04/12/15

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15-17

BABASAHEB BHIMRAO AMBEDKAR UNIVERSITY
Department of Applied Statistics
M.Sc. Semester I Examination (Session 2015-17)
Probability and Distribution Theory (MAS 102)

Time: 3 Hours

Max. Marks: 70

All the notation and terminologies used in the paper are the same as that discussed in the lectures.

Section A (30 marks)

Attempt any six parts of the following question. Each part of the question is of 5 marks.

1. (i) State and prove Holder's inequality.

   (ii) State and prove Markov inequality. With the help of Markov inequality deduce Chebyshev's inequality.

   (iii) An electronic device has n transistors and the device will flash red light if one or more transistors fail. When red light flashes the device is examined to detect the failed transistor(s). During a specified period of operation, each of the transistors independently works with probability p of failing. Obtain the probability of exactly r failures, $1 \leq r \leq n$.

   (iv) State and prove Jensen's inequality.

   (v) Discuss, in short, the noncentral t distribution.

   (vi) Discuss the discrete uniform distribution with an example.

   (vii) Discuss the negative binomial distribution with an example.

Section B (40 marks)

Attempt any four questions. Each question is of 10 marks.

2. Let the joint pdf of random variables X and Y be

$$f(x,y) = \begin{cases} \frac{1}{\alpha^2}\exp\left(-(x+y)/\alpha\right), & \text{if } x>0, y>0, \alpha>0, \\ 0, & \text{otherwise.} \end{cases}$$

   Find the distribution of $\frac{1}{2}(X-Y)$. Also find the marginal distribution of X.

3. (a) A two dimensional random variable (X,Y) have bivariate distribution given by

$$P(X=x, Y=y) = \frac{x^2+y}{32}, \quad \text{for } x = 0,1,2,3, \text{ and } y = 0,1.$$

   Find the marginal distributions of X and Y.

   (b) A two dimensional random variable (X,Y) have joint probability mass function

$$P(X=x, Y=y) = \frac{1}{27}(2x+y),$$

   where x and y can assume only the integer values 0, 1 and 2. Find the conditional distribution of Y given $X=x$.

4. Let $X_1,\ldots,X_n$ be independent random variables having exponential distributions with respective parameters $\theta_1,\ldots,\theta_n$. Derive the moment generating function of X_i, $i=1,\ldots,n$. Define $Z=\min\{X_1,\ldots,X_n\}$. Prove that Z has exponential distribution with parameter $\sum_{i=1}^{n}\theta_i$.

5. Let the random variable X follows noncentral chi-square distribution with pdf

$$f_X(x) = e^{-\nu^2/2} \sum_{s=0}^{\infty} \frac{(\nu^2/2)^s}{s!} \frac{e^{-x/2} x^{(k/2)+s-1}}{\Gamma(\frac{k}{2}+s) 2^{(k/2)+s}}, \quad x > 0.$$

Show that the moment generating function of X is given by

$$M_X(t) = (1-2t)^{-k/2} e^{\nu^2 t/(1-2t)}.$$

6. Using the moment generating function, or otherwise, derive the mean and variance of the random variable X defined in Question No. 5.

7. What do you mean by compound distribution? Let the variate X be $P(M)$, where the parameter M itself is $G(\lambda, r)$ with r as an integer. Find the unconditional distribution of X.

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