CONTRIBUTIONS TO INFERENTIAL PROCEDURES FOR SOME RELIABILITY CHARACTERISTICS

ABSTRACT

of

THESIS

SUBMITTED TO

BABASAHEB BHIMRAO AMBEDKAR UNIVERSITY
(A CENTRAL UNIVERSITY)
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BHIMRAO
AMBEDKAR
UNIVERSITY

LUCKNOW

प्रज्ञा शील करुणा

ESTABLISHED 1996

FOR THE AWARD OF DEGREE OF

Doctor of Philosophy

IN

STATISTICS

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Enrolment No. - 728/18 2021

ABSTRACT

The term reliability of an item or a system can be defined as its ability to perform a required function properly for a given period of time under the stated operating conditions. Further, the reliability function $R(t)$ can be defined as the probability of failure-free operation until time t. Thus, if the random variable (r.v.) X denotes the lifetime of an item; then $R(t) = P(X > t)$. Here, greater is the reliability, more is the life of the item.

Another measure of reliability was introduced by Birnbaum (1956), known as "stress-strength " reliability. Under stress-strength setup, let the r.v. X denotes the amount of strength of a component and the r.v. Y represents the random stress applied; then the reliability function is given by $P = P(X > Y)$. It means that the component will fail if the stress exceeds the strength and P serves as the reliability parameter when the amount of strength is more than the stress.

In this thesis, reliability inference based on complete sample case, Type II censoring, Sampling Scheme of Bartholomew (1963) and random censoring is purposed for different distributions and family of distributions.

In Chapter 1, the term reliability and the various related concepts are discussed in detail. We have also discussed different censoring schemes, some of the important concepts and results related to classical and Bayesian theory of estimation and different R packages used in our work.

In Chapter 2, we consider Chen distribution and derive Uniformly Minimum Variance Unbiased (UMVU) estimators and Maximum Likelihood (ML) estimators of the parameter λ, hazard rate $h(t)$ and the two measures of reliability, namely $R(t) = P(X > t)$, where X denotes the lifetime of an item and $P = P(X > Y)$, which represents the reliability of an item or system of random strength X subject to random stress Y, under Type II censoring scheme and the sampling scheme

of Bartholomew. We also develop interval estimates of the reliability measures. Testing procedures for the hypotheses related to different parametric functions have also been developed. A comparative study of different methods of point estimation and average confidence length has been carried out through simulation studies. The analysis of a real data set is presented for illustration purpose.

In Chpter 3, we consider Kumaraswamy-G distributions and derive UMVU estimators and ML estimators of the two measures of reliability, namely $R(t) = P(X > t)$ and $P = P(X > Y)$ under Type II censoring scheme and sampling scheme of Bartholomew (1963). We also develop interval estimates of the reliability measures. A comparative study of different methods of point estimation has been done through simulation studies. The analysis of a real data set is presented for illustration purpose.

In Chapter 4, the Classical and Bayesian estimation procedures for Kumaraswamy distribution under random censoring scheme are considered. Maximum likelihood estimates of the parameters, reliability function, failure rate function, and Mean Time to System Failure are derived. Further, Asymptotic confidence intervals for the parameters based on the observed Fisher's information matrix are obtained. Bayesian estimates for the parameters and the reliability characteristics are obtained using importance Sampling and Gibbs sampling technique. Highest posterior density credible intervals for the parameters are constructed using Markov Chain Monte Carlo (MCMC) method. Expected Time on Test of experiment based on random censoring is also discussed. To compare the efficiencies of the three estimates developed, a Monte Carlo simulation study is also carried out. Finally, the analysis of randomly censored real data set is discussed for the illustration purposes.

In Chapter 5, a generalization of positive exponential family of distributions developed by Liang (2008) is taken into consideration. Its properties are studied. Two measures of reliability are discussed, namely $R(t) = P(X > t)$ and $P = P(X > Y)$. UMVU estimators, ML estimators and Method of Moment (MM) estimators are developed for the reliability functions. The performances of three types of estimators are compared through Monte Carlo Simulation. Real life data sets are also analysed.

In Chapter 6, a weighted generalization of positive exponential family of distributions is taken into consideration and its properties are studied. Considering two measures of reliability, namely $R(t) = P(X > t)$ and $P = P(X > Y)$, their UMVU estimators, ML estimators and MM estimators are developed and the performance of the estimators are investigated using Monte Carlo Simulation. We investigate two empirical data sets to illustrate the proposed approach.

References

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