusive convection with soret effect in couple stress liquids. More recently, Gaikwad and Kamble [43] have studied theoretically, the cross diffusion effects on convective instability in porous media and Gaikwad et al. [44] have performed a study on double diffusive convection in a binary viscoelastic fluid saturated porous layer with Soret effect and internal heat source. Therefore, in the present paper, we have carried out linear and nonlinear stability analyses and studied the effect of internal heat and Soret parameter on Darcy-Brinkman convection in a binary viscoelastic fluid saturated porous layer.

2. MATHEMATICAL FORMULATION

Consider a viscoelastic fluid saturated porous layer, confined between two infinitely extended horizontal planes at $z=0$ and $z=d$ heated from below and cooled from above. An internal heat source term has been included in the energy equation. A cartesian frame of reference is chosen in such a way that the origin lies on the lower plane and the z-axis as vertical upward. An adverse temperature gradient is applied across the porous layer and the lower and upper planes are kept at temperature $T_0+\Delta T$, and T_0 with concentration $S_0+\Delta S$ and S_0 respectively. The governing equations are as given

$$\nabla . q = 0 \tag{1a}$$

$$\left(1+\overline{\lambda_1}\frac{\partial}{\partial t}\right)\left(\frac{\rho_0}{\varepsilon}\frac{\partial q}{\partial t}+\nabla p-\rho g\right)=\left(1+\overline{\lambda_2}\frac{\partial}{\partial t}\right)\left(\mu_c\nabla^2 q-\frac{\mu}{\kappa}q\right) \tag{1b}$$

$$\gamma\frac{\partial T}{\partial t}+\left(q\nabla\right)T=K_{11}\nabla^2 T+Q\left(T-T_0\right) \tag{1c}$$

$$\varepsilon\frac{\partial S}{\partial t}+\left(q\nabla\right)S=K_{22}\nabla^2 S+K_{21}\nabla^2 T \tag{1d}$$

$$\rho=\rho_0\left[1-\beta_T\left(T-T_0\right)+\beta_S\left(S-S_0\right)\right] \tag{1e}$$

where the physical variables have their usual meanings as given in the nomenclature. The externally imposed the thermal and solutal boundary conditions are given by

$$\begin{aligned} &T=T_0+\Delta T;\ \ at\ \ z=0\ \ and\ T=T_0;\ \ at\ z=d;\\ &S=S_0+\Delta S;\ \ at\ z=0\ \ and\ \ S=S_0;\ \ at\ z=d; \end{aligned} \tag{2}$$

2.1. BASIC SOLUTION

At this state the velocity, pressure, temperature and density profiles are given by

$$q_b=0, p=p_b(z), T=T_b(z), S=S_b(z), \rho=\rho_b(z). \tag{3}$$

Substituting Eq. (3) in Eq. (1a-1e), we get the following relations:

$$\frac{dp_b}{dz}=-\rho_b g, \tag{4}$$

$$K_{11}\frac{d^2T_b}{dz^2}+Q(T_b-T_0)=0, \tag{5}$$

$$\frac{d^2S_b}{dz^2}=0, \tag{6}$$

$$\rho_b=\rho_0\left[1-\beta_T(T_b-T_0)+\beta_S(S_b-T_0)\right]. \tag{7}$$

The solution of Eq. (5), subject to the boundary condition (2),is given by

$$T_b=T_0+\Delta T\,\frac{sin\sqrt{R_i}\left(1-\frac{z}{d}\right)}{sin\sqrt{R_i}}. \tag{8}$$

The solution of Eq. (5), subject to the boundary condition (2),

$$S_b=S_0+\Delta S\left(1-\frac{z}{d}\right) \tag{9}$$

Now, we superimpose finite amplitude perturbations on the basic state in the form:

$$q=q_b+q',\ T=T_b+T',\ p=p_b+p',\ S=S_b+S',\ \rho=\rho_b+\rho', \tag{10}$$

We get the following set of equations:

$$\nabla.q'=0$$

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left(\frac{\rho_0}{\varepsilon}\frac{\partial q'}{\partial t}+\nabla p'-\rho(\beta_T T'-\beta_S S')g\right)=\left(1+\lambda_2\frac{\partial}{\partial t}\right)\left(\mu_c\nabla^2 q'-\frac{\mu}{\kappa}q'\right)$$

$$\gamma\frac{\partial T'}{\partial t}+(q'.\nabla)T'+w'\frac{\partial T_b'}{\partial z}=K_{11}\nabla^2T'+QT'$$

$$\varepsilon\frac{\partial S'}{\partial t}+(q'.\nabla)S-w'\frac{\Delta S}{d}=K_{22}\nabla^2S'+K_{21}\nabla^2T'$$

$$\rho'=-\rho_0\left[\beta_T T'+\beta_S S'\right]$$

Infinitesimal perturbation was applied to the basic state of the system and then the pressure term was eliminating by taking curl twice of Eq. (1b). The above resulting equations are non-dimensionalized using the following transformations,

$$(x',y',z')=(x^*,y^*,z^*)d,\ t'=t^*(\frac{\gamma d^2}{K_{11}}),\ q=\frac{K_{11}}{d}q^*,\ (u,v,w)=(u^*,v^*,w^*)(\frac{K_{11}}{d}),\ T'=(\Delta T)T^*,\ S'=(\Delta S)S^* \tag{11}$$

T_b , S_b in dimensionless forms are given as

$$T_b=(1-z),\ S_b=(1-z) \tag{12}$$

The non dimensionalized equations (on dropping the asterisks for simplicity) are

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left(\frac{1}{Pr_D}\frac{\partial}{\partial t}\nabla^2w-Ra_T\nabla_1^2T+Ra_S\nabla_1^2S\right)-\left(1+\lambda_2\frac{\partial}{\partial t}\right)\left(D_a\nabla^4w-\nabla^2w\right)=0 \tag{13}$$

$$\left[\frac{\partial}{\partial t}-\nabla^2-R_i+q.\nabla\right]T-w=0$$

(14)

$$\left[\varepsilon_n\frac{\partial}{\partial t}-\nabla^2\frac{1}{L_e}\right]S+(q.\nabla)S-S_r\nabla^2T-w=0 \tag{15}$$

where $Pr_D=\frac{\varepsilon\gamma\nu d^2}{K_{11}K}$ is Darcy-Prandtl number, $Ra_T=\frac{\beta_T g\Delta SKd}{\nu K_{11}}$ is the thermal Rayleigh number, $Ra_S=\frac{\beta_S g\Delta SKd}{\nu K_{11}}$ is the solute Rayleigh number, $R_i=\frac{Qd^2}{K_{11}}$ is the internal Rayleigh parameter, $\lambda_1=(\frac{K_{11}}{\gamma d^2})\overline{\lambda_1}$ is relaxation parameter , $\lambda_2=(\frac{K_{11}}{\gamma d^2})\overline{\lambda_2}$ is retardation parameter, $L_e=\frac{K_{11}}{K_{22}}$ is Lewis number, $S_r=\frac{K_{21}\Delta T}{K_{11}\Delta S}$ the soret parameter, $\varepsilon_n=\frac{\varepsilon}{\gamma}$ normalized porosity. The above system will be solved by considering stress free and isothermal boundary conditions as given below:

$$w=\frac{\partial^2w}{\partial z^2}=T=S=0\ \text{ at }\ z=0, z=1. \tag{16}$$

3. LINEAR STABILITY ANALYSIS

In order to do the linear stability analysis of the system, Eq. (13)-(15) subject to the boundary condition given in Eq.(16), we use time dependent periodic disturbance in horizontal plane as

$$(w,T,S)=(W,\Theta,\phi)exp\left(i\left(lx+my\right)+\sigma t\right), \quad (17)$$

Where α are horizontal wave number and $\sigma=\sigma_r+i\sigma_j$ is growth rate. Substituting eq. (17) into the lineralized eq. (13)-(15). We obtain

$$(1+\lambda_1\sigma)\left(\frac{\sigma}{\Pr_D}\left(D^2-a^2\right)W+a^2Ra_T\Theta-a^2Ra_S\phi\right)-(1+\lambda_2\sigma)\left(D^2-a^2\right)\left[D_a\left(D^2-a^2\right)^2-1\right]W=0 \quad (18)$$

$$\left[\sigma-(\mathrm{D}^2 - \mathrm{a}^2) - \mathrm{Ri}\right]\Theta-\mathrm{W}=0 \quad (19)$$

$$\left[\varepsilon_n\sigma+\frac{D^2-a^2}{L_e}\right]\phi-W-(D^2-a^2)S_r\Theta=0. \quad (20)$$

Where $a^2=l^2+m^2$. The boundary conditions (16) are now

$$W=\frac{\partial^2 W}{\partial z^2}=\Theta=\phi=0 \;\; on \;\; z=0, z=1. \quad (21)$$

We assume the solution W,Θ,ϕ as

$\left(W,\Theta,\phi\right)=\left(W_0,\Theta_0,\phi_0\right)\mathrm{Sin}\,n\pi z$ (n=1,2,3........),

The most unstable mode corresponds to n = 1 (fundamental mode). Therefore, substituting Eq. (21) with n = 1 into Eq. (18)-(20), we obtain a matrix form $A\,x=0$ as

$$\begin{pmatrix} \left(\frac{\sigma}{\Pr_D}+\Lambda\left(D_a\delta^2+1\right)\right)\delta^2 & -a^2Ra_T & a^2Ra_S \\ -1 & \left(\sigma+\delta^2-R_i\right) & 0 \\ -1 & S_r\delta^2 & \varepsilon_n\sigma+\frac{\delta^2}{Le} \end{pmatrix}\begin{pmatrix} W_0 \\ \Theta_0 \\ \Phi_0 \end{pmatrix}=0. \quad (22)$$

The thermal Rayleigh number can be expressed as

$$Ra_T=\frac{\delta^2}{a^2}\left(\frac{\sigma}{Pr_D}+\wedge(D_a\delta^2+1)\right)(\sigma+\delta^2-R_i)+\frac{\sigma-R_i+\delta^2(1-S_r)}{\varepsilon_n\sigma+\frac{\delta^2}{L_e}}Ra_S$$

(23)

Where $\delta^2=\pi^2+a^2, \Lambda=\frac{1+\lambda_2\sigma}{1+\lambda_1\sigma}$. The growth rate σ is in general a complex quantity such that $\sigma=\sigma_r+i\;\sigma_i$. The system with $\sigma_r<0$ is always stable, while for $\sigma_r>0$ it will become unstable. For neutral stability state $\sigma_r=0$

3.1. STATIONARY STATE

We now set $\sigma=0$ at the margin of stability. The expressed for the thermal Rayleigh number of the system for a stationary mode of convection is as given below:

$$Ra_T^{st}=\frac{\pi^2+a^2}{a^2}(1+D_a\delta^2)(\delta^2-R_i)+\frac{\left(\delta^2(1-S_r)-R_i\right)L_e}{\delta^2}Ra_S,$$

(24)

It is important to note that the critical wave number $a=a_c^{St}$, where $a_c^{St}=\sqrt{S}$ satisfied the following equation

$$2D_aS^3+(3D_a\pi^2+1)S^2-\pi^4(D_a\pi^2+1)=0 \quad (25)$$

In the absence of Soret effect i.e. $S_r=0$ Eq. (24) becomes

$$Ra_T^{st} = \frac{\pi^2 + a^2}{a^2}(1 + D_a\delta^2)(\delta^2 - R_i) + \frac{(\delta^2 - R_i)L_e}{\delta^2} Ra_S. \quad (26)$$

For the system without internal-heating, i.e., $R_i = 0$ we get

$$Ra_T^{st} = \frac{(\pi^2 + a^2)^2}{a^2}(1 + D_a\delta^2) + L_e Ra_S \quad (27)$$

This is exactly the same as obtained by Swamy et al. [30]. When $D_a \to 0$

$$Ra_T^{st} = \frac{(\pi^2 + a^2)^2}{a^2} + L_e Ra_S \quad (28)$$

In case of single component fluid, the Solutal Rayleigh number is zero i.e. $Ra_S = 0$, we have

$$Ra_T^{st} = \frac{(\pi^2 + a^2)^2}{a^2} \quad (29)$$

Which is the classical result obtained by Horton and Rogers [2] and Lapwood [3] for single component fluid in porous layer.

3.2. OSCILLATORY STATE

We set $\sigma = i\sigma_i$ in Eq. (23) and clear the complex quantities from the denominator, to obtain

$$Ra_T^{osc} = \Delta_1 + i\sigma_i\Delta_2. \quad (30)$$

Where

$$\Delta_1 = \frac{\delta^2}{a^2}\left[\left(\delta^2 - R_i\right)\left(D_a\delta^2 + 1\right)\left(\frac{1 + \lambda_1\lambda_2\sigma^2}{1 + \lambda_1{}^2\sigma^2}\right) - \sigma^2\left(\frac{1}{\Pr_D} + \frac{\left(D_a\delta^2 + 1\right)\left(\lambda_2 - \lambda_1\right)}{1 + \lambda_1{}^2\sigma^2}\right)\right] + \frac{\varepsilon_n\sigma^2 + \delta^4 L_e{}^{-1}(1 - S_r)\delta^2 L_e{}^{-1}R_i}{\left(\delta^2 L_e{}^{-1}\right)^2 + \varepsilon_n{}^2\sigma^2} Ra_S$$

$$\Delta_2 = \frac{\delta^2}{a^2}\left[\left(\delta^2 - R_i\right)\left(\frac{1}{\Pr_D} + \frac{\left(D_a\delta^2 + 1\right)\left(\lambda_2 - \lambda_1\right)}{1 + \lambda_1{}^2\sigma^2}\right) + \left(D_a\delta^2 + 1\right)\left(\frac{1 + \lambda_1\lambda_2\sigma^2}{1 + \lambda_1{}^2\sigma^2}\right)\right] + \frac{\delta^2 L_e{}^{-1} - \varepsilon_n\left(\delta^2\left(1 - S_r\right) - R_i\right)}{\left(\delta^2 L_e{}^{-1}\right)^2 + \varepsilon_n{}^2\sigma^2} Ra_S.$$

For oscillatory mode $\Delta_2 = 0$ and $\sigma_i \neq 0$, which is not given for brevity. The thermal Rayleigh number for oscillatory mode is given as:

$$Ra_T^{osc} = \Delta_1 \quad (31)$$

4. NONLINEAR STABILITY ANALYSIS

In this section, we study the nonlinear stability analysis using minimal truncated Fourier series. For simplicity, we confine ourself only to two dimensional rolls, so that all the physical quantities are independent of y. Introducing the stream function ψ as $u = \frac{\partial\psi}{\partial z}$, w= $-\frac{\partial\psi}{\partial x}$ and taking curl of Eq. (1 b) to eliminate pressure term we get

$$\left(1 + \lambda_1\frac{\partial}{\partial t}\right)\left(\frac{1}{Pr_D}\frac{\partial}{\partial t}\nabla^2\psi + Ra_T\frac{\partial T}{\partial x} - Ra_S\frac{\partial S}{\partial x}\right) = \left(1 + \lambda_2\frac{\partial}{\partial t}\right)\left(D_a\nabla^4\psi - \nabla^2\psi\right) \quad (32)$$

$$\left(\frac{\partial}{\partial t} - \nabla^2 - R_i\right)T - \frac{\partial(\psi, T)}{\partial(x, z)} + \frac{\partial\psi}{\partial x} = 0 \quad (33)$$

$$\left(\varepsilon_n\frac{\partial}{\partial t} - L_e{}^{-1}\nabla^2\right)S - \frac{\partial(\psi, S)}{\partial(x, z)} + \frac{\partial\psi}{\partial x} - S_r\nabla^2 T = 0 \quad (34)$$

It is to be noted that the effect of nonlinearity is to distort the temperature concentration fields through the interaction of ψ and T , ψ and S . As a result a component of the form $Sin(2\pi z)$ will be generated. A minimal Fourier series which describes the finite amplitude convection is given by

$$\psi = A_1(t)sin(ax)sin(\pi z), \quad (35)$$

$$T = A_2(t)cos(ax)sin(\pi z) + A_3(t)sin(2\pi z), \quad (36)$$

$$S = A_4(t)cos(ax)sin(\pi z) + A_5(t)sin(2\pi z), \quad (37)$$

Where the amplitudes $A_1(t)$, $A_2(t)$, $A_3(t)$, $A_4(t)$, $A_5(t)$ are functions of time and are to be determined. Substituting above expressions in Eq. (13)-(15) and equating the like terms, the following set of nonlinear autonomous differential equations were obtained

$$\frac{dA_1}{dt} = B \tag{38}$$

$$\frac{dB}{dt} = -\frac{Pr_D}{\delta^2\lambda_1}\left[\left(\frac{\delta^2}{Pr_D} + \lambda_2 D_a\delta^4 + \delta^2\lambda_2\right)B + \delta^2\left(1 + D_a\delta^2\right)A_1 + aRa_T A_2 - aRa_S A_4 + a\lambda_1 Ra_T\frac{dA_2}{dt} - a\lambda_1 Ra_S\frac{dA_4}{dt}\right] \tag{39}$$

$$\frac{dA_2}{dt} = -[aA_1 + (\delta^2 - R_i)A_2 + \pi a A_1 A_3] \tag{40}$$

$$\frac{dA_3}{dt} = (R_i - 4\pi^2)A_3 + \frac{\pi a}{2}A_1 A_2 \tag{41}$$

$$\frac{dA_4}{dt} = -\frac{1}{\varepsilon_n}(aA_1 + L_e^{-1}\delta^2 A_4 + \pi a A_1 A_5 + S_r\delta^2 A_2) \tag{42}$$

$$\frac{dA_5}{dt} = -\frac{1}{\varepsilon_n}(4\pi^2 L_e^{-1}A_5 - \frac{\pi a}{2}A_1 A_4 + 4\pi^2 S_r A_3) \tag{43}$$

4.1. STEADY FINITE AMPLITUDE MOTIONS

We set $\frac{\partial}{\partial t} = 0$, the above system becomes

$$B=0 \tag{44}$$

$$\delta^2(1 + D_a\delta^2)A_1 + aRa_T A_2 - aRa_S A_4 = 0 \tag{45}$$

$$aA_1 + (\delta^2 - R_i)A_2 + \pi a A_1 A_3 = 0 \tag{46}$$

$$(R_i - 4\pi^2)A_3 + \frac{\pi a}{2}A_1 A_2 = 0 \tag{47}$$

$$aA_1 + L_e^{-1}\delta^2 A_4 + \pi a A_1 A_5 + S_r\delta^2 A_2 = 0 \tag{48}$$

$$4\pi^2 L_e^{-1}A_5 - \frac{\pi a}{2}A_1 A_4 + 4\pi^2 S_r A_3 = 0 \tag{49}$$

Numerical method was used to solve the above nonlinear differential equation to find the amplitudes. On solving for the amplitudes in terms of A_1 ,we obtain A_2, A_3, A_4, A_5 .

4.2. STEADY HEAT AND MASS TRANSPORTS

In the study of this type problem, quantification of heat and mass transport is very important.

If H and J are the rate of heat and mass transport per unit area,then

$$H = -K_{11}\left\langle\frac{\partial T_{total}}{\partial z}\right\rangle_{z=0} \tag{50}$$

$$J = -K_{21}\left\langle\frac{\partial T_{total}}{\partial z}\right\rangle_{z=0} - K_{22}\left\langle\frac{\partial S_{total}}{\partial z}\right\rangle_{z=0} \tag{51}$$

Where the angular bracket corresponds to a horizontal average and

$$T_{total} = T_0 - \Delta T\frac{z}{d} + T(x,z,t) \tag{52}$$

$$S_{total} = S_0 - \Delta S\frac{z}{d} + S(x,z,t) \tag{53}$$

Substituting Eq. (36)-(37) into Eq. (53) and using the resultant Eq. (50), (51) we get

$$H = \frac{K_{11}\Delta T}{d}(1-2\pi A_3) \quad (54)$$

$$J = \frac{K_{22}\Delta S}{d}\left[(1-2\pi A_5) + S_r L_e (1-2\pi A_3)\right] \quad (55)$$

The Nusselt number and Sherwood number , which denotes the rate of heat and mass transports respectively, are defined by

$$Nu = \frac{H}{\frac{K_{11}\Delta T}{d}} = (1-2\pi A_3) \quad (56)$$

$$Sh = \frac{J}{\frac{K_{22}\Delta S}{d}} = (1-2\pi A_5) + S_r L_e (1-2\pi A_3) \quad (57)$$

Using the expressions Eq.(54)-(55), and substituting A_3, A_5 into Eq. (56,57) ,finally the expressions for N_u , S_h are obtained.

5. RESULTS AND DISCUSSION

This paper investigates the combined effect of internal heating and Soret parameter on stationary and oscillatory convection in a porous medium saturated with a binary viscoelastic fluid and discusses the effects of various parameters on the onset of double diffusive convection. The expressions for the stationary and oscillatory modes of convection for different values of the parameters such as Prantdl number, relaxation parameter, retardation parameter, solute Rayleigh number, Lewis number, Soret parameter and Darcy number are computed, and the results are depicted in figures. The neutral stability curves in the (Ra_T, a) plane for various parameter values are as shown in Fig. 1 and Fig. 2. We fixed the values for the parameters as $\Pr_D = 10$, $D_a = .1$, $R_i = 3$ $\lambda_1 = .8$, $\lambda_2 = .1$, $Ra_S = 100$, $L_e = 2$, $S_r = .05$, and except the varying parameter.

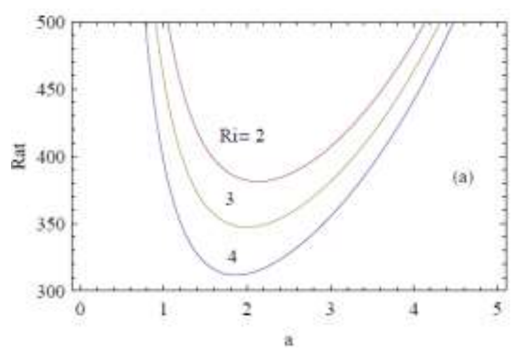

Fig. 1: Variation of Stationary rayleigh number with wave number for the different values of *Ri*

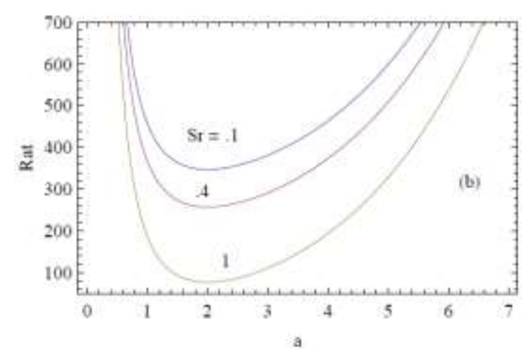

Fig. 1: Variation of Stationary rayleigh number with wave number for the different values of *Sr*

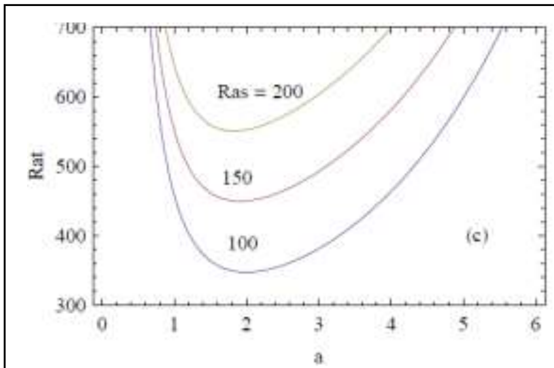

Fig. 1: Variation of Stationary rayleigh number with wave number for the different values of *Ras*

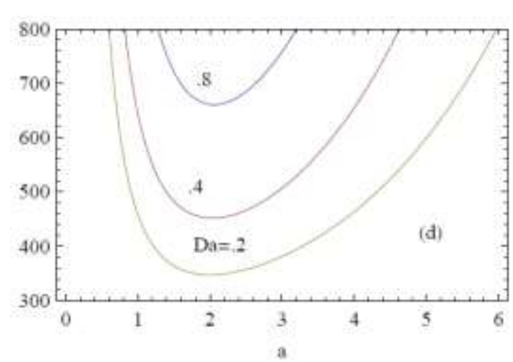

Fig. 1: Variation of Stationary rayleigh number with wave number for the different values of D_a

From Figs.1, 2(a), it is observed that increasing the value of internal heat source R_i, decreases the values of stationary and oscillatory Rayleigh number, which means that the effect of increasing the internal heat source R_i is to destabilize the system. In Figs.1, 2(b), the effect of Soret parameter (S_r) is depicted, respectively for both stationary and oscillatory convection. It is found that an increment in the value of Soret parameter decreases the value of Rayleigh numbers for both stationary and oscillatory mode of convection, thus onset of convection takes place at an early point. Figs.1, 2(c) depicts the effect of solute Rayleigh number Ra_S on the onset of convection. We find that the effect of increasing the value of Ra_S is to increase the value of Rayleigh number Ra_T thus stabilizing the system in both stationary and oscillatory modes. Further, Figs.1, 2(d) show that the effect of increasing the Darcy number, D_a is to increase the value of Rayleigh number Ra_T, thus stabilizing the system that is the onset of convection will take place at a later point. However, the effect of increasing the Lewis number L_e is found to increase the value of Rayleigh number for stationary mode and decrease the value for oscillatory modes, thus to stabilize the stationary mode of convection and destabilize the oscillatory convection [Figs.1, 2(e)].

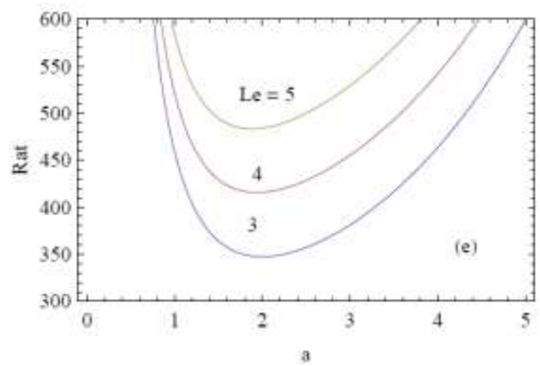

Fig. 1: Variation of Stationary rayleigh number with wave number for the different values of *Le*

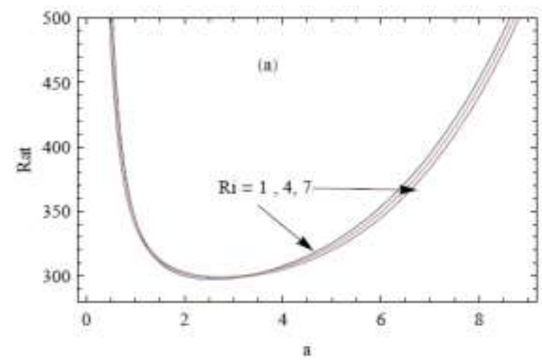

Fig. 2: Variation of Oscillatory rayleigh number with wave number for the different values of *Ri*

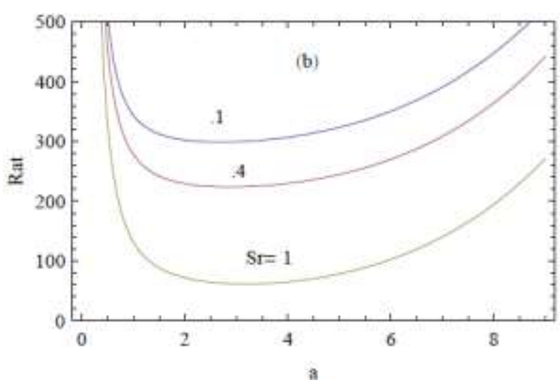

Fig. 2: Variation of Oscillatory rayleigh number with wave number for the different values of *Sr*

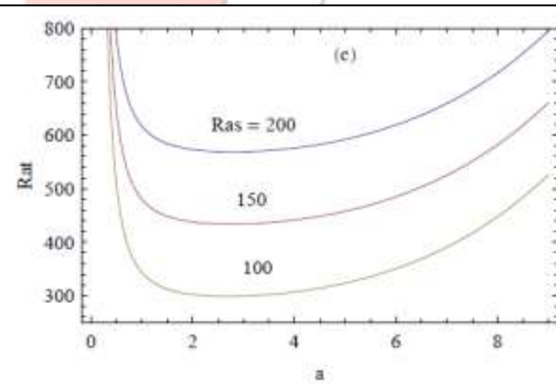

Fig. 2: Variation of Oscillatory rayleigh number with wave number for the different values of *Ras*

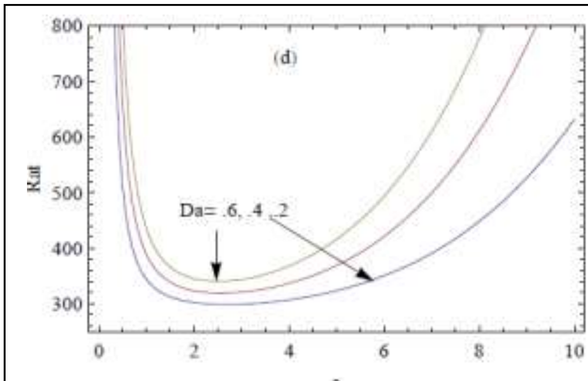

Fig. 2: Variation of Oscillatory rayleigh number with wave number for the different values of *Da*

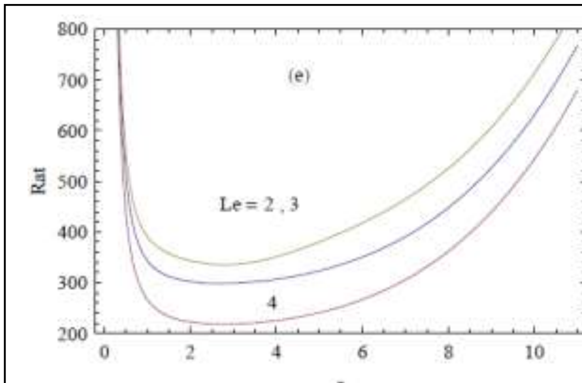

Fig. 2 : Variation of Oscillatory rayleigh number with wave number for the different values of *Le*

Also, from Figs. 2(f, g), we find that the oscillatory Rayleigh number decreases on increasing the value of the relaxation parameter λ_1 and Prandtl number $\Pr_D$, indicating that the effect of relaxation parameter and the Prandtl number is to destabilize the system. Thus, the oscillatory convection takes place at an early point. However, from Fig.2 (h), the effect of retardation parameter λ_2 is found to stabilize the system, thus opposite to that due to λ_1.

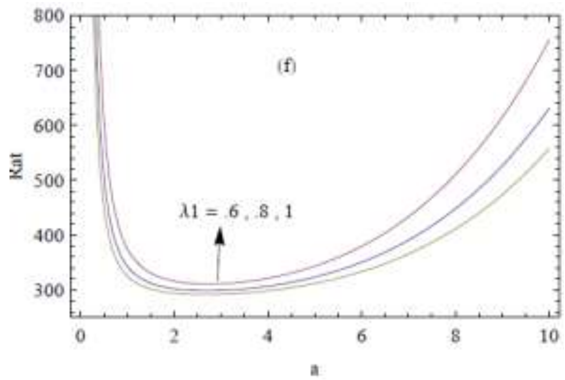

Fig. 2: Variation of Oscillatory rayleigh number with wave number for the different values of λ_1

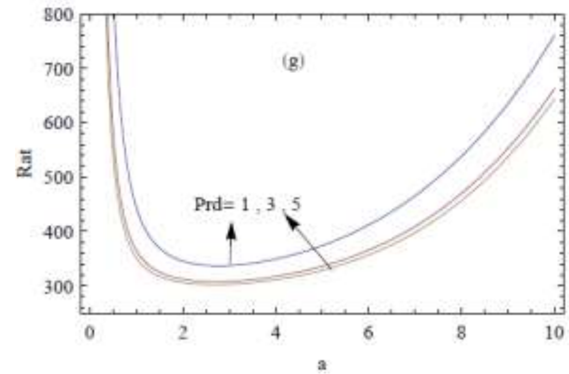

Fig. 2: Variation of Oscillatory rayleigh number with wave number for the different values of *Prd*

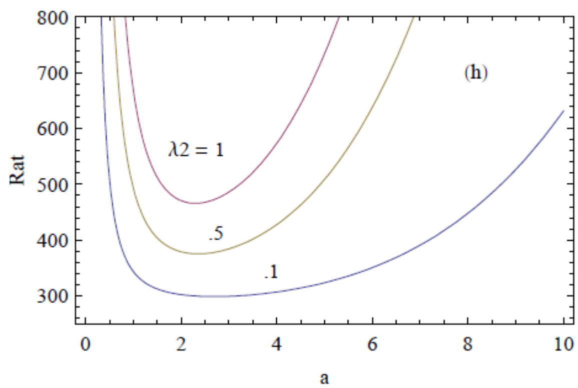

Fig. 2 : Variation of Oscillatory rayleigh number with wave number for the different values of λ_2

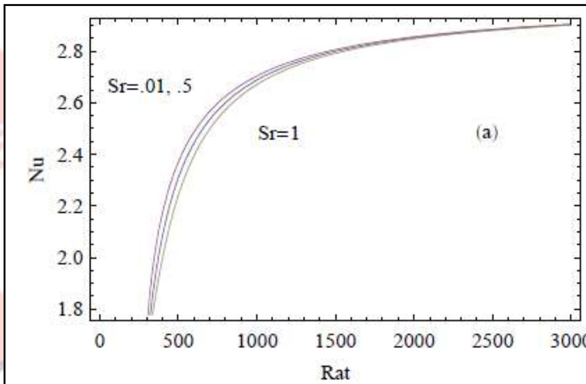

Fig. 3: Variation of Nusselt number with rayleigh number for the different values of Sr

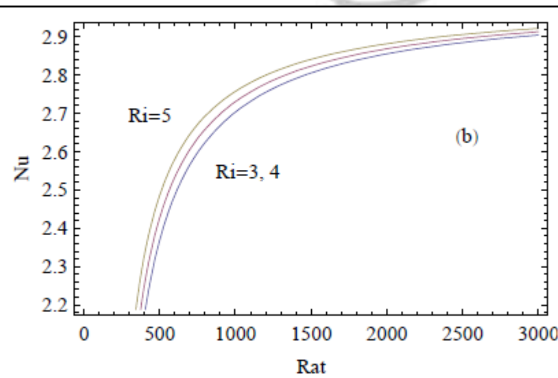

Fig. 3: Variation of Nusselt number with rayleigh number for the different values of Ri

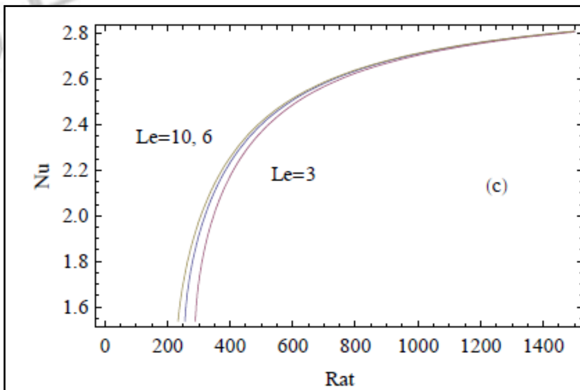

Fig. 3: Variation of Nussult number with rayleigh number for the different values of Le

Now, we fix the values of the parameters as $\mathrm{Ra}_S = 100$, $L_e = 2$, $D_a = .1$, $S_r = .05$ and $R_i = 3$ to compute the heat and mass transports across the porous medium. The results have been obtained for steady state motion, in terms of the Nussult and Sherwood numbers and depicted in the Figs.3, 4 respectively. It is found that the steady state values of N_u and S_h approach 3 as Ra_T increases. Further, it is found from Figs.3,4(a) that the value of N_u decreases, while that of S_h increases on increasing the values of Soret parameter S_r This shows that the effect of Soret parameter is to decrease the heat transport, thus stabilizing the system and increase the mass transport in the system. In Figs. 3(b, c) and 4(b, c), it is found that heat and mass transports increase on increasing R_i and L_e , thus destabilizing the system. However, Ra_S has a stabilizing effect on the system as heat and mass transport decrease on increasing the value of Ra_S [Fig.3, 4(d)].

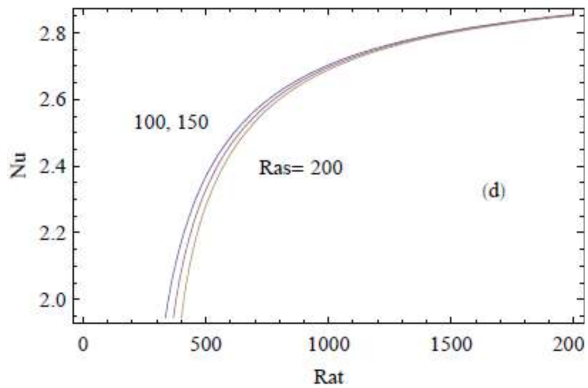

Fig. 3: Variation of Nussult number with rayleigh number for the different values of Ras

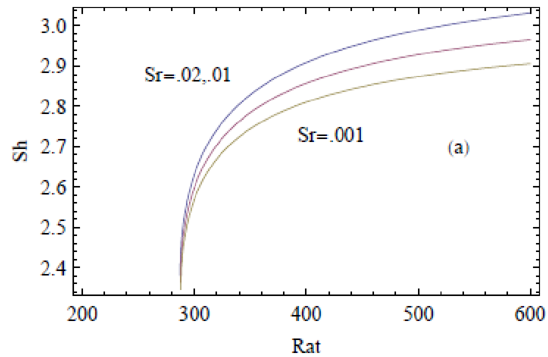

Fig. 4: Variation of Sherewood number with rayleigh number for the different values of *Sr*

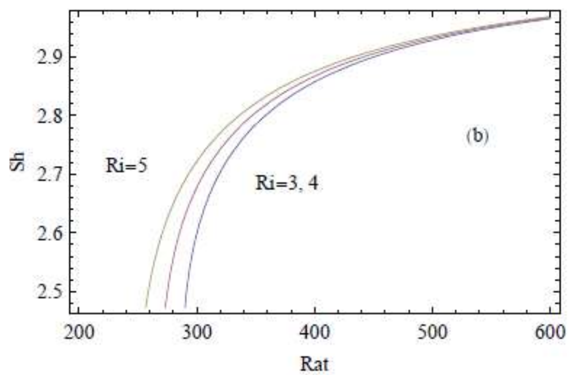

Fig. 4: Variation of Sherewood number with rayleigh number for the different values of Ri

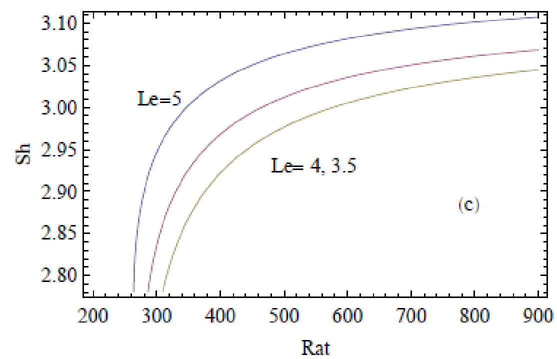

Fig. 4: Variation of Sherewood number with rayleigh number for the different values of Le

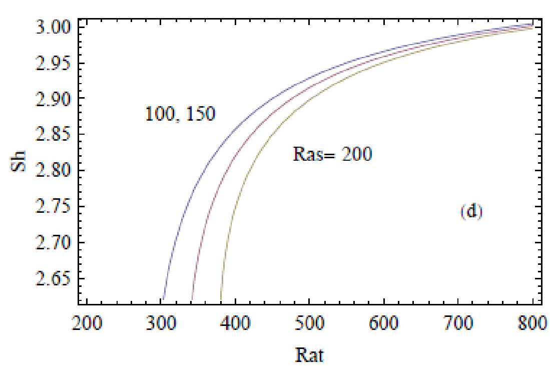

Fig. 4: Variation of Sherewood number with rayleigh number for the different values of Ras

6. CONCLUSIONS

Effects of Soret parameter and internal heat source on double diffusive convection in a binary viscoelastic fluid saturated porous layer, heated and salted from below, is investigated analytically using linear and nonlinear stability analysis. Following conclusions are drawn:

1) The Internal heat source R_i and Soret parameter S_r have destabilizing effect on the system in both stationary and oscillatory modes of convection.
2) The Darcy number D_a and Solute Rayleigh number Ra_S have stabilizing effect on the both stationary and oscillatory convection.
3) The Lewis number L_e has stabilizing effect on stationary mode of convection while destabilizing effect on oscillatory mode of convection.
4) Relaxation parameter λ_1 and Prandtl number Pr_D have destabilizing effect, while retardation parameter λ_2 has stabilizing effect on the oscillatory convection.
5) Increments in Lewis number L_e and internal Rayleigh number R_i increase, while in Ra_S decrease heat and mass transports in the system.
6) Effect of Soret parameter S_r is to decrease the heat transfer and increase the mass transfer in the system.

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