STUDY OF NEUTRINOLESS DOUBLE BETA DECAY WITHIN PHFB MODEL

SUMMARY

THESIS SUBMITTED FOR THE AWARD OF THE DEGREE

OF

Doctor of Philosophy

In

Applied Physics

By

Yash Kaur Singh

Enrolment No.: 265/13

Under the Supervision of

Dr. Ramesh Chandra

Department of Applied Physics

School for Physical Sciences

Babasaheb Bhimrao Ambedkar University, Lucknow

U.P., (India) – 226025

2018

CONTENTS

Chapter 1

Introduction

The story of neutrinos as well as weak interaction is rather checkered and quite exciting due to their enigmatic nature. In nature, the primordial neutrinos were produced during the first three minutes of the big-bang in which the universe was created 13.7 ± 0.2 Gyr ago. Among all the particles that constitute the Universe, neutrinos are the most common and the most weird particles. The first evidence for the physical existence of neutrinos came from the study of beta (β) decay. In order to explain the continuous β-spectrum, Pauli in 1930 proposed that in the β decay process, the electron is emitted together with a massless and chargeless particle of spin 1/2.

The neutrinos are very elusive and hardly interact with the matter. Raymond Davis Jr. was the first person to look into the heart of a star by capturing the neutrinos produced in the sun's core and spread out across the space. In 1967, Raymond Davis started looking for neutrinos produced by the nuclear fusion reactions in the Sun's core using a huge tank of cleaning fluid (C_2Cl_4) which was placed deep underground to shield it from cosmic rays. However, he could only detect about one third of what was predicted by the theoretical calculation of John N. Bahcall using solar models. This discrepancy between experimental finding and theoretical calculation was called the "mystery of missing neutrinos". Other underground detectors also found similar neutrino shortfall.

In 1957, Bruno Pontecorvo, inspired by the possibility of $K^0 - \overline{K}^0$ oscillation, proposed the possibility of transitions neutrino→antineutrino provided lepton number is not conserved [Pontecorvo (1957)]. This was the first idea about neutrino oscillation. Independently, Maki, Nakagawa and Sakata in 1962 proposed that neutrinos might change from one flavor into another (flavor oscillations) [Maki *et al.* (1962)]. The mass as well as nature (Dirac or Majorana) of neutrinos is not completely known till today, and the neutrinos are the most enigmatic particles in the fermionic sector of the *SM*.

1.1 Mass and nature of neutrinos

All known fundamental fermions other than neutrinos are Dirac particles due to the conservation of electric charge. On the other hand, it is natural to treat the massive neutrinos like any other fermion and therefore they should be 4-component Dirac spinors. However, an important difference between the neutrinos and other fermions is that they are neutral particles. This suggests a new theoretical possibility that the neutrinos might also be 2-component Majorana spinors.

According to the Standard Model (SM) the neutrinos should have zero mass, however, there was no valid reason for the same. The neutrino oscillations have been confirmed in atmospheric, solar, accelerator and reactor neutrinos and thereby establishing the fact that the neutrinos have mass. However, the neutrino oscillation data provide only mass squared difference and the actual neutrino mass cannot be extracted. On the other hand, the study of tritium single β decay and $\beta\beta$ decay together can provide sharpest limits on the mass and nature of the electron neutrino.

1.2 The Nuclear $\beta\beta$ decay

The nuclear $\beta\beta$ decay is a rare second order weak transition between two isobars having even Z-even N configuration and differing in nuclear charge by two units. The $\beta\beta$ decay candidates are stable against single β decay either due to energy conservation or angular momentum mismatch. There are two modes of $\beta\beta$ decay accompanied by the electron emission, one involving the emission of two neutrinos ($2\nu\beta^-\beta^-$ decay) given as

$$^A_ZX \rightarrow ^A_{Z+2}Y + 2e^- + 2\overline{\nu}_e \tag{1.1}$$

and the other neutrinoless double beta ($0\nu\beta^-\beta^-$) decay expressed as

$$ {}^{A}_{Z}X \rightarrow {}^{A}_{Z+2}Y + 2e^- + 2\overline{\nu}_e \tag{1.2} $$

The former conserves the lepton number L exactly and is an allowed process within the *SM*. In the latter one, the lepton number is violated by two units and it has the potential to throw some light on physics beyond the *SM*.

1.2.1 $2\nu\beta^-\beta^-$ decay and validity of nuclear models

The nuclear $\beta\beta$ decay is a second order process in weak interaction. The inverse half-life of $2\nu\beta^-\beta^-$ decay for the $0^+ \rightarrow J_f^+$ transition is given by

$$ \left[T_{1/2}^{2\nu}(0^+ \rightarrow J_f^+)\right]^{-1} = G_{2\nu}(J_f^+)\left|M_{2\nu}(J_f^+)\right|^2 \tag{1.3} $$

where $M_{2\nu}(J_f^+)$ is the nuclear transition matrix element (NTME) and given as

$$ M_{2\nu}(J_f^+) = \frac{1}{\sqrt{s}}\sum_N \frac{\left\langle J_f^+ \left\| \boldsymbol{\sigma}\tau^+ \right\| 1_N^+ \right\rangle \left\langle 1_N^+ \left\| \boldsymbol{\sigma}\tau^+ \right\| 0^+ \right\rangle}{[E_N - (E_I + E_F)/2]^s} \tag{1.4} $$

with $s = \{1+2\delta_{J2}\}$ and the integrated kinematical factor $G_{2\nu}(J_f^+)$ can be calculated with good accuracy [Doi *et al* (1985), (1992)].

The half-life $T_{1/2}^{2\nu}$ of $2\nu\beta^-\beta^-$ decay has been already measured for the $0^+ \rightarrow 0^+$ transition for eleven nuclei out of 35 possible candidates. Using the experimental half-life $T_{1/2}^{2\nu}$ and accurately known integrated kinematical factor $G_{2\nu}$, the values of $M_{2\nu}(0^+)$ can be extracted directly from Eq.(1.3). Hence, the validity of different nuclear models can be tested through the calculation of $M_{2\nu}(0^+)$. Further, it is observed that in all cases of $2\nu\beta^-\beta^-$ decay, the NTMEs $M_{2\nu}(0^+)$ are sufficiently quenched. The main motive of all theoretical calculations is to understand the physical mechanism responsible for the observed suppression of $M_{2\nu}(0^+)$.

Over the past few years, several nuclear models have been employed to calculate the $2\nu\beta^-\beta^-$ decay rate in 2n mechanism. They can be broadly classified into three categories, namely shell model and its variants, quasiparticle random phase approximation (QRPA) and its extensions and alternative models. The details about these models -their advantages as well as shortcomings- have been discussed excellently by Suhonen and Civitarese (1998) and Faessler and Simkovic (1998).

1.2.2 $0\nu\beta^-\beta^-$ decay and physics beyond the *SM*

The $0\nu\beta^-\beta^-$ decay mode is far more interesting since it violates the conservation of lepton number L by two units ($\Delta L = 2$) and is not allowed in the *SM*. The virtual neutrino exchange mechanism of $0\nu\ \beta^-\beta^-$ decay, the emission of a neutrino and its absorption as an antineutrino, demands that neutrino and antineutrino to be the same (Majorana particle). The violation of parity maximally in weak interactions adds an additional requirement that the emitted Majorana neutrino should reverse its helicity. Such a reversal might be caused by the neutrino mass (m_ν) and/or might occur explicitly through an admixture of right-handed current in weak interactions. The study of $\beta\beta$ decay in general and $0\nu\beta^-\beta^-$ decay in particular is a convenient tool to test the following important ramifications vis-a-vis constraints on parameters of various gauge theoretical models beyond the *SM,* namely (i) lepton number violation, (ii) mass and charge conjugation properties of the electron-neutrino and (iii) possible right handed admixtures in the weak leptonic current.

The $0\nu\beta^-\beta^-$ decay can be studied mainly in three types of models, namely Left-right symmetric models ($LRSM$), Majoron models and R-parity violating Supersymmetric models (R_p-violating $MSSM$). Further, the $0\nu\beta^-\beta^-$ decay can verify issues like compositeness, leptoquarks, sterile neutrinos and violation of weak equivalence principle.

1.3 Objective and motivation of the work

In the present work, our aim is to study the $0^+ \rightarrow 0^+$ transition of $0\nu\beta^-\beta^-$ decay of potential $\beta^-\beta^-$ emitters in the mass range $A = 90 - 150$, namely ^{94,96}Zr, ^{100}Mo, ^{110}Pd, ^{128,130}Te and ^{150}Nd isotopes within mechanisms involving light Majorana neutrino mass and right handed current to extract the effective light electron-neutrino mass $< m_\nu >$, the effective weak coupling constants $< \lambda >$ and $< \eta >$ for coupling of right-handed leptonic current with right-handed and left-handed nucleonic currents, respectively. We also aim to study the $0^+ \rightarrow 2^+$ transition of $2\nu\beta^-\beta^-$ decay of ^{94,96}Zr, ^{104}Ru, ^{100}Mo, ^{110}Pd, ^{128,130}Te and

^{150}Nd isotopes. The relevant NTMEs are calculated in projected Hartree-Fock Bogoliubov (PHFB) model in conjunction with the four parametrizations of pairing plus quadrupole-quadrupole plus hexadecapole-hexadecapole ($PPQQHH$) interaction, namely $PPQQ1$, $PPQQHH1$, $PPQQ2$ and $PPQQHH2$ parametrizations.

The present thesis is organized as follows. In Chapter 2, we have calculated the yrast spectra, reduced $B(E2{:}0^+ \rightarrow 2^+)$ transition probabilities, deformation parameters β_2 and gyromagnetic factors $g(2^+)$ of ^{94,96}Zr, ^{94,96,100}Mo, ^{100,104}Ru, ^{104,110}Pd, ^{110}Cd, ^{128,130}Te, ^{128,130}Xe, ^{150}Nd and ^{150}Sm nuclei and compared them with the available experimental data to test the "goodness of the PHFB wave functions". In Chapter 3, the same PHFB wave functions are employed to study the $2\nu\beta^-\beta^-$ decay of ^{94,96}Zr, ^{100}Mo, ^{104}Ru, ^{110}Pd, ^{128,130}Te and ^{150}Nd isotopes for the $0^+ \rightarrow 2^+$ transition. In the same chapter, we have also studied the effect of deformation on NTMEs $M_{2\nu}(2^+)$. Subsequently, we have studied the $0\nu\beta^-\beta^-$ decay of ^{94,96}Zr, ^{100}Mo, ^{110}Pd, ^{128,130}Te and ^{150}Nd nuclei for the $0^+ \rightarrow 0^+$ transition and extracted the limits on gauge parameters $< m_\nu >$, $< \lambda >$ and $< \eta >$ in Chapter 4. The effect of deformation on NTMEs of $0\nu\beta^-\beta^-$ decay has also been studied in the same chapter. Finally, the concluding remarks are given in Chapter 5.

Chapter 2

Spectroscopic properties of nuclei participating in $\beta^-\beta^-$ decay

In the present chapter we have tested the "goodness of wave functions" prior to the calculation of NTMEs for $\beta^-\beta^-$ decay by calculating the yrast spectra, reduced $B(E2{:}0^+ \to 2^+)$ transition probabilities, deformation parameters β_2 and gyromagnetic factors $g(2^+)$ and comparing them with the available experimental data. In Section 2.1, a brief outline of the PHFB model is given. The formalism to calculate the spectroscopic properties is presented in Section 2.2. We compare the calculated yrast spectra and electromagnetic properties with the available experimental data for ^{94,96}Zr, ^{94,96,100}Mo, ^{100,104}Ru, ^{104,110}Pd, ^{110}Cd, ^{128,130}Te, ^{128,130}Xe, ^{150}Nd and ^{150}Sm nuclei in Section 2.3. Finally, the conclusions are given in Section 2.4.

2.1 The PHFB model

The PHFB calculation is carried out through the following procedure.

(1) The HFB intrinsic state $|\Phi_K\rangle$ is obtained through a self-consistent solutions of the HFB Hamiltonian [Baranger (1963), Villars (1966) and Goodman (1979)].

(2) States with good angular momentum **J** are projected out from the axially symmetric HFB intrinsic state $|\Phi_K\rangle$ using the standard technique given by Onishi and Yosida (1966).

2.1.1 HFB theory

The nuclear many-body Hamiltonian for non-relativistic point nucleons neglecting three and many-nucleon forces is given by

$$H = \sum_{\alpha\beta} \langle\alpha|\ T\ |\beta\rangle a_\alpha^\dagger a_\beta + \frac{1}{4}\sum_{\alpha\beta\gamma\delta} \langle\alpha\beta|\ V\ |\gamma\delta\rangle a_\alpha^\dagger a_\beta^\dagger a_\delta a_\gamma \tag{2.1}$$

The HFB equations are obtained by equating to zero the off diagonal bilinear quasiparticle part of the transformed Hamiltonian after making a canonical transformation from a_α's to quasi-fermions q_α's.

The axially symmetric HFB intrinsic state $|\Phi_0\rangle$ can be written as

$$|\Phi_0\rangle = \prod_{im} (u_{im} + v_{im} b_{im}^\dagger b_{i\bar{m}}^\dagger)|0\rangle \tag{2.2}$$

$$= N\ exp\left(\frac{1}{2}\sum_{\alpha\beta} f_{\alpha\beta} a_{\alpha,m}^\dagger a_{\alpha,-m}^\dagger |0\rangle\right). \tag{2.3}$$

where the creation operators $b_{im}^\dagger$ and $b_{i\bar{m}}^\dagger$ are defined as

$$b_{im}^\dagger = \sum_\alpha C_{i\alpha,m} a_{\alpha,m}^\dagger \quad \text{and} \quad b_{i\bar{m}}^\dagger = \sum_\alpha (-1)^{l+j-m} C_{i\alpha,m} a_{\alpha,-m}^\dagger \tag{2.4}$$

with

$$f_{\alpha\beta} = \sum_i C_{i,j_\alpha m_\alpha} C_{i,j_\beta m_\beta} \left(\frac{v_{im_\alpha}}{u_{im_\alpha}}\right) \delta_{m_\alpha - m_\beta} \tag{2.5}$$

where N is a normalization constant.

2.1.2 Projection of angular momentum

The intrinsic state $|\Phi_K\rangle$ can be expanded in terms of states of good angular momentum **J** as

$$|\Phi_K\rangle = \sum_J a_J |\Psi_{JK}\rangle \tag{2.6}$$

Applying the rotation operator $R(\Omega)$ and multiplying by rotation matrix, D_{MK}^J, and integrating, one gets

$$|\Psi_{JM}\rangle = \frac{2J+1}{8\pi^2 a_J} \int d\Omega D_{MK}^J(\Omega)\hat{R}(\Omega)\ |\Phi_K\rangle \tag{2.7}$$

Restricting to the axially symmetric HFB intrinsic state $|\Phi_0\rangle$ with $K = 0$, one obtains

$$|\Psi_{J0}\rangle = P_{00}^J |\Phi_0\rangle = \frac{2J+1}{8\pi^2} \int D_{00}^J(\Omega) R(\Omega) |\Phi_0\rangle \, d\Omega \quad (2.8)$$

2.2 Spectroscopic properties of yrast states

In this section, we present the expressions to calculate various nuclear spectroscopic properties, namely yrast energy spectra, reduced $B(E2{:}J_i \rightarrow J_f)$ transition probabilities [Dixit *et al.* (2002)], deformation parameter β_2 [Raman *et al* (2001)] and magnetic dipole moments $\mu(J)$ [Rath *et al.* (1988)].

2.3 Results and discussions

In case of $A \leq 110$ nuclei, we treat the doubly even ^{76}Sr ($N = Z = 38$) nucleus as an inert core with the valence space spanned by $1p_{1/2}$, $2s_{1/2}$, $1d_{3/2}$, $1d_{5/2}$, $0g_{7/2}$, $0g_{9/2}$ and $0h_{11/2}$ orbits for protons and neutrons. The set of single particle energies (SPE's) used here are $\varepsilon(1p_{1/2}) = -0.8$, $\varepsilon(0g_{9/2}) = 0.0$, $\varepsilon(1d_{5/2}) = 5.4$, $\varepsilon(2s_{1/2}) = 6.4$, $\varepsilon(1d_{3/2}) = 7.9$, $\varepsilon(0g_{7/2}) = 8.4$ and $\varepsilon(0h_{11/2}) = 8.6$ (in MeV) for proton and neutrons.

For $A \geq 128$ nuclei, the doubly even ^{100}Sn ($N = Z = 50$) nucleus has been treated as an inert core with the valence space spanned by $2s_{1/2}$,$1d_{3/2}$, $1d_{5/2}$, $1f_{7/2}$, $0g_{7/2}$, $0h_{9/2}$ and $0h_{11/2}$ orbits for protons and neutrons. The set of single particle energies (SPE's) used here are (in MeV) $\varepsilon(1d_{5/2}) = 0.0$, $\varepsilon(2s_{1/2}) = 1.4$, $\varepsilon(1d_{3/2}) = 2.0$, $\varepsilon(0g_{7/2}) = 4.0$, $\varepsilon(0h_{11/2}) = 6.5$ (4.8 for ^{150}Nd and ^{150}Sm), $\varepsilon(1f_{7/2}) = 12.0$ (11.5 for ^{150}Nd and ^{150}Sm), $\varepsilon(0h_{9/2}) = 12.5$ (12.0 for ^{150}Nd and ^{150}Sm) for proton and neutrons.

We use a Hamiltonian with Pairing plus Quadrupole-Quadrupole plus Hexadecapole-Hexadecapole (*PPQQHH)* type of effective two-body interaction. The Hamiltonian is explicitly written as

$$H = H_{sp} + V(P) + \zeta_{qq} \left[V(QQ) + V(HH) \right] \quad (2.9)$$

where H_{sp} denotes the single particle Hamiltonian. The strength parameter ζ_{qq} is arbitrary and the final results are obtained by setting $\zeta_{qq} = 1$. The purpose of introducing ζ_{qq} is to study the role of deformation by varying the strength of $QQHH$ interaction.

In $PPQQ1$ [Chandra et al. (2005), Singh et al. (2007)] and $PPQQHH1$ [Chandra et al. (2009)] parametrizations the strengths of the like particle components of the QQ interaction was taken as: $\chi_{2pp} = \chi_{2nn} = -0.0105$ MeV b^{-4}, where b is oscillator parameter and an optimum yrast spectra of ^{94,96}Zr, ^{94,96,98,100}Mo, ^{98,100,104}Ru, ^{104,110}Pd, ^{110}Cd, ^{128,130}Te, ^{128,130}Xe, ^{150}Nd and ^{150}Sm nuclei was obtained by varying the strength of proton-neutron (pn) component of the QQ interaction χ_{2pn} for a given model space, SPE's, G_p, G_n and χ_{2pp}. All these input parameters were kept fixed to calculate other nuclear spectroscopic properties. In present work we use the other two parametrizations, namely $PPQQ2$ and $PPQQHH2$ parametrizations in which χ_{2pn} is taken as twice of $\chi_{2pp}(= \chi_{2nn})$ i.e. $\chi_{2pn} = 2\chi_{2pp} = 2\chi_{2nn}$ and varying the three parameters together to obtain the E_{2^+} of above nuclei in optimum agreement with the experimental values.

2.3.1 Yrast spectra

In Table 2.1, we present the theoretically calculated as well as experimentally observed yrast energies [Sakai (1984)] for the E_{2^+} to E_{6^+} states of the above mentioned nuclei. It can be seen that the agreement between the theoretically calculated and experimentally observed E_{2^+} is quite good.

2.3.2 Electromagnetic properties

In Table 2.2, the calculated as well as the experimentally observed reduced $B(E2{:}0^+ \rightarrow 2^+)$ transition probabilities, deformation parameter β_2 and the gyromagnetic factors $g(2^+)$ of the above mentioned nuclei are presented. It is noted that there is an overall agreement between the theoretically calculated and experimentally observed electromagnetic properties.

2.4 Conclusions

A reasonable agreement between the calculated and observed electromagnetic properties of nuclei suggests that the PHFB wave functions generated by fixing χ_{2pn} to reproduce E_{2^+} are quite reliable. Hence, we proceed to calculate NTMEs $M_{2\nu}(2^+)$ and half-lives $T_{1/2}^{2\nu}(2^+)$ of $2\nu\beta^-\beta^-$ decay for the $0^+ \rightarrow 2^+$ transition of ^{94,96}Zr, ^{100}Mo, ^{104}Ru, ^{110}Pd, ^{128,130}Te and ^{150}Nd isotopes using the same set of PHFB wave functions.

Table 2.1: Excitation energies E_{J^π} (MeV) of $J^\pi = 2^+$, 4^+ and 6^+ yrast states of ^{94,96}Zr, ^{94,96,100}Mo, ^{100,104}Ru, ^{104,110}Pd, ^{110}Cd, ^{128,130}Te, ^{128,130} Xe, ^{150}Nd and ^{150}Sm nuclei along with the experimental values.

Nuclei		*PPQQ*2	*PPQQHH*2	*PPQQ*1 [1]	*PPQQHH*1 [2]	Experiment [3]
^{94}Zr	E_{2^+}	0.9065	0.9178	0.9182	0.9165	0.9183
	E_{4^+}	2.0559	2.1332	1.9732	1.9657	1.4688
	E_{6^+}	3.3018	3.1486	2.7993	2.8087	
^{94}Mo	E_{2^+}	0.8706	0.8731	0.8715	0.8713	0.8711
	E_{4^+}	1.9875	1.9864	1.9685	1.9682	1.5737
	E_{6^+}	3.3439	3.3520	3.3136	3.3283	2.4234
^{96}Zr	E_{2^+}	1.7508	1.7351	1.7570	1.7541	1.7507
	E_{4^+}	3.5748	3.6415	3.5269	3.6296	3.1202
	E_{6^+}	-	6.6679	9.7261	9.3686	
^{96}Mo	E_{2^+}	0.7703	0.7631	0.7779	0.7817	0.7782
	E_{4^+}	2.0299	1.9957	2.0373	0.0220	1.6281
	E_{6^+}	3.5750	3.5029	3.5776	3.5300	2.4406
^{100}Mo	E_{2^+}	0.5369	0.5428	0.5356	0.5357	0.5355
	E_{4^+}	1.4636	1.5088	1.4719	1.4766	1.1359
	E_{6^+}	2.6500	2.7605	2.6738	2.6893	
^{100}Ru	E_{2^+}	0.5397	0.5300	0.5395	0.5402	0.5396
	E_{4^+}	1.5576	1.5605	1.5591	1.5847	1.2265
	E_{6^+}	2.8890	2.9247	2.8940	2.9629	2.0777
^{104}Ru	E_{2^+}	0.3595	0.3579	0.3580	0.3578	0.35799
	E_{4^+}	1.1382	1.1388	1.1339	1.1385	0.8885
	E_{6^+}	2.2357	2.2493	2.2280	2.2486	1.5563
^{104}Pd	E_{2^+}	0.5516	0.5477	0.5552	0.5560	0.5558
	E_{4^+}	1.5486	1.5713	1.5729	1.6138	1.32359
	E_{6^+}	2.8144	2.8956	2.8790	2.9954	2.2498

Table 2.1 continued

Nuclei		$PPQQ2$	$PPQQHH2$	$PPQQ1$ [1]	$PPQQHH1$ [2]	Experiment [3]
^{110}Pd	E_{2^+}	0.3789	0.3673	0.3737	0.3738	0.3738
	E_{4^+}	1.1783	1.1429	1.1563	1.1583	0.9208
	E_{6^+}	2.2796	2.2159	2.2254	2.2359	1.5739
^{110}Cd	E_{2^+}	0.6605	0.6477	0.6576	0.6585	0.6577
	E_{4^+}	1.8818	1.8906	1.8709	1.8921	1.5424
	E_{6^+}	3.4054	3.4849	3.3865	3.4728	2.4799
^{128}Te	E_{2^+}	0.7442	0.7435	0.7436	0.7435	0.7432
	E_{4^+}	2.0307	2.0021	2.0458	2.0130	1.4971
	E_{6^+}	3.6888	3.6267	3.7363	3.6647	1.8111
^{128}Xe	E_{2^+}	0.4390	0.4506	0.4511	0.4420	0.4429
	E_{4^+}	1.3570	1.3737	1.4263	1.3444	1.0329
	E_{6^+}	2.5915	2.6016	2.7976	2.5404	1.7370
^{130}Te	E_{2^+}	0.8407	0.8392	0.8393	0.8395	0.8395
	E_{4^+}	1.7645	1.8019	1.7741	1.8085	1.6325
	E_{6^+}	3.0718	3.1214	3.0833	3.1283	1.8145
^{130}Xe	E_{2^+}	0.5283	0.5366	0.5385	0.5384	0.5361
	E_{4^+}	1.5210	1.5234	1.5496	1.5268	1.2046
	E_{6^+}	2.7326	2.7355	2.7831	2.7400	1.9444
^{150}Nd	E_{2^+}	0.1297	0.1308	0.1307	0.1300	0.13012
	E_{4^+}	0.4288	0.4335	0.4320	0.4305	0.3815
	E_{6^+}	0.8892	0.9025	0.8960	0.8958	
^{150}Sm	E_{2^+}	0.3335	0.3319	0.3328	0.3359	0.3339
	E_{4^+}	1.0126	1.0164	1.0156	1.0290	0.7733
	E_{6^+}	1.9040	1.9253	1.9185	1.9504	1.2788

References:

[1] Chandra *et al.* (2005), Singh *et al.* (2007) [2] Chandra *et al.* (2009)
[3] Sakai (1984)

Table 2.2: Comparison of calculated and experimentally observed reduced transition probabilities $B(E2:0^+ \rightarrow 2^+)$, β_2 parameters and g factors $g(2^+)$ of ^{94,96}Zr, ^{94,96,100}Mo, ^{100,104}Ru, ^{104,110}Pd, ^{110}Cd, ^{128,130}Te, ^{128,130} Xe, ^{150}Nd and ^{150}Sm nuclei. The $B(E2)$ is calculated in units of e^{2}b^2 for effective charge $e_p = 1 + e_{eff}$ and $e_n = e_{eff}$. The $g(2^+)$ has been calculated in units of nuclear magneton for$g_l^\pi = 1.0$, $g_l^\nu = 0.0$ and $g_s^\pi = g_s^\nu = 0.60$. Here (a), (b), (c) and (d) denote $PPQQ2$, $PPQQHH2$, $PPQQ1$ and $PPQQHH1$ parametrizations, respectively.

		$B(E2{:}0^+ \rightarrow 2^+)$ (e^{2}b^2)		β_2		$g(2^+)$ (nm)	
		Theory	Experiment[1]	Theory	Experiment[1]	Theory	Experiment[2]
^{94}Zr	(a)	0.095	0.066±0.014*	0.109	0.090±0.010	0.565	-0.329±0.015[3]
	(b)	0.085	0.081±0.017	0.102		0.068	-0.26±0.06
	(c)[4]	0.081	0.056±0.014	0.100		0.121	-0.05±0.05
	(d)[5]	0.097		0.110		0.112	
^{94}Mo	(a)	0.241	0.203±0.004*	0.164	0.1509±0.0015	0.348	
	(b)	0.237	0.230±0.040	0.163		0.345	
	(c)[4]	0.232	0.270±0.035	0.161		0.343	
	(d)[5]	0.232		0.161		0.343	
^{96}Zr	(a)	0.061	0.055±0.022*	0.085	0.080±0.017	0.324	
	(b)	0.063		0.087		0.320	
	(c)[4]	0.060		0.085		0.254	
	(d)[5]	0.063		0.087		0.297	
^{96}Mo	(a)	0.338	0.271±0.005*	0.192	0.1720±0.0016	0.552	
	(b)	0.323	0.310±0.047	0.188		0.532	
	(c)[4]	0.335	0.302±0.039	0.191		0.563	
	(d)[5]	0.317		0.186		0.535	
^{100}Mo	(a)	0.512	0.516±0.010*	0.230	0.2309±0.0022	0.490	0.34±0.18
	(b)	0.527	0.511±0.009	0.233		0.519	
	(c)[4]	0.515	0.526±0.026	0.231		0.477	
	(d)[5]	0.493		0.226		0.467	
^{100}Ru	(a)	0.490	0.490±0.005*	0.215	0.2148±0.0011	0.363	0.47±0.06
	(b)	0.487	0.493±0.003	0.214		0.371	0.51±0.07
	(c)[4]	0.488	0.494±0.006	0.214		0.355	
	(d)[5]	0.484		0.214		0.363	

Table 2.2 continued

		$B(E2{:}0^+ \to 2^+)$ (e^2b^2)		β_2		$g(2^+)$ (nm)	
		Theory	Exp.[1]	Theory	Exp.[1]	Theory	Exp.[2]
^{104}Ru	(a)	0.911	0.820±0.012*	0.285	0.2707±0.0020	0.339	0.41±0.05
	(b)	0.892	0.93±0.06	0.282		0.346	
	(c)[4]	0.912	1.04±0.16	0.285		0.339	
	(d)[5]	0.890		0.282		0.345	
^{104}Pd	(a)	0.570	0.535±0.035*	0.216	0.209±0.007	0.483	0.46±0.04
	(b)	0.578	0.61±0.09	0.217		0.491	0.40±0.05
	(c)[4]	0.571	0.535±0.035	0.216		0.439	0.38±0.04
	(d)[5]	0.586		0.219		0.458	
^{110}Pd	(a)	0.621	0.870±0.040*	0.217	0.257±0.006	0.514	0.37±0.03
	(b)	0.611	0.780±0.120	0.215		0.525	0.35±0.03
	(c)[4]	0.614	0.820±0.080	0.216		0.478	
	(d)[5]	0.604		0.214		0.489	
^{110}Cd	(a)	0.582	0.450±0.020*	0.201	0.1770±0.0039	0.386	0.31±0.07
	(b)	0.619	0.504±0.040	0.208		0.390	0.285±0.055
	(c)[4]	0.548	0.467±0.019	0.196		0.358	
	(d)[5]	0.522		0.191		0.377	
^{128}Te	(a)	0.378	0.383±0.006*	0.135	0.1363±0.0011	0.516	0.35±0.04
	(b)	0.383	0.380±0.009	0.136		0.526	0.31±0.04
	(c)[4]	0.381	0.378±0.007	0.136		0.514	
	(d)[5]	0.384		0.136		0.526	
^{128}Xe	(a)	0.778	0.750±0.040*	0.187	0.1836±0.0049	0.410	0.41±0.07
	(b)	0.734	0.790±0.040	0.182		0.430	0.31±0.03
	(c)[4]	0.819	0.890±0.230	0.192		0.400	
	(d)[5]	0.729		0.181		0.439	
^{130}Te	(a)	0.288	0.295±0.007*	0.117	0.1184±0.0014	0.682	0.33±0.08
	(b)	0.304	0.290±0.011	0.120		0.668	0.29±0.06
	(c)[4]	0.289	0.260±0.050	0.117		0.679	
	(d)[5]	0.304		0.120		0.667	

Table 2.2 continued

		$B(E2{:}0^+ \rightarrow 2^+)$ (e^2b^2)		β_2		$g(2^+)$ (nm)	
		Theory	Exp.[1]	Theory	Exp.[1]	Theory	Exp.[2]
^{130}Xe	(a)	0.632	0.65±0.05*	0.167	0.169±0.007	0.461	0.38±0.07
	(b)	0.606	0.631±0.048	0.163		0.476	0.31±0.04
	(c)[4]	0.624	0.640±0.160	0.166		0.463	
	(d)[5]	0.605		0.163		0.478	
^{150}Nd	(a)	2.581	2.760±0.040*	0.276	0.2853±0.0021	0.637	0.422±0.039
	(b)	2.640	2.640±0.080	0.279		0.621	0.322±0.009
	(c)[4]	2.580	2.670±0.100	0.276		0.636	
	(d)[5]	2.632		0.279		0.622	
^{150}Sm	(a)	2.010	1.350±0.030*	0.236	0.1931±0.0021	0.578	0.385±0.027
	(b)	2.088	1.470±0.090	0.240		0.593	0.411±0.032
	(c)[4]	2.056	1.440±0.150	0.238		0.592	
	(d)[5]	2.098		0.241		0.604	

*Average $B(E2)$ values [Raman *et al.* (2001)]

References:

[1] Raman *et al.* (2001)
[2] Raghavan (1989)
[3] Speidel *et al.* (2002)
[4] Chandra *et al.* (2005), Singh *et al.* (2007)
[5] Chandra *et al.* (2009)

Chapter 3

$2\nu\beta^-\beta^-$ decay of ^{94,96}Zr, ^{100}Mo, ^{104}Ru, ^{110}Pd, ^{128,130}Te and ^{150}Nd isotopes for the $0^+ \rightarrow 2^+$ transition

The $0^+ \rightarrow 2^+$ transition of $2\nu\beta^-\beta^-$ decay has not been experimentally observed so far and only limits are available. The marked variation in the theoretically calculated NTMEs $M_{2\nu}(2^+)$ for the $0^+ \rightarrow 2^+$ transition using different nuclear models is a general feature [Suhonen and Civitarese (1998)]. Hence, the observation of the $0^+ \rightarrow 2^+$ transition of $2\nu\beta^-\beta^-$ decay can constrain the validity of different nuclear models employed in the calculation of NTMEs. Alternatively, a reliable theoretical prediction will supplement the experimental designing and planning to study this particular mode of $2\nu\beta^-\beta^-$ decay. In Section 3.1, we outline the theoretical formalism to calculate the half life $T_{1/2}^{2\nu}(2^+)$ of $2\nu\beta^-\beta^-$ decay. In Section 3.2, the results of $2\nu\beta^-\beta^-$ decay of ^{94,96}Zr, ^{100}Mo, ^{104}Ru, ^{110}Pd, ^{128,130}Te and ^{150}Nd nuclei for the $0^+ \rightarrow 2^+$ transition are given and discussed. The final conclusions are given in Section 3.3.

3.1 Theoretical formalism

The theoretical formalism to calculate the half-life for the $0^+ \rightarrow 2^+$ transition of $2\nu\beta^-\beta^-$ decay $T_{1/2}^{2\nu}(0^+ \rightarrow 2^+)$ in 2n mechanism has been given in refs. [Tomoda (1991), Haxton and Stephenson Jr. (1984), Doi *et al.* (1985)]. We briefly outline steps to derive the 2ν $\beta^-\beta^-$ decay rate formula following the notations of Doi *et al.* (1985).

3.1.1 Effective Hamiltonian for β^- decay

The effective weak interaction Hamiltonian, H_W, due to W boson exchange is given by

$$H_W = \left(\frac{G}{\sqrt{2}}\right)\left[j_{L\mu}J_L^{\mu\dagger} + \kappa j_{L\mu}J_R^{\mu\dagger} + \eta j_{R\mu}J_L^{\mu\dagger} + \lambda j_{R\mu}J_R^{\mu\dagger}\right] + h.c. \tag{3.1}$$

where the coupling constants κ, η and λ are small ($\ll 1$) parameters and $G = 1.16637 \times 10^{-5}$ GeV^{-2}.

3.1.2 Decay rate of $2\nu\beta^-\beta^-$ mode for the $0^+ \to 2^+$ transition

The half life for the $0^+ \to 2^+$ transition of $2\nu\beta^-\beta^-$ decay $T_{1/2}^{2\nu}(2^+)$ in 2n mechanism is given by

$$\left[T_{1/2}^{2\nu}(2^+)\right]^{-1} = G_{2\nu}(2^+)\left|M_{2\nu}(2^+)\right|^2 \tag{3.2}$$

where the integrated kinematical factor $G_{2\nu}(2^+)$ has been calculated with good accuracy [Pahomi *et al.* (2014)]. The model dependent nuclear transition matrix element NTME $M_{2\nu}(2^+)$ is given by

$$M_{2\nu}(2^+) = \sqrt{\frac{1}{3}}\sum_N \frac{\left\langle 2_F^+ \left\|\sigma\tau^+\right\| 1_N^+\right\rangle \left\langle 1_N^+ \left\|\sigma\tau^+\right\| 0_I^+\right\rangle}{\left[E_0 + E_N - E_I\right]^3} \tag{3.3}$$

where

$$E_0 = \frac{1}{2}(E_I - E_F) = \frac{1}{2}Q_{\beta\beta} + m_e \tag{3.4}$$

Presently, the summation over the intermediate states is performed using the summation method [Civitarese and Suhonen (1993), Chandra *et al.* (2005)].

3.1.3 NTME $M_{2\nu}(2^+)$ in the PHFB model

Using the summation method, one obtains the following expression for the NTME $M_{2\nu}(2^+)$ of $2\nu\beta^-\beta^-$ decay for the $0^+ \to 2^+$ transition.

$$\begin{aligned} M_{2\nu}(2^+) &= \sum_{\pi,\nu} \frac{\langle \Psi_{00}^{J_f=2} || [\boldsymbol{\sigma} \otimes \boldsymbol{\sigma}]^{(2)} \tau^+\tau^+ || \Psi_{00}^{J_i=0} \rangle}{[E_0 + \varepsilon(n_\pi, l_\pi, j_\pi) - \varepsilon(n_\nu, l_\nu, j_\nu)]^3} \\ &= \left[n_{(Z,N)}^{Ji=2} n_{(Z+2,N-2)}^{J_f=0} \right]^{-1/2} \int_0^\pi n_{(Z,N),(Z+2,N-2)}(\theta) \sum_\mu \begin{bmatrix} J_i & 2 & J_f \\ -\mu & \mu & 0 \end{bmatrix} d_{\mu 0}^{J_i}(\theta) \\ &\times \sum_{\alpha\beta\gamma\delta} \frac{\left\langle \alpha\beta \left| [\boldsymbol{\sigma} \otimes \boldsymbol{\sigma}]^{(2)} \tau^+\tau^+ \right| \gamma\delta \right\rangle}{[E_0 + \varepsilon_\alpha(n_\pi, l_\pi, j_\pi) - \varepsilon_\gamma(n_\nu, l_\nu, j_\nu)]^3} \\ &\times \sum_{\varepsilon\eta} \left[\left(1 + F_{Z,N}^{(\pi)}(\theta) f_{Z+2,N-2}^{(\pi)*} \right) \right]_{\varepsilon\alpha}^{-1} \left(f_{Z+2,N-2}^{(\pi)*} \right)_{\varepsilon\beta} \\ &\times \left[\left(1 + F_{Z,N}^{(\nu)}(\theta) f_{Z+2,N-2}^{(\nu)*} \right) \right]_{\gamma\eta}^{-1} \left(F_{Z,N}^{(\nu)*} \right)_{\eta\delta} sin\theta d\theta \end{aligned} \tag{3.5}$$

where

$$n^J = \int_0^\pi \left[det \left(1 + F^{(\pi)} f^{(\pi)\dagger} \right) \right]^{1/2} \left[det \left(1 + F^{(\nu)} f^{(\nu)\dagger} \right) \right]^{1/2} d_{00}^J(\theta) sin(\theta) d\theta \tag{3.6}$$

and

$$n_{(Z,N),(Z+2,N-2)}(\theta) = \left[det \left(1 + F_{Z,N}^{(\nu)} f_{Z+2,N-2}^{(\nu)\dagger} \right) \right]^{1/2} \times \left[det \left(1 + F_{Z,N}^{(\pi)} f_{Z+2,N-2}^{(\pi)\dagger} \right) \right]^{1/2} \tag{3.7}$$

The $\pi(\nu)$ represents the proton (neutron) of nuclei involved in the $2\nu\beta^-\beta^-$ decay process. The matrices $f_{Z,N}$ and $F_{Z,N}(\theta)$ are given by

$$f_{Z,N} = \sum_i C_{ij_\alpha,m_\alpha} C_{ij_\beta,m_\beta} \left(v_{im_\alpha} / u_{im_\alpha} \right) \delta_{m_\alpha,-m_\beta} \tag{3.8}$$

$$F_{Z,N}(\theta) = \sum_{m'_\alpha m'_\beta} d_{m_\alpha,m'_\alpha}^{j_\alpha}(\theta) d_{m_\beta,m'_\beta}^{j_\beta}(\theta) f_{j_\alpha m'_\alpha, j_\beta m'_\beta} \tag{3.9}$$

The results of PHFB calculations are summarized by amplitudes (u_{im}, v_{im}) and expansion coefficients $C_{ij,m}$. The required NTME $M_{2\nu}$ is calculated as follows. In the first step, matrices $f_{Z,N}$ and $F_{Z,N}(\theta)$ given by Eqs. (3.8) and (3.9) are setup for the nuclei involved in the $2\nu\beta^-\beta^-$ decay making use of 20 Gaussian quadrature points in the range (0, π). Finally, the required NTME can be calculated in a straightforward manner using the Eq. (3.5).

3.2 Results and discussions

Our aim is to study the $2\nu\beta^-\beta^-$ decay of ^{94,96}Zr, ^{100}Mo, ^{104}Ru, ^{110}Pd, ^{128,130}Te and ^{150}Nd isotopes for the $0^+ \to 2^+$ transition using $PPQQ1$, $PPQQHH1$, $PPQQ2$ and $PPQQHH2$ parametrizations. In Table 3.1, we compile all the available experimental and the theoretical results along with our calculated average $M_{2\nu}(2^+)$ using the PHFB approach with above four parametrizations and corresponding half-lives $T^{2\nu}_{1/2}(2^+)$ of above mentioned nuclei for the $0^+ \to 2^+$ transition. The phase space factors $G_{2\nu}(2^+)$ have been calculated by Pahomi *et al.* (2014) for most of the prospective $2\nu\beta^-\beta^-$ emitters. However, the $G_{2\nu}(2^+)$ of ^{94}Zr and ^{104}Ru isotopes are not available. We calculate them by adopting the prescription of Suhonen and Civitarese (1998).

3.2.1 Deformation effects

To investigate the effect of deformation for the $0^+ \to 2^+$ transition, the NTMEs $M_{2\nu}(2^+)$ are calculated by keeping the deformation for parent nuclei fixed at $\zeta_{qq} = 1$ and changing the deformation of daughter nuclei by varying ζ_{qq} in the range $0.0 - 1.5$. It can be observed that in all cases but for ^{128,130}Te, the largest NTMEs correspond to the $|\Delta\beta_2|$ close to zero. With further increase in deformation, the NTMEs decrease with increase in $|\Delta\beta_2|$.

3.3 Conclusions

A substantial variation in the calculated NTMEs $M_{2\nu}(2^+)$ in different nuclear models shows that the validity of these nuclear models can not be uniquely established in the absence of experimental results. Further work is necessary both in the experimental as well as theoretical front to judge the relative applicability, success and failure of various models used so far for the study of $\beta\beta$ decay processes.

Table 3.1: Theoretically calculated NTME $M_{2\nu}(2^+)$ and half-life $T^{2\nu}_{1/2}(2^+)$ for the $0^+ \rightarrow 2^+$ transition of ^{94,96}Zr, ^{100}Mo, ^{104}Ru, ^{110}Pd, ^{128,130}Te and ^{150}Nd nuclei along with experimental half-lives $T^{2\nu}_{1/2}(2^+)$. "*" denotes the present calculation with average NTME.

Nuclei	Theory				Experiment	
	Model	Ref.	$\lvert M_{2\nu}(2^+)\rvert$	$T^{2\nu}_{1/2}(2^+)$ (y)	$T^{2\nu}_{1/2}(2^+)$ (y)	Ref.
^{94}Zr	PHFB	*	9.884×10^{-5}	1.505×10^{37}	$>1.3\times 10^{19}$	[1]
	QRPA†	[2]	0.0170	5.088×10^{32}	$>3.4\times 10^{19}$	[3]
	QRPA‡	[2]	0.0155	6.120×10^{32}		
^{96}Zr	PHFB	*	9.978×10^{-5}	6.723×10^{25}	$>2.0\times 10^{18}$	[1]
	QRPA	[4]	(0.005-0.038)	2.677×10^{22}-	$>4.1\times 10^{19}$	[5]
				4.635×10^{20}	$>7.9\times 10^{19}$	[4]
	QRPA	[6]	1.113×10^{-4}	5.403×10^{25}		
	QRPA	[7]	0.011	5.532×10^{21}		
	RQRPA†	[8]	0.011	5.532×10^{21}		
	RQRPA‡	[8]	0.010	6.693×10^{21}		
	RQRPA	[9]		$(1.1\text{-}1.4)\times10^{21}$		
	SRPA	[10]	3.117×10^{-4}	6.889×10^{24}		
^{100}Mo	PHFB	*	1.887×10^{-5}	1.924×10^{27}	$>1.5\times 10^{20}$	[11]
	QRPA	[12]	0.033	6.290×10^{20}	$>5.0\times 10^{20}$	[13]
	QRPA	[6]	1.814×10^{-4}	2.081×10^{25}	$>2.3\times 10^{21}$	[14]
	QRPA	[7]	0.0078	1.126×10^{22}	$>1.6\times 10^{21}$	[15]
	RQRPA	[9]		$(1.0\text{-}1.1)\times10^{22}$		
	SRPA	[10]	1.482×10^{-3}	3.119×10^{23}		
	SU(3)$^+$	[16]	7.3×10^{-5}	1.285×10^{26}		
	SU(3)$^{++}$	[16]	1.53×10^{-4}	2.926×10^{25}		
	MCM	[17]		$(5.3\text{-}13)\times10^{20}$		
^{104}Ru	PHFB	*	3.634×10^{-5}	7.867×10^{32}		
	QRPA	[6]	3.736×10^{-3}	7.444×10^{28}		
	QRPA†	[2]	0.00792	1.656×10^{28}		
	QRPA‡	[2]	0.00811	1.580×10^{28}		

Table 3.1 continued

Nuclei	Theory				Experiment	
	Model	Ref.	$\|M_{2\nu}(2^+)\|$	$T_{1/2}^{2\nu}(2^+)$ (y)	$T_{1/2}^{2\nu}(2^+)$ (y)	Ref.
^{110}Pd	PHFB	*	1.192×10^{-4}	5.731×10^{27}	$>2.9\times10^{20}$	[18]
	QRPA	[6]	6.671×10^{-3}	1.830×10^{24}		
	QRPA†	[2]	0.0112	6.492×10^{23}		
	QRPA‡	[2]	0.00766	1.388×10^{24}		
	SRPA	[19]	5.621×10^{-3}	2.577×10^{24}		
^{128}Te	PHFB	*	2.068×10^{-6}	1.636×10^{35}	$>4.7\times 10^{21}$	[20]
	QRPA	[6]	3.055×10^{-4}	7.498×10^{30}		
	QRPA	[7]	0.00287	8.496×10^{28}		
	SRPA	[19]	1.022×10^{-3}	6.700×10^{29}		
^{130}Te	PHFB	*	1.341×10^{-6}	1.201×10^{30}	$>4.5\times 10^{21}$	[20]
	QRPA	[6]	8.272×10^{-5}	3.155×10^{26}	$>1.6\times 10^{21}$	[21]
	QRPA	[7]	0.00016	8.433×10^{25}		
	SRPA	[19]	4.088×10^{-3}	1.292×10^{23}		
^{150}Nd	PHFB	*	5.864×10^{-6}	8.940×10^{26}	$>8.0\times 10^{18}$	[5]
	SU(3)	[22]	5.38×10^{-5}	1.062×10^{25}	$>9.1\times 10^{19}$	[23]

†WS basis; ‡AWS basis; $^+$Spherical occupation wave functions; $^{++}$Deformed occupation wave functions

References:

[1] Norman (1987)
[2] Suhonen (2011)
[3] Dokania *et al.* (2017)
[4] Barabash *et al.* (1996)
[5] Arpesella *et al.* (1994)
[6] Raduta *et al.* (2007)
[7] Unlu (2013), (2014)
[8] Toivanen and Suhonen (1997)
[9] Schweiger *et al.* (1998)
[10] Stoica and Mihut (1996)
[11] Kudomi *et al.* (1992)
[12] Suhonen and Civitarese (1994)
[13] Blum *et al.* (1992)
[14] Barabash *et al.* (1993)
[15] Barabash *et al.* (1995)
[16] Hirsch *et al.* (1995)
[17] Suhonen (1998)
[18] Lehnert *et al.* (2016)
[19] Stoica (1994)
[20] Bellotti *et al.* (1987)
[21] Barabash *et al.* (2001)
[22] Hirsch *et al.* (1995*a*)
[23] Arpesella *et al.*(1999)

Chapter 4

$0\nu\beta^-\beta^-$ decay of ^{94,96}Zr, ^{100}Mo, ^{110}Pd, ^{128,130}Te and ^{150}Nd nuclei within mechanisms involving light neutrino mass and right-handed current

Observation of the lepton number L violating neutrinoless double beta ($0\nu\beta\beta$) decay is the most pragmatic approach to establish the Majorana nature of neutrinos. In $0\nu\beta\beta$ decay, the neutrino emitted from a nucleon is to be absorbed by another nucleon implying the existence of Majorana neutrino with finite mass. Alternatively, the occurrence of $0\nu\beta\beta$ decay is also possible with the coexistence of right-handed $V + A$ and left-handed $V - A$ currents.

In Section 4.1, we present a brief theoretical formalism to study $0\nu\beta^-\beta^-$ decay involving light Majorana neutrino mass and right handed current. The calculated NTMEs required to study $0\nu\beta^-\beta^-$ decay of ^{94,96}Zr, ^{100}Mo, ^{110}Pd, ^{128,130}Te and ^{150}Nd isotopes for the $0^+ \to 0^+$ transition and the uncertainties in NTMEs are presented in Section 4.2. Further, the extracted limits on the effective light Majorana neutrino mass $< m_\nu >$, the effective weak coupling of right-handed leptonic current with right-handed hadronic current $< \lambda >$, and the effective weak coupling of right-handed leptonic current with left-handed hadronic current $< \eta >$ from the largest available limits on half-lives of $0\nu\beta^-\beta^-$

decay $T_{1/2}^{(0\nu)}(0^+ \to 0^+)$ are presented in the same section. Finally, conclusions are given in Section 4.3.

4.1 Theoretical formalism

The general form of weak interaction Hamiltonian H_W is given by

$$H_W = \left(\frac{G}{\sqrt{2}}\right)\left[j_{L\mu}J_L^{\mu\dagger} + \kappa j_{L\mu}J_R^{\mu\dagger} + \eta j_{R\mu}J_L^{\mu\dagger} + \lambda j_{R\mu}J_R^{\mu\dagger}\right] + h.c. \tag{4.1}$$

where $j_{L,R}$ and $J_{L,R}$ are left and right handed leptonic and hadronic currents respectively and κ, η and λ are the parameters for the admixture of $V + A$ currents.

Using the standard approximations of Doi *et al.* (1985), with CP conservation, the rate for the $0^+ \to 0^+$ transition of $0\nu\beta^-\beta^-$ decay is given by

$$\begin{aligned}\left[T_{1/2}^{(0\nu)}\right]^{-1} &= \frac{|\langle m_\nu\rangle|^2}{m_e}C_{mm} + \frac{|\langle m_\nu\rangle|}{m_e}\langle\lambda\rangle C_{m\lambda} + \frac{|\langle m_\nu\rangle|}{m_e}\langle\eta\rangle C_{m\eta} \\ &\quad + \langle\lambda\rangle^2 C_{\lambda\lambda} + \langle\eta\rangle^2 C_{\eta\eta} + \langle\lambda\rangle\langle\eta\rangle C_{\lambda\eta}\end{aligned} \tag{4.2}$$

where

$$\langle m_\nu\rangle = \sum\nolimits_i' U_{ei}^2 m_i \tag{4.3}$$

$$\langle\lambda\rangle = \lambda\left|\sum\nolimits_i' \left(\frac{g_V'}{g_V}\right) U_{ei}V_{ei}\right| \tag{4.4}$$

$$\langle\eta\rangle = \eta\left|\sum\nolimits_i' U_{ei}V_{ei}\right| \tag{4.5}$$

and the nuclear structure factors C_{xy} are given by product of appropriate NTMEs and ten phase space factors. In the present calculation, we take the phase space factors calculated by Štefánik *et al.* (2015), reevaluated at $g_A = 1.2701$.

4.2 Results and discussions

We have calculated sets of twelve NTMEs, namely $M_{\omega F}$, M_{qF}, $M_{\omega GT}$, M_{qGT}, M_{qT}, M_P and M_R employing four different parametrizations of the two body effective interaction, namely $PPQQ1$, $PPQQHH1$, $PPQQ2$, $PPQQHH2$ and three different parametrizations of the SRC due to Miller-Spencer parametrization (SRC1), Argonne NN (SRC2) and CD-Bonn potentials (SRC3). The average and uncertainty of these NTMEs are presented in Table 4.1 and 4.2. Subsequently, sets of twelve nuclear structure factors C_{mm}, $C_{m\lambda}$, $C_{m\eta}$, $C_{\lambda\lambda}$, $C_{\lambda\lambda}$ are computed and their averages are given in Table 4.3. Using the average nuclear structure factors $\overline{C}_{mm}$, $\overline{C}_{\lambda\lambda}$, $\overline{C}_{\eta\eta}$, on-axis limits on the effective mass of light neutrino $\langle m_\nu \rangle$, the effective weak coupling of right-handed leptonic current with right-handed hadronic current $< \lambda >$, and the effective weak coupling of right-handed leptonic current with left-handed hadronic current $< \eta >$ are extracted from the largest observed limits on half-lives $T_{1/2}^{(0\nu)}$ of $0\nu\beta^-\beta^-$decay and presented in Table 4.4.

4.2.1 Deformation effect

To quantify the effect of deformation on M_α ($\alpha = \omega F, qF, \omega GT, qGT, qT, P$ and R), the quantity

$$D_\alpha = \frac{M_\alpha(\zeta_{qq} = 0)}{M_\alpha(\zeta_{qq} = 1)} \tag{4.6}$$

has been defined as the ratio of M_α at zero deformation ($\zeta_{qq} = 0$) and full deformation ($\zeta_{qq} = 1$) [Chaturvedi *et al.* (2008)]. In the mass range $A = 90 - 150$, the NTMEs M_α are suppressed by factor of about 2–7 (D_{qT} and D_P are suppressed by a factor of about 25 and 14, respectively) due to deformation effects.

4.3 Conclusions

The maximum uncertainty in $M_{\omega F,qF}$, $M_{\omega GT,qGT}$ and M_P is about 15% albeit the standard deviation of M_P for ^{150}Nd is about 40%. In the case of M_R, the maximum uncertainty is about 30%. The NTMEs M_{qT} are quite uncertain. Using the average nuclear structure factors $\overline{C}_{mm}$, $\overline{C}_{\lambda\lambda}$, and $\overline{C}_{\eta\eta}$, the most stringent on-axis extracted limits on $\langle m_\nu \rangle$, $\langle \lambda \rangle$, and $\langle \eta \rangle$ from the largest observed limits on half-lives $T_{1/2}^{0\nu}$ of ^{130}Te isotope are 0.33 eV, 4.57×10^{-7} and 4.72×10^{-9}, respectively.

Table 4.1: Average values for NTMEs $\overline{M}_{\alpha}$ (uncertainty $\Delta\overline{M}_{\alpha}$) ($\alpha = \omega F$, qF, ωGT, qGT) for the $0\nu\beta^{-}\beta^{-}$ decay of ^{94,96}Zr, ^{100}Mo, ^{110}Pd, ^{128,130}Te and ^{150}Nd isotopes.

Nuclei	$M_{\omega F}$	M_{qF}	$M_{\omega GT}$	M_{qGT}
^{94}Zr	0.569(0 .066)	0.627(0 .058)	-3.119(0 .312)	-3.841(0 .318)
^{96}Zr	0.443(0.050)	0.470(0.055)	-2.303(0.230)	-2.799(0.183)
^{100}Mo	1.004(0.130)	1.115(0.156)	-4.985(0.516)	-6.081(0.483)
^{110}Pd	1.102(0.150)	1.259(0.185)	-5.618(0.596)	-7.068(0.591)
^{128}Te	0.587(0.061)	0.699(0.065)	-2.849(0.335)	-3.541(0.325)
^{130}Te	0.642(0.081)	0.779(0.114)	-3.140(0.360)	-3.969(0.455)
^{150}Nd	0.456(0.071)	0.567(0.094)	-2.134(0.324)	-2.819(0.398)

Table 4.2: Average values for NTMEs $\overline{M}_{\alpha}$ (uncertainty $\Delta\overline{M}_{\alpha}$) ($\alpha = qT$, P, R) for the $0\nu\beta^{-}\beta^{-}$ decay of ^{94,96}Zr, ^{100}Mo, ^{110}Pd, ^{128,130}Te and ^{150}Nd isotopes.

Nuclei	M_{qT}	M_{P}	M_{R}
^{94}Zr	0.021(0.065)	2.382(0.207)	-2.274(0 .664)
^{96}Zr	0.050(0.024)	2.296(0.121)	-1.874(0.542)
^{100}Mo	0.050(0.067)	3.966(0.245)	-3.832(1.097)
^{110}Pd	0.065(0.073)	4.731(0.241)	-4.474(1.279)
^{128}Te	0.189(0.015)	1.091(0.156)	-2.541(0.753)
^{130}Te	0.084(0.005)	1.474(0.073)	-2.686(0.758)
^{150}Nd	0.033(0.011)	0.260(0.106)	-1.801(0.545)

Table 4.3: Average nuclear structure factors $\overline{C}_{mm}$, $\overline{C}_{m\lambda}$, $\overline{C}_{m\eta}$, $\overline{C}_{\lambda\lambda}$, and $\overline{C}_{\eta\eta}$ for the $0\nu\beta^{-}\beta^{-}$ decay of ^{96}Zr, ^{100}Mo, ^{110}Pd, ^{130}Te and ^{150}Nd isotopes.

Nuclei	C_{mm}	$C_{m\lambda}$	$C_{m\eta}$	$C_{\lambda\lambda}$	$C_{\eta\eta}$	$C_{\lambda\eta}$
^{96}Zr	4.37×10^{-13}	-2.26×10^{-13}	5.02×10^{-11}	1.51×10^{-12}	1.15×10^{-8}	-1.54×10^{-12}
^{100}Mo	1.62×10^{-12}	-8.45×10^{-13}	1.80×10^{-10}	4.76×10^{-12}	3.52×10^{-8}	-4.63×10^{-12}
^{110}Pd	6.42×10^{-13}	-2.48×10^{-13}	9.14×10^{-11}	8.21×10^{-13}	1.45×10^{-8}	-7.76×10^{-13}
^{130}Te	6.09×10^{-13}	-2.69×10^{-13}	6.97×10^{-11}	1.21×10^{-12}	1.19×10^{-8}	-1.10×10^{-12}
^{150}Nd	1.32×10^{-12}	-6.85×10^{-13}	1.05×10^{-10}	4.55×10^{-12}	1.89×10^{-8}	-4.08×10^{-12}

Table 4.4: Experimental limits on half-lives $T_{1/2}^{(0\nu)}$ and the extracted on-axis limits on the effective mass of light neutrino $\langle m_\nu \rangle$, $\langle \lambda \rangle$, and $\langle \eta \rangle$ for the $0\nu\beta^-\beta^-$ decay of ^{96}Zr, ^{100}Mo, ^{110}Pd, ^{130}Te and ^{150}Nd isotopes. Predicted half-lives $T_{1/2}^{(0\nu)}$ of $0\nu\beta^-\beta^-$decay for two sets of parameters (i) $\langle m_\nu \rangle = 50$ meV (Case I) and (ii) $\langle m_\nu \rangle = 50$ meV, $\langle \lambda \rangle = 10^{-7}$ and $\langle \eta \rangle = 10^{-9}$ (Case II).

Nuclei	$T_{1/2}^{(0\nu)}$(Ex)	Ref.	$< m_\nu >$	$< \lambda >$	$< \eta >$	$T_{1/2}^{(0\nu)}$(I)	$T_{1/2}^{(0\nu)}$(II)
^{96}Zr	9.2×10^{21}	[1]	8.09	8.53×10^{-6}	1.00×10^{-7}	2.39×10^{26}	3.00×10^{25}
^{100}Mo	1.1×10^{24}	[2]	0.38	4.39×10^{-7}	5.23×10^{-9}	6.45×10^{25}	9.34×10^{24}
^{110}Pd	6.0×10^{17}	[3]	8.27×10^{2}	1.43×10^{-3}	1.10×10^{-5}	1.63×10^{26}	2.84×10^{25}
^{130}Te	4.0×10^{24}	[4]	0.33	4.57×10^{-7}	4.72×10^{-9}	1.72×10^{26}	2.95×10^{25}
^{150}Nd	2.0×10^{22}	[5]	3.17	3.35×10^{-6}	5.36×10^{-8}	7.88×10^{25}	1.25×10^{25}

References:

[1] Argyriades *et al.* (2010) [2] Arnold *et al.* (2015) [3] Winter (1952)
[4] Alduino *et al.* (2016) [5] Arnold *et al.* (2016)

Chapter 5

Conclusions

In the present work, our aim was to study the $2\nu\beta^-\beta^-$ decay for the $0^+ \rightarrow 2^+$ transition as well as the $0\nu\beta^-\beta^-$ decay for the $0^+ \rightarrow 0^+$ transition within mechanisms involving light Majorana neutrino mass and right handed current of some potential nuclei in the mass range $A = 90 - 150$. The required NTMEs for both the modes of $\beta^-\beta^-$ decay have been calculated by employing the PHFB model in conjunction with four different parametrizations of pairing plus multipolar effective two body interaction, namely $PPQQ1$, $PPQQHH1$, $PPQQ2$, $PPQQHH2$ and three different parametrizations of the SRC due to Miller-Spencer parametrization, Argonne NN and CD-Bonn potentials. Prior to the calculation of NTMEs of $0\nu\beta^-\beta^-$ decay, the "goodness of the wave functions" has been checked by calculating nuclear spectroscopic properties, namely the yrast spectra, reduced $B(E2{:}0^+ \rightarrow 2^+)$ transition probabilities, β_2 parameters and g-factors $g(2^+)$ of the nuclei under going $\beta^-\beta^-$ decay and comparing them with the experimental values. After obtaining an overall agreement between the calculated and experimentally observed values, the NTMEs $M_{2\nu}(2^+)$ and half-lives $T_{1/2}^{2\nu}(2^+)$ of $2\nu\beta^-\beta^-$ decay for the $0^+ \rightarrow 2^+$ transition of the ^{94,96}Zr, ^{100}Mo, ^{104}Ru, ^{110}Pd, ^{128,130}Te and ^{150}Nd nuclei have been calculated and the results have been compared with the other existing theoretical calculations as well as experimental results.

Finally, the NTMEs of $0\nu\beta^-\beta^-$ decay of ^{94,96}Zr, ^{100}Mo, ^{110}Pd, ^{128,130}Te and ^{150}Nd for the $0^+ \rightarrow 0^+$ transition within mechanisms involving the light Majorana neutrino, and right handed $V+A$ current have been calculated using the same PHFB wave functions. We have extracted limits on the effective light Majorana neutrino mass $< m_\nu >$, the effective weak coupling of right-handed leptonic current with right-handed hadronic current $< \lambda >$ and the effective weak coupling of right-handed leptonic current with left-handed hadronic current $< \eta >$ from the observed limit on half-life $T^{0\nu}_{1/2}$.

Uncertainty in NTMEs

In order to extract lepton number violating gauge-theoretical parameters accurately, reliable calculation of NTMEs is required. On the one hand, there is a large variation in the calculated NTMEs in different nuclear models. Even in the same type of generic model, the calculated NTMEs have noticeable uncertainty. Further, there is no objective way to judge the correctness of these theoretical calculations. The uncertainty in NTMEs is mainly due to different approaches employed in the existing theoretical calculations [Engel (2015)]. The two basic ingredients of any nuclear model are the model space and the effective two-body interaction. In the absence of any general guiding principles to fix these two ingredients, there is no clear cut prescription in practice for this purpose. Hence, the nuclear models employed to calculate the NTMEs use different model spaces and different effective interactions. In addition, the basic approach to fix the single particle energies and parameters of the two-body interactions are quite different even for the same model space. Moreover, Šimkovic *et al.* (2009) have shown that the choice of g_A and SRC is also source of uncertainty in the NTMEs.

Improvements in the PHFB model

The trend of results in the present version of PHFB model suggests that it is necessary to incorporate a number of improvements in our study of $\beta^-\beta^-$ decay in general and $2\nu\beta^-\beta^-$ decay in particular. The main improvements are the inclusion of proton-neutron pairing in the Hamiltonian, use of complex Bogoliubov transformation in the HFB formalism, number projection, band mixing and use of a large model space in conjunction with appropriate two-body effective interaction.

Bibliography

[1] C. Alduino *et al.,* Phys. Rev. C 93, 045503 (2016).

[2] J. Argyriades *et al.* Nucl. Phys. A 847, 168,(2010).

[3] R. Arnold *et al.* Phys. Rev. D 92, 072011 (2015).

[4] R. Arnold *et al.* Phys. Rev. D 94, 072003 (2016).

[5] C. Arpesella, A.S. Barabash, E. Bellotti, C. Brofferio, E. Fiorini, P.P. Sverzellati, and V.I. Umatov, Europhys. Lett. 27, 29 (1994).

[6] C. Arpesella, E. Bellotti, N. Ferrari, L. Zanotti, Nucl. Phys. B (Proc. Suppl.) 70, 249 (1999).

[7] A. S. Barabash, F. T. Avignoue III, C. K. Guerard, R. L. Brodzinski, H. S. Miley, J. H. Reeves, and V. I. Umatov, Proc. 3-rd Int. Syrup. on Weak and Electromagn. Interactions in Nuclei WEIN-92. Dubna, Russia, June 16-22, 1992 p.582 (World Sci. Publ. Co., 1993).

[8] A. S. Barabash *et al.,* Z. Phys. A 352, 231 (1995*a*).

[9] A. S. Barabash, R. Gurriaran, F. Hubert, Ph. Hubert, J. L. Reyss, J. Suhonen, V. I. Umatov, J. Phys. G 22, 487 (1996).

[10] A. S. Barabash, F. Hubert, Ph. Hubert, V.I. Umatov, Eur. Phys. J. A 11, 143 (2001).

[11] M. Baranger, Phys. Rev. 120, 957 (1960); Cargese lectures in theoretical physics, 1962 (W.A. Benjamin,New York, 1963).

[12] E. Bellotti, C. Cattadori, O. Cremonesi, E. Fiorini, C. Liguori, A. Pullia, P. P. Sverzellati and L. Zanotti, Euro. Phys. Lett. 3, 889 (1987).

[13] A. Bohr and B. R. Mottelson, Nuclear Structure Vol. II p. 356 (New York, Benjamin, 1975).

[14] D. Blum *et al*, Phys. Lett. B 275, 506 (1992).

[15] R. Chandra, J. Singh, P. K. Rath, P. K. Raina and J. G. Hirsch, Euro. Phys. J. A 23, 223 (2005).

[16] R. Chandra, K. Chaturvedi, P.K. Rath, P.K. Raina, J.G. Hirsch, Europhys. Lett. 86, 32001 (2009).

[17] K. Chaturvedi, R. Chandra, P.K. Rath, P.K. Raina, and J. G. Hirsch, Phys. Rev. C 78, 054302 (2008).

[18] O. Civitarese and J. Suhonen, Phys. Rev. C 47, 2410 (1993).

[19] B. M. Dixit, P. K. Rath and P. K. Raina, Phys. Rev. C 65, 034311 (2002); ibid 67, 059901 (E) (2003).

[20] M. Doi, T. Kotoni and E. Takasugi, Prog. Theor. Phys. Suppl. 83, 1 (1985).

[21] N. Dokania *et al.,* Eur. Phys. J. A 53: 74 (2017).

[22] A. Faessler and F. Simkovic, J. Phys. G 24, 2139 (1998).

[23] A. L. Goodman, "Advances in Nuclear Physics", Ed. J. W. Negele and E. Voget (Plenum, New York, 1979).

[24] W. C. Haxton and G. J. Stephenson Jr., Prog. Part. Nucl. Phys. 12, 409 (1984).

[25] J. G. Hirsch, O. Castanos, P. O. Hess and O. Civitarese, Phys. Rev. C 51, 2252 (1995).

[26] J. G. Hirsch, O. Castanos, P. O. Hess and O. Civitarese, Nucl. Phys. A 589, 445 (1995a).

[27] N. Kudomi, H. Ejiri, K. Nagata, K. Okada, T. Shibata, T. Shima, and J. Tanaka, Phys. Rev. C 461, 2132 (1992).

[28] B. Lehnert *et al*, J. Phys G: Nucl. Part. Phys. 43, 115201 (2016).

[29] Z. Maki, M. Nakagawa and S. Sakata, Prog. Theor. Phys. 28, 870 (1962).

[30] E. B. Norman, D. M. Meekhof, Phys. Lett. B 195, 126 (1987).

[31] N. Onishi and S. Yoshida, Nucl. Phys. A260, 226 (1966), Nucl. Phys. 80, 367 (1966).

[32] T. E. Pahomi, A. Neacsu, M. Mirea, S. Stoica, Romanian Reports in Physics, 66, 370 (2014)

[33] B. Pontecorvo, Zh. Eksp. Teor. Fiz. 33, 549 (1957).

[34] A. A. Raduta, C. M. Raduta, Phys. Lett. B 647, 265 (2007).

[35] P. Raghavan, Atomic Data and Nuclear Data Table, 42, 189 (1989).

[36] S. Raman, C. W. Nestor Jr. and P. Tikkanen, Atomic Data and Nuclear Data Table, 78, 1 (2001).

[37] P. K. Rath and S. K. Sharma, Phys. Rev. C 38, 2928 (1988).

[38] M. Sakai, Atomic Data and Nuclear Data Tables 31, 400 (1984).

[39] J. Schweiger, F. Simkovic, A. Faessler, W.A. Kominski, Phys. Rev. C 57, 1738 (1998).

[40] S. Singh, R. Chandra, P. K. Rath, P. K. Raina, J. G. Hirsch, Eur. Phys. J. A 33, 375 (2007).

[41] D. Štefánik, R. Dvornický, F. Šimkovic, and P. Vogel, Phys. Rev. C 92, 055502 (2015).

[42] S. Stoica, Phys. Rev. C 49, 2240 (1994).

[43] S. Stoica and I. Mihut, Nucl. Phys. A 602, 197 (1996); S. Stoica, Phys. Lett. B 350, 152 (1995).

[44] J. Suhonen, O. Civitarese, Phys. Rec. C 49, 3055 (1994).

[45] J. Suhonen, O. Civitarese, Phys. Rep. 300, 123 (1998).

[46] J. Suhonen, Nucl. Phys. A 864, 63 (2011).

[47] J. Toivanen and J. Suhonen, Phys. Rev. C 55, 2314 (1997).

[48] T. Tomoda, Rep. Prog. Phys. 54, 53 (1991).

[49] S. Unlu , Chin. Phy. Lett. 31, 042101(2014); Phys. Scr. 87, 045202 (2013).

[50] F. Villars, Proceedings of International School of Physics, "Enrico Fermi" Course 36 edited by C. Bloch Academic, New York (1966).

[51] R. G. Winter, Phys. Rev. 85, 687 (1952).